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DESIGN OF MACHINE ELEMENTS

(Theory & Problems)

B.D. Shiwalkar

Denett

Design of Machine Elements

(Theory & Problems)

For Engineering Students & Design Engineers in Field

In S.I. Units

B.D. Shiwalkar
B.E. (Hons.), M.S. (USA), F.I.E., M.I.S.T.E.,
Ex. Professor of Mechanical Engineering Dept.,
Visvesvaraya National Institute of Technology,
NAGPUR

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PREFACE

The author has great pleasure in presenting to the readers the second volume of the set of two volumes on design of machine elements which the author wanted to publish. The two volumes are :

1. Design Data for Machine Elements
2. Design of Machine Elements (Theory & Problems)

Out of this the first volume has been published long back and is being extensively used by the student, teachers and practicing engineers. However, there were some difficulties in the use of the data book because of the absence of illustrative solved problems with the help of the data book. Hence the second volume whose publishing was long overdue is finally being published and the author hopes that the data book will now be used more effectively and willingly by everybody concerned.

The book essentially consists of three parts a) Theory b) Solved problems and c) Unsolved problems for various commonly used machine elements. In the solved problems, the reference to the tables & figures of data book is given as T for Tables 7 F for figures. For example : T-II-7 means table no. 7 from chapter II. Similarly F-9-2 means figure No. 2 from chapter IX of data book. The tables and figures of the second volume are referred as table No. And figure No. Both the volumes are in S. I. units. Every effort has been made to reduce the errors. However, the readers are most welcome to inform the errors to the author.

B. D. Shiwalkar

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It is impossible to acknowledge are contributors to this book. The ideas and information have been drawn from numerous books, articles, persons in the design field and from personal experience of the author who has taught the subject for almost thirty five years. However, the book is influenced to a great extent by the following books.

1. V. L. Maleev & J. B. Hartman --- Machine Design.
2. J. E. Shigley --- Mechanical Engg. Design
3. R.C. Juvinall --- Fundamentals of Machine Component Design.
4. Hall, Holowenko, Laughlin --- Theory & Problems of Machine Design.
5. R. M. Phelan --- Fundamentals of Mechanical Design.
6. P. H. Black --- Machine Design

The author also wishes to acknowledge the valuable suggestions received from colleagues and friends. Finally the author wishes to thank Shri Prafulla Vaidya and Shri Bamnote who helped the author in the preparation of the manuscript and proof correction. The publishers deserve mention for publishing the book at a fast pace.

B. D. Shiwalkar

Dedicated to

MY PARENTS

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NOTATION

Note : Unless otherwise stated, the notations indicate following.

SYMBOL	MEANING	UNITS
A.a	Area	mm^2
B,b	breadth	mm
C,c	Distance from neutral axis to extreme fiber	mm
D,d	Diameter	mm
E	Modulus of Elasticity	N/mm^2, MPa or GPa
e	Eccentricity	mm
F	Force	N
G	Modulus of rigidity	N/mm^2 , MPa or GPa
g	Acceleration due to gravity	m/sec^2
H,h	Height	mm
I	Moment of inertia	mm^4
J	Polar moment of inertia	mm^4
K	Radius of gyration	mm
k	Thermal conductivity	KJ/m hr^{0}C
L,l	Length	mm
M	Bending moment	N-m, N-mm
m	Module	mm
N	Revolutions per minute in R.P.M.	
P,p	Pressure	N/mm^2, MPa
	Power	Watts
Q	Oil flow	mm^3/sec
Qs	Side flow of oil	mm^3/sec
R,r	Radius	mm
S	Strength	N/mm^2, MPa
T	Torque	N-m, N-mm
t	Thickness	mm
V	Velocity	m/min, m/sec
W	Weight	N
w	Width	mm
Y	Lewis from factor for gears ..	
y	Distance fromeneutral axis	mm
Z	Section modulus	mm^3
Zk	Kinematic viscosity	m^2/s
α	Torsional strain ..	
δ	Deflection	mm
ε	Linear strain	
θ	Angle Radian	
μ	Coefficient of friction	
μ	Poissions ratio	
ρ	Density	kg/m^3
η	Efficiency	
σ	Induced Normal Stress	N/mm^2, MPa
τ	Induced Sheare stress	N/mm^2, MPa
	SUFFIX	
b	Bending	
c	Compression, Contact	
d	Design value	
i	Inside	
o	Outside	
r	Radial	
s	Shear, Shaft	
t	Thensile	
u	Ultimate	
x	In X direction	
y	In Y direction, yield	
e	Endurance	

Chapter 1

STRESS ANALYSIS

1.1 Introduction :

This book is concerned with the design and analysis of machine components. Since these are load carrying members, an analysis of loads is of fundamental importance. A sophisticated stress or deflection analysis is of little value if it is based on incorrect loads. A component cannot perform satisfactorily unless it's design is based on realistic operating loads. The determination of appropriate loads is often a difficult and challenging initial step in the design of a machine. However the discussion of methods used to determine operating loads is beyond the scope of this book.

1.2 Free Body Diagram :

The operating external loads act directly on some components of a machine which are then transmitted to other components. With the use of equilibrium equations it is possible to show the forces or loads acting on any one component when isolated from other components. These are called *free body diagrams.* The loads will be generally of three types as shown in the Fig. (1.1)

(a) Forces normal to the cross section called axial loads.

(b) Forces parallel to the cross section & perpendicular to the axis called transverse loads.

(c) Forces parallel to the cross section but having an eccentricity with respect to the axis.

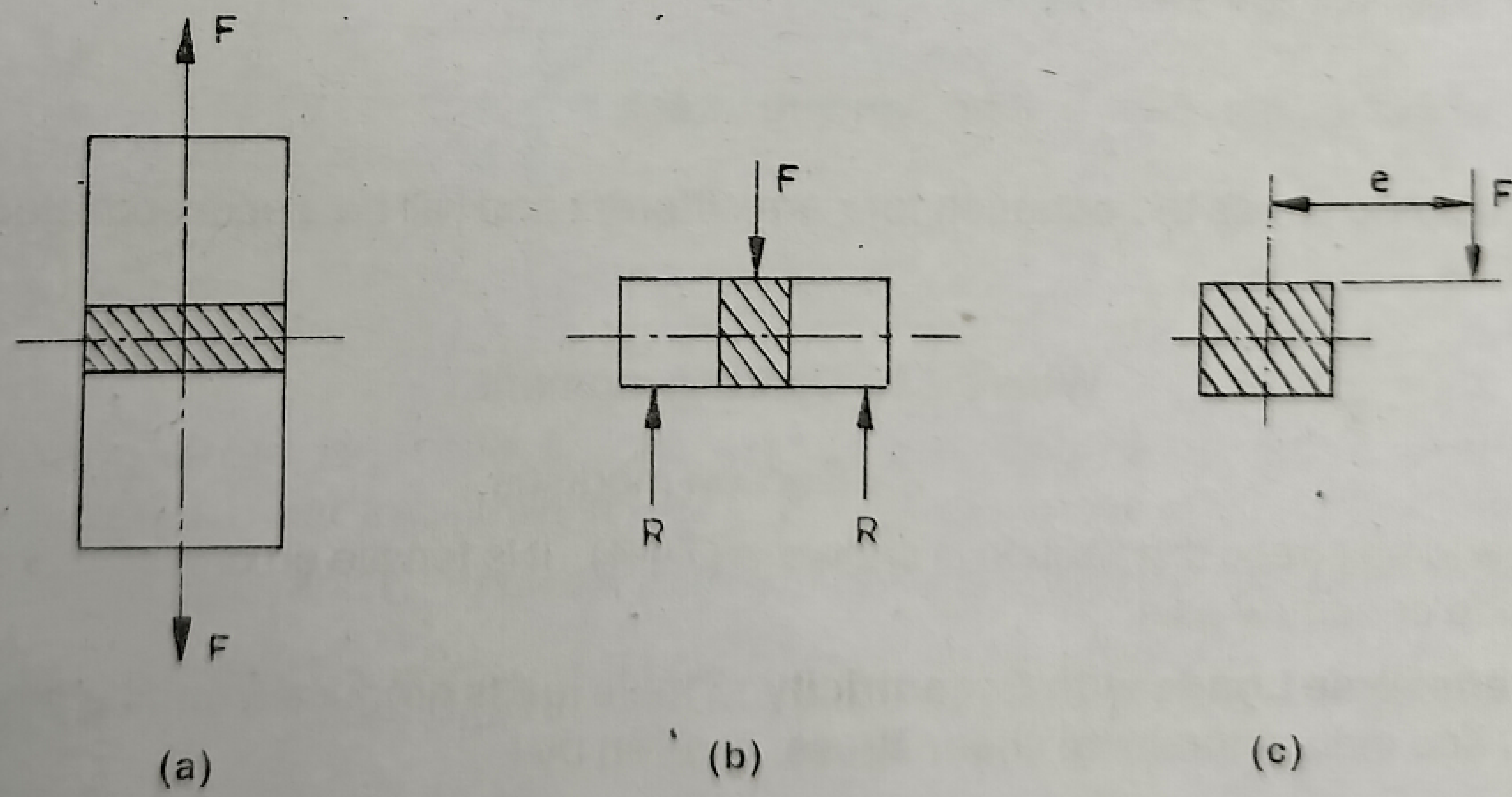


Fig. 1.1 Type of loads

Once the external loads applied to a member have been determined the next item of interest is often the resulting stresses. The stresses induced within the member as a whole are called *body stresses* as distinguished from *surface* or *contact stresses* in localized regions where external loads are applied. The equations for stresses are generally given for body stresses. Further the loads are generally of three types :

1. Static loads which do not change in magnitude or direction over time.
2. Fatigue or variable loads which change in magnitude and/or direction over time.
3. Impact loads when the loads are applied with shock.

The methods of design and formulae for stress and deflection analysis of each type are different and will be discussed seperately. This book as well as the data book follows the convention of reserving the capital letters S with subscripts, for the strength of materials and using Greek letters σ and τ for induced normal and shear stress respectively.

For example :

S_{ut} -- Ultimate tensile strength
S_{ys} -- Yield strength in shear
S_{ds} -- Design or allowable stress in shear
σ_t -- Induced tensile stress

1.3 Simple Stress in Static Loading :

The three types of loads shown in Fig. 1.1 will induce the following stresses. (T-I-4)

(a) Axial Loads : They will produce normal stresses which may be tensile or compressive and are given by :

σ_t or $\sigma_c = \frac{F}{A}$ Where F = Force
A = Area of cross section.

For compressive loads when the length of the member is large compared to its radius of gyration, there is a possibility of its buckling. These slender members are then called *coloumns* and they have to be designed so that buckling does not take place.

(b) Transverse Loads : The effect of transverse loads depends on the distance between the support reactions and the load. If the distance is negligible it induces pure shear stress; if the distance is substantial it induces a bending stress in addition to the shear stress. These members are called *beams*. These stresses are given by :

shear stress $\tau = \frac{F}{A}$; if no bending stress.

With bending stress the equation for τ are different and will be discussed later. The bending stress is given by :

$\sigma_b = \frac{M}{Z}$ Where M = Bending moment.
Z = Section modulus.

The bending stress distribution is shown in (T-I-4). It is tensile on one side and compressive on the other side of neutral axis.

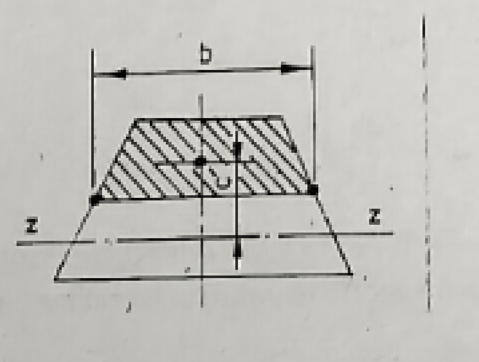
(c) Transverse Loads with Eccentricity : These loads produce a turning moment or torque on the section and induce torsional shear stress, τ, given by :

$\tau = \frac{T}{Z_p}$ Where T = torque,
Z_p = Polar section modulus.

The distribution of shear stress in shown in (T-I-4). In addition they produce direct shear stress as in (b).

1.4 Shear Stresses in Beams :

Most beams have both shear forces and bending moment present. It is only occassionally that we encounter beams subjected to pure bending i.e. beams having zero shear stress. Although the average shear stress in beams in equal to $\frac{F}{A}$, the maximum shear stress is substantially higher. The formulae for these stresses as per (T-I-5) is



$\tau = \frac{F A' C'}{I_z b}$ Where, A' = Cross hatched area.
C' = Distance of C.G. of A' from N.A.
I_z = Moment of inertia of whole section about zz, the neutral axis.

Fig. 1.2

General expressions for four commonly used sections are given in T-I-5 with the maximum shear stress.

1.5 Bending of Curved Beams :

For a straight beam made of homogeneous material with the same modulus of elasticity in tension and compression and stressed within the elastic limits, the stress at any section at a distance y from neutral axis is given by

$\sigma_b = \frac{My}{I}$ Where, I = Moment of inertia of the section about neutral axis.
M = Bending moment acting on the section.

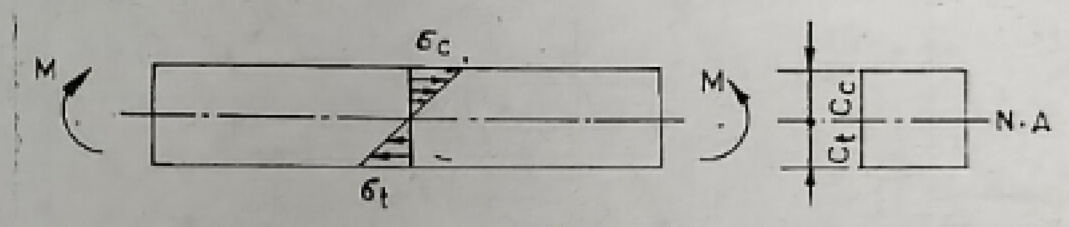


Fig. 1.3

The bending stress is tensile on one side of N.A. and compressive on the other side. The stress distribution is linear as shown in Fig. (1.3). the distances of the farthest sections on the two sides are denoted by C_t and C_c to give the maximum tensile and compressive stresses.

$\sigma_t\,(\text{max.}) = \frac{M\,C_t}{I}$ $\sigma_c\,(\text{max.}) = \frac{M\,C_c}{I}$

The quantity $\frac{I}{C}$ is called section modulus and is denoted by Z. For symmetrical section, $C_t = C_c$. Hence the tensile and compressive stresses are equal and given by

$\sigma_t = \sigma_c = \frac{M}{Z}$

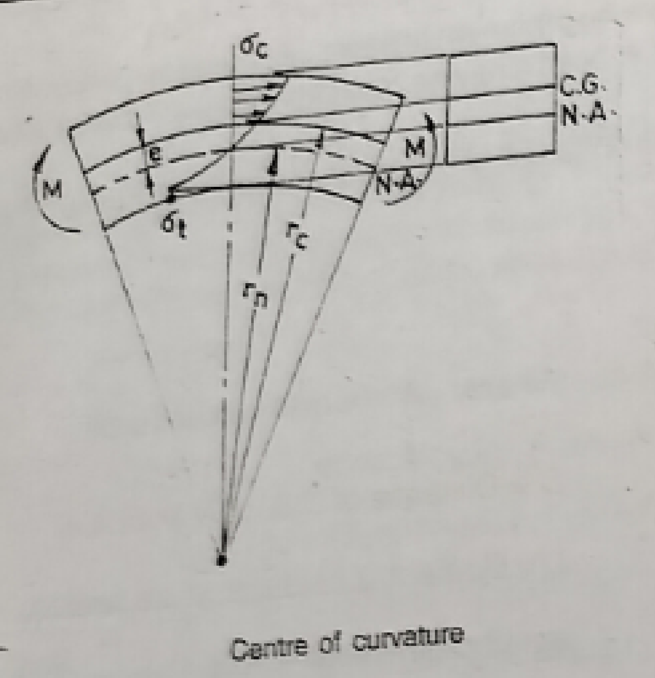


Fig. 1.4

For curved beams the stress distribution does not remain linear but becomes non-linear as shown in Fig. (1.4).

The stresses at the inner and outer fibres are given by (T-I-25).

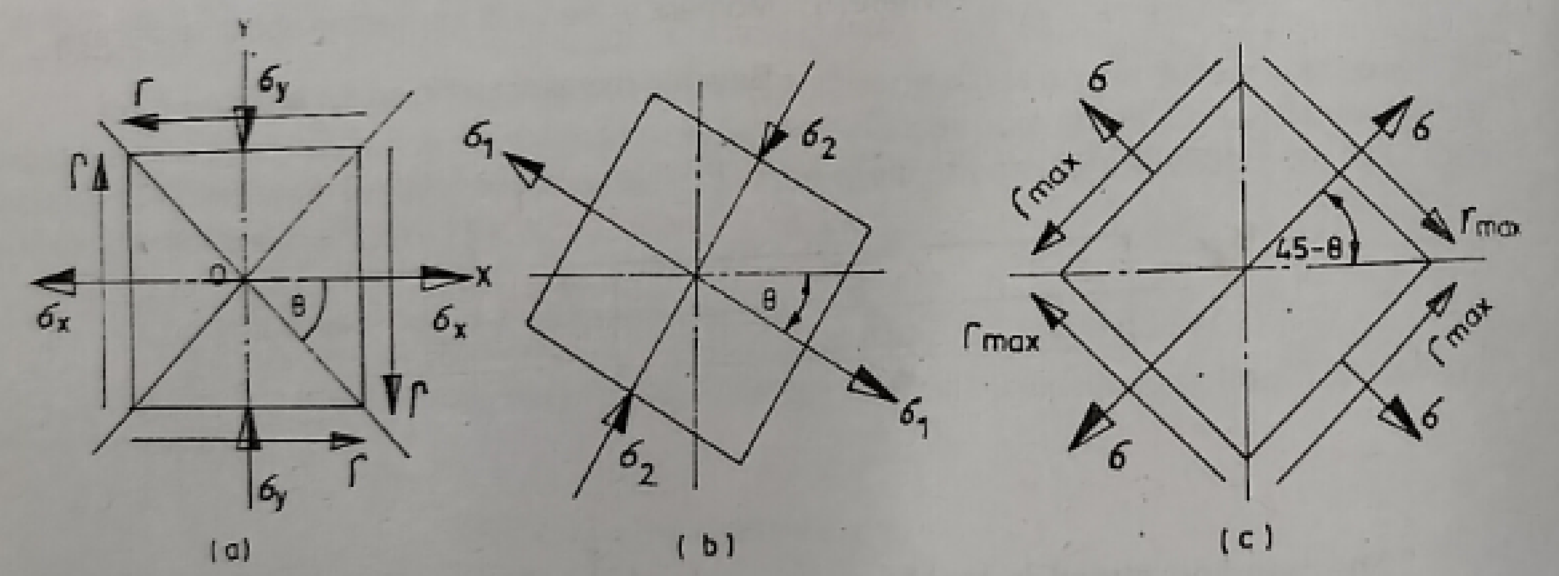
$$\sigma_i = \frac{M\,C_i}{Ae\,r_i};$$

$$\sigma_o = \frac{-M\,C_o}{Ae\,r_o}$$

The value of r_n for commonly used sections is given in T-I-25 from which the value of e can be calculated.

1.6 Combined Stresses & Mohr's Circle :

Most of the machine components are not loaded with only one type of load but they are subjected to a combination of axial loads and transverse loads, which induce normal stresses and shear stresses. Most commonly occuring case is normal stresses in two perpendicular directions plus a shear stress as shown in Fig. (1.5). The resultant effect of these stresses in any other plane making an angle ϕ with X axis can be found out.



(a) (b) (c)

Fig. 1.5

It can be shown that for a particular value of ϕ the shear stress becomes zero and only normal stress is acting. This plane and a plane perpendicular to this plane are called *principle planes* and these normal stresses are called *principle stresses.* Similarly the shear stress will also be maximum for some value of ϕ. It is always necessary to find the principle stresses and maximum shear stress because materials have different strengths in tension and shear. The state of stress can be represented by a Mohr's circle. The procedure for plotting Mohr's circle will now be given below.

Ref. (T-I-6) and Fig. (1.6).

Step 1 : Plot σ_x = OA and σ_y = OB, along the X axis. Note that tensile stresses are treated as positive and compressive as negative. Hence they are plotted on two sides of the origin O.

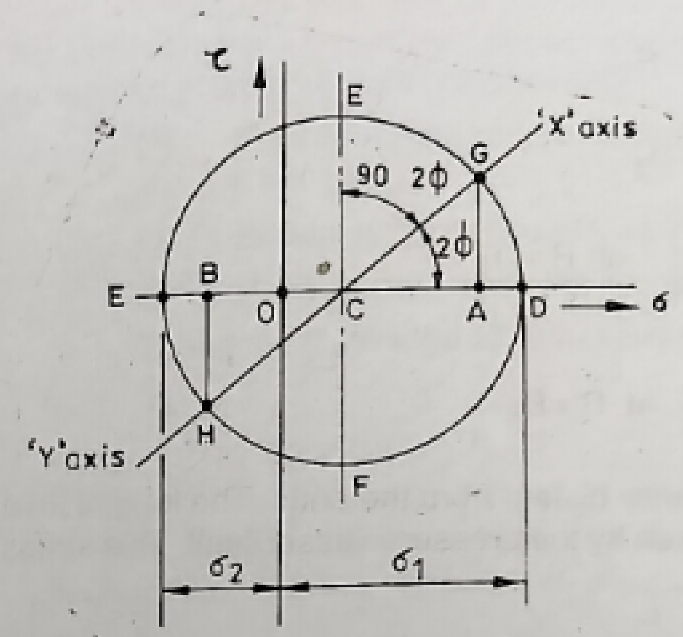


Fig. 1.6

Step 2 : Plot the shear stress τ = AG at A and τ = BH at B. For shear stress, if it is acting clockwise on the right face it is taken as positive and plotted above σ axis.

Step 3 : Join G to H. the line GH bisects AB at C which becomes the centre of the Mohr's Circle. The radius of the circle is given by CG. Draw the circle. It intersects the σ axis at D and E. For further reference, CG becomes X axis and CH, Y axis in Mohr's Circle.

Step 4 : The principal stresses are given by OD and OE and are denoted by σ_1 and σ_2. Angle GCD = 2ϕ.

Step 5 : The principle planes will be located at an angle ϕ clockwisse from X axis. Note that the angles in the space diagram are half that in the Mohr's Circle.

Step 6 : The maximum shear stress, τ_{max} = CE and occurs in planes making an angle of (45 – ϕ) counter clockwise w.r.t X axis. The normal stress acting on these planes is given by OC.

The principal stresses and maximum shear stress along with the planes in which they are acting are also shown in Fig. (1.6).

The analytical expression for the principal stresses & max. shear stress are given below (T-I-6).

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \;;$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$\tan 2\phi = \frac{2\tau}{\sigma_x - \sigma_y} \;;\quad \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$\theta = \phi \pm 45^0$$

The use of these equations & method of analysis will be shown with the help of solved problems.

1.7 Stresses in Cylinders :

Cylindrical pressure vessels, hydraulic cylinders, gun barrel and pipes carrying fluids at high pressures develop both radial and tangential stresses with values which are dependent upon the radius of the element under consideration. In determining σ_R and σ_t we make use of the assumption that the longitudinal elongation is constant around the circumference of the cylinder. The expressions for σ_R and σ_t when the cylinder is subjected to an internal pressure P_i and external pressure P_o, along with their distribution are given in (T-I-22). The graphical method of finding these stresses are given in (T-I-23). The stresses in compound cylinders are also given in (T-I-23) and (T-I-22). Generally $P_o = 0$. The expression for σ_R and σ_t with only P_i acting, are given below :

$$\sigma_R = \frac{R_1^2 P_i}{R_2^2 - R_1^2}\left(1 - \frac{R_2^2}{R^2}\right); \qquad \sigma_R = -P_i \text{ at } R = R_1.$$

$$= 0 \text{ at } R = R_2.$$

$$\sigma_t = \frac{R_1^2 P_i}{R_2^2 - R_1^2}\left[1 + \frac{R_2^2}{R^2}\right]; \qquad \sigma_t = P_i\left[\frac{R_1^2 + R_2^2}{R_2^2 - R_1^2}\right] \text{ at } R = R_1$$

$$= \frac{2R_1^2 P_i}{R_2^2 - R_1^2} \text{ at } R = R_2$$

These equations apply only to sections significantly distant from the ends. The longitudinal stresses exist when the end reactions to pressure are taken by the pressure vessel itself. This stress is given by

$$\sigma_l = \frac{P_i R_1^2}{R_2^2 - R_1^2}$$

When the wall thickness of a cylindrical pressure vessel is less than one tenth of its internal diameter the radial stress is quite small compared to the tangential stress. Under these conditions, the tangential stress can be assumed to be uniformly distributed across the wall thickness. When this assumption is made, the vessel is called a *thin cylinder*. The examples of this are large tanks, pipes, engine cylinders etc. The tangential and longitudinal stresses in these cylinders are given by (T-I-22).

$$\sigma_t = \frac{PD_i}{2t} \text{ and } \sigma_l = \frac{PD_i}{4t}$$

The use of these equations will be illustrated with the help of solved problems.

1.8 Thermal Stresses :

These stresses occur due to change in temperature of a body which results in the change of its dimensions. If the body is unrestrained, no stresses will develop. But if it is restrained, a compressive stress will be induced which is given by

$\sigma_c = \alpha(\Delta T)E$ Where α = Coefficient of thermal expansion

ΔT = Temperature change

E = Modulus of elasticity

These stresses are not thermal stresses in real sense because they are induced due to the restraint for expansion. A *thermal stress* is one which arises because of the existence of a temperature gradient in a body. The most common examples are engine cylinders and cylinder heads, heat exchangers etc. The equations for thermal stresses in plates and cylinders are given in (T-I-9). The use of these equations will be illustrated with the help of solved problems.

1.9 Impact Loads :

The previous articles have dealt with static loading. We now turn to the more commonly ecountered case of dynamic loading. The criteria for deciding whether to treat the loads as static or dynamic, is the ratio of the time required to apply the load (i.e. to increase if from zero to it's full value) to the natural period of vibration. If the time of loading is greater than three times the natural period. It can be assumed as *static load*. On the other hand if the loading time is less than half the natural period, it is definitely an *impact* or *dynamic load*. In between is grey area. An important difference between static and impact loading is that statically loaded parts must be designed desired to carry loads, where as parts subjected to impact must be designed to absorb energy.

Dynamic loading includes both *impact*, the subject of this article and *fatigue* which will be discussed in the next article. Impact loading also called shock loading may be divided into three categories in order of increasing severity.

(a) Rapidly moving loads of essentially constant magnitude as produced by a vehicle crossing a bridge.

(b) Suddenly applied loads, as those resulting from an explosion and

(c) Direct impact loads, as produced by a pile driver, drop forge or vehicle crash.

These are illustrated schematically in the Fig. (1.7).

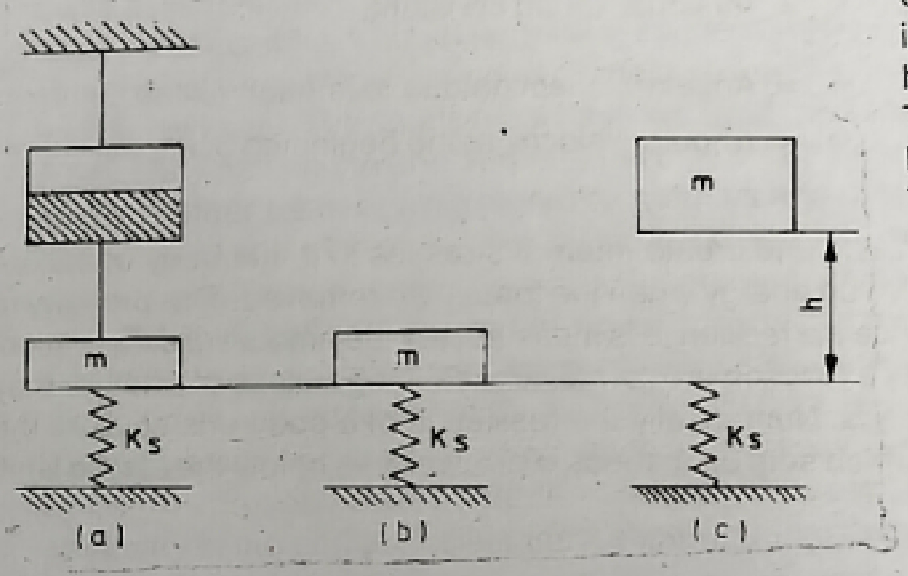


Fig. 1.7

Out of these the load is applied gradually in case (a) because of the dashpot and hence it can be considered static loading. The case (b) is special case of (c) in which h = o. Hence we will analyse only case (c). Though the figure shows the impact load to be compressive, it can be tensile, bending, torsional or a combination of these.

The figure shows the idealised version of a freely falling mass impacting a structure which is represented by a spring; this is appropriate because all structures have some elasticity. To derive the simplified equations for stress and deflection the assumptions made are :

1. The mass of the structure (spring) is negligible.
2. The deflections within the mass itself are negligible.
3. Damping is negligible.

Keeping these assumptions in mind and equating the potential energy given up by all falling mass with the elastic energy absorbed by the spring, we get :

$mg(h + \delta) = \frac{1}{2} K_s \delta^2$ Where h = height of free fall

δ = deflection of spring

K_s = spring stiffness.

Therefore the maximum force acting on the spring called equivalent static load, F_e is given by

$$F_e = K_s \delta$$

If the deflection due to W = mg, is given by δ_{st} we get $K_s = \frac{W}{\delta_{st}}$

$$\therefore W(h + \delta) = \frac{W}{2}\frac{\delta^2}{\delta_{st}} \quad \text{If } h >> \delta, \text{ then } \delta = \sqrt{2h\delta_{st}}$$

Solving these equations gives,

$$\delta = \delta_{st}\left[1 + \sqrt{1 + \frac{2h}{\delta_{st}}}\right], \quad F_e = W\left[1 + \sqrt{1 + \frac{2h}{\delta_{st}}}\right]$$

In some cases it is more convenient to express the velocity of impact, v in place of height of fall h. These are related to each other by

$$h = \frac{v^2}{2g}$$

Hence we can use the same equations after finding equivalent h. Once the equivalent static load Fe is found the methods of stress analysis are same. This will be illustrated with the help of solved problems. The equations for stress and deflection under impact loading are given in (T-I-7). In case of torsional impact these equations become.

$$\frac{1}{2} I \omega^2 = \frac{1}{2} K_{ts}\, \theta^2$$

Where K_{ts} = Torsional spring constant.

θ = Angular deflection due to impact.

ω = Angular velocity at the beginning of impact.

1.10 Resilience :

When external forces deform a body and create internal stresses in it, the body can store some of the energy. Also, the body can give up energy when the forces are removed. This property of a material is generally called resilience. Since the resilience is measured by the amount of deformation, it is also called *strain energy*. Resilience is a function of internal stresses, regardless of whether they are induced by static loads or dynamic loads. Numerically the resilience of a body U is equal to the work required to produce a deformation which sets up a stress σ or τ which is below the elastic limit.

Thus $U = \frac{1}{2} F \delta$

The ability of a material to withstand impact without permanent deformation when acting as a rigid spring, is indicated by its *modulus of resilience* designated as u. It is defined as the strain energy stored up in a unit volume of the material at the elastic limit. The modulii of resilience for various types of loading are given in (T-I-8).

1.11 Fatigue Loads :

When the loads are changing in magnitude or direction, the loading is called fatigue loading. The variation is generally cyclic and is of three types as shown in Fig. (1.8).

(a) Reversed, (b) Repeated, (c) Fluctuating or alternating.

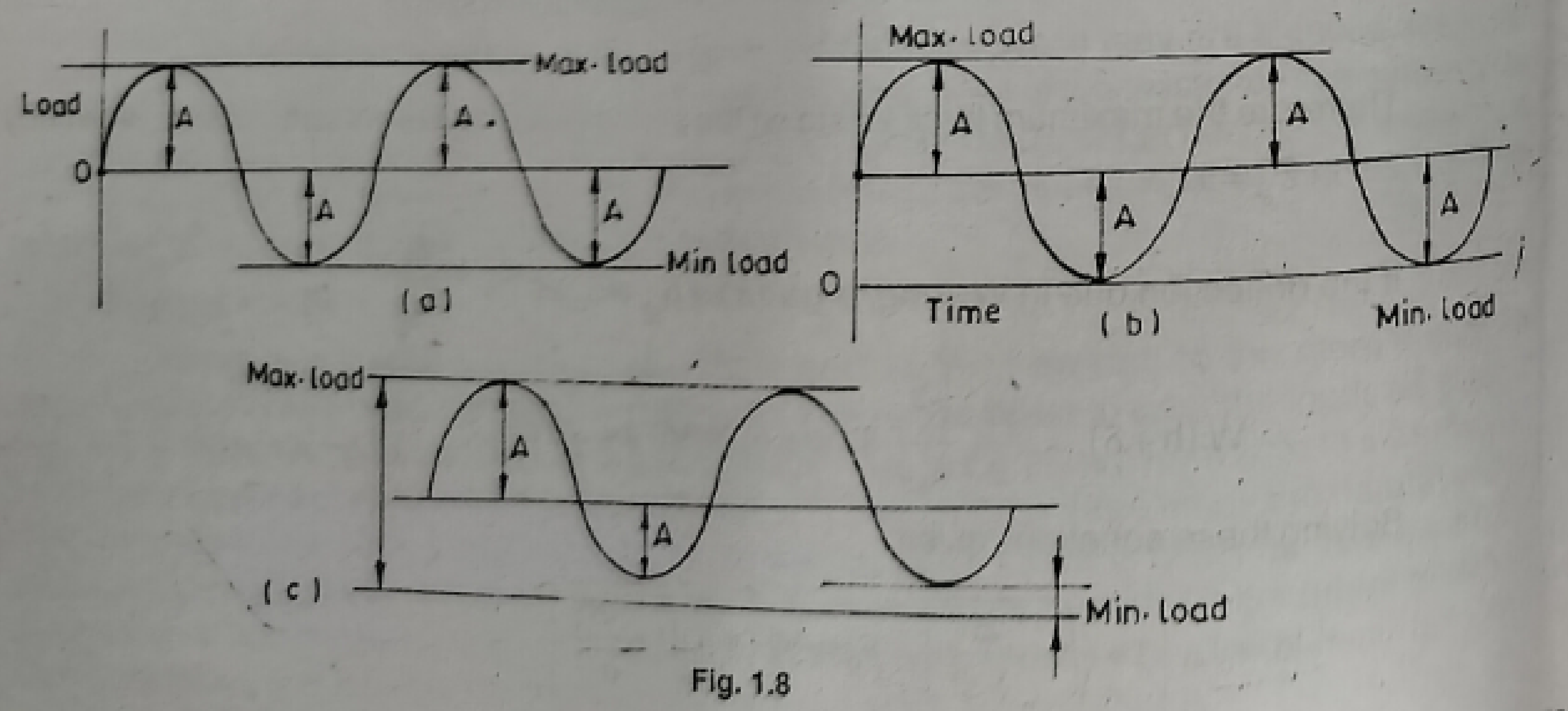


Fig. 1.8

In all the cases,

$$\text{Mean load} = \frac{\text{Max. load} + \text{Min. load}}{2} \quad ; \quad \text{Variable load} = \frac{\text{Max. load} - \text{Min. load}}{2}$$

The loads may cause tension, compression, bending or torsion. The stress analysis for the two portions of the loads -- mean and variable -- is done on the same lines as for static loads. The combined effect of the mean and variable portion for designing the parts will be discussed in chapter 4.

1.12 Stress Concentration :

In the development of the basic stress equations for tension, compression, bending and torsion it was assumed that no irregularities occured in the member under consideration. But it is quite difficult to design a machine without permitting some changes in the cross section of the members. Rotating shafts must have steps designed on them so that the bearings can be properly located and they can take thrust loads. The shafts must have key slots machined onto them for securing pulleys and gears. A bolt has head on the end and threads on the other end, both of which are abrupt changes in cross section. Other parts require holes, oil grooves and notches of various kinds. Any discontinuity in a machine part alters the stress distribution in the neighbourhood of the discontinuity so that the elementary stress equations no longer describe the state of stress in the part. Such discontinuities are called *stress-raisers* and the regions in which they occur are called *areas of stress concentration*. The nominal stress as calculated from equations given earlier are multiplied by a factor to obtain the maximum stress occuring at the discontinuity. This factor is called *stress concentration factor* and is always greater than one. The magnitude of this factor depends on the shape or geometry of the part as well as the property of the material especially the notch sensitivity. The factors depending only on the geometry of the part are called theoretical or geometrical stress concentration factor and denoted by K_t or K_{ts}. Their values are plotted in (F-I-1) to (F-I-6) of data book for commonly used shapes. When we consider the effect of material properties also, it is called fatigue stress concentration factor, K_f because they are important for fatigue loading. The relationship between K_t and K_f is given by

$$K_f = 1 + q\,(K_t - 1)$$ where q = Notch sensitivity factor.

The use of K_t and K_f for design purposes will be discussed in chapter 4. The use of K_t for calculation of stresses will be illustrated in the solved problems.

1.13 Hertz Contact Stresses :

A state of triaxial stress seldom arises in design. An exception to this occurs when two bodies having curved surfaces are pressed against one another. When this happens, point or line contact changes to area contact and the stress developed in both bodies is three dimensional. Contact stress problems arise in the contact of a wheel and rail, a cam and its follower, mating gear teeth and in rolling element bearings. To guard against the possibility of surface failure in such cases, it is necessary to have means of computing stress states which result from loading one body against another. Generally, two types of problems arise in practice which are discussed below.

(a) Sphere on Sphere : Where two spheres of radii R_1 and R_2 are pressed together with a force F, the theoretical point contact is converted into a circular area of contact of radius a. The compressive stress distribution is elliptical with the max. value σ_c (max.). The relations for 'a' and σ_c (max.) are given in (T-I-10) with the value of $\mu_1 = \mu_2 = 0.3$ (steel). The general expressions are given below. These expressions can also be used for the contact of a sphere with a plane surface and a sphere with a spherical cavity as given in (T-I-10).

If we define a quantity Δ as follows :

$$\Delta = \left(\frac{1-\mu_1^2}{E_1}\right) + \left(\frac{1-\mu_2^2}{E_2}\right)$$

Then $$a = .908 \left[\frac{F\Delta}{\frac{1}{R_1}+\frac{1}{R_2}}\right]^{\frac{1}{3}}$$

$$\sigma_c\,(\text{max.}) = 0.578 \left[\frac{F\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^2}{\Delta^2}\right]^{\frac{1}{3}}$$

(b) Cylinder On Cylinder : In this case the contacting elements are two cylinders of length 'L' and radii R_1 and R_2. When a force 'F' is applied to them, the theoretical line contact is converted into a rectangular contact area of width '2b' and lenght 'L'. The equations for the maximum compressive stress σ_c (max.) and half width 'b' are given in (T-I-10) for steel on steel i.e. $\mu_1 = \mu_2 = 0.3$.

These expression can also be used for the contact of a cylinder against a plane surface and a cylinder against a cylindrical cavity as given in (T-I-10). The general expressions are :

$$b = 1.13 \left[\frac{F\Delta}{L\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}\right]^{\frac{1}{2}}$$

$$\sigma_c\,(\text{max.}) = 0.564 \left[\frac{F}{L\Delta}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right]^{\frac{1}{2}}$$

The use of these equations will be illustrated with the help of solved problems.

SOLVED PROBLEMS

Prob. 1.1 : A horizontal member ABC is hinged at point A and is supported by a cable at C as shown in Fig. A vertical downward load of 20 kN is applied at B. Determine the maximum stresses induced in member ABC and also indicate the position of maximum stresses.

Solution : Let T be the tension in the cable. So the FBD of the member ABC is as shown below :

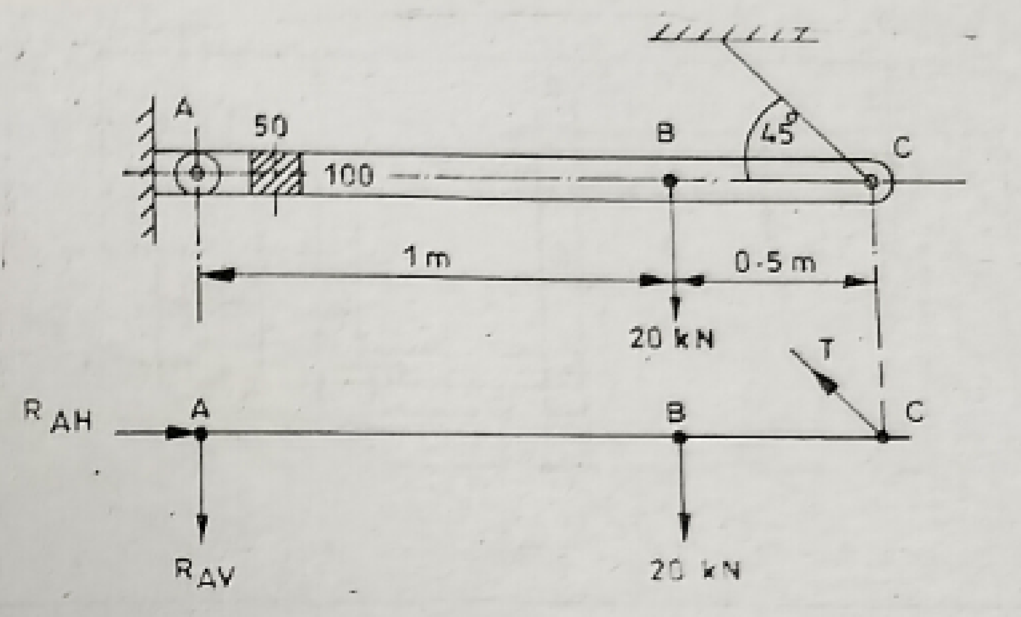


Fig.

$\Delta\ \Sigma M_A = 20 \times 1000 - T \cos 45 \times 1500 = 0$

$\therefore\ T \cos 45 = 13.33$ kN

$\therefore\ T = 18.85$ kN

Area of C.S. $A = 100 \times 50 = 5000\ \text{mm}^2$

$\therefore$ Max. stress will occur at B. The stresses are :

i) Direct compressive stress, $\sigma_c = \frac{T \cos 45}{A} = \frac{13.33 \times 10^3}{5000} = 2.66$ MPa

ii) Bending stress $\sigma_b = \frac{M}{Z}$. $\quad M = 13.33 \times 10 \times 500 = 6.66 \times 10^6$ N-mm.

$= \frac{6.66 \times 10^6}{.83 \times 10^5} \quad Z = \frac{bh^2}{6} = \frac{50 \times 100^2}{6} = .83 \times 10^5$

$= 80$ MPa

On the top of section at B, σ_b will be compressive. So the total compressive stress $= \sigma_c + \sigma_b = 2.66 + 80 = 82.66$ MPa.

So the maximum stress at the top of the section = 82.66 MPa (con)

Prob. 1.2 : The figure shows a cránk loaded by a force F = 1300 N which causes twisting and bending of a 20 mm diameter shaft fixed to the support.

(a) Draw free body diagrams of the shaft AB and arm BC and compute the values of all forces, moments and torques which act on them.

(b) Compute the maximum torsional stress and maximum bending stress in the arm BC and indicate where these act.

(c) Locate a stress element on the top surface of the shaft at A and calculate all the stress components which act upon this element.

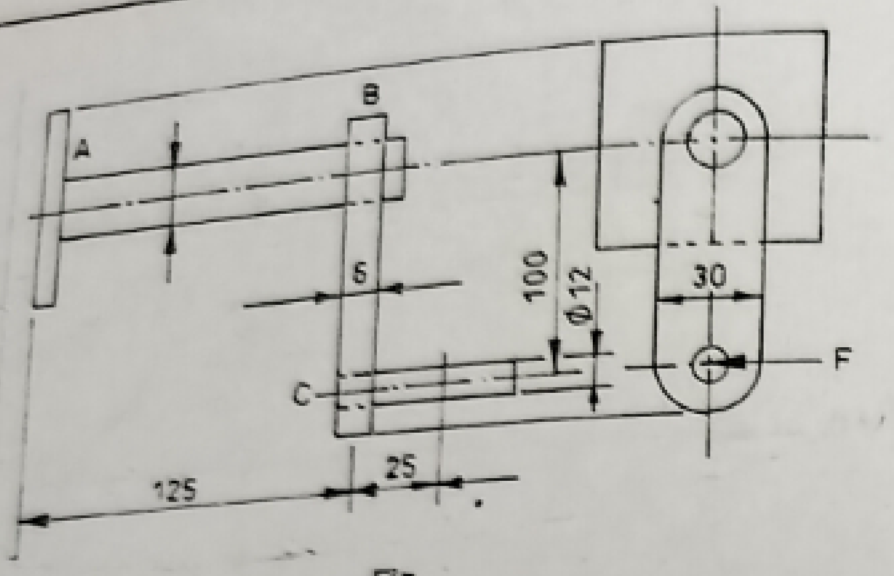


Fig.

Calculations

Ref.

T-I-1

The free body diagram of arm BC is shown below.

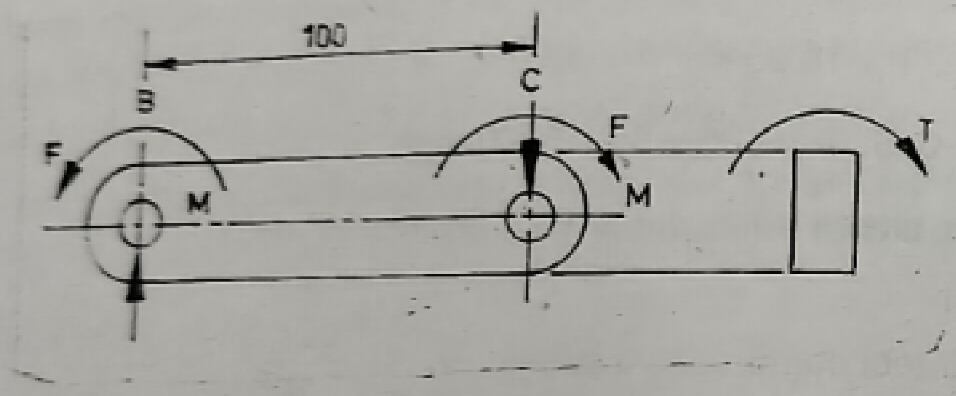


Fig.

It is acted upon by a torque T and bending moment M in addition to a direct shear force F.

Torque, T = 1300 × 25 = 32,500 N-m.

M = 1300 × 100 = 13 × 10⁴ N-m.

The maximum stresses will occur at B. Section at B is shown below.

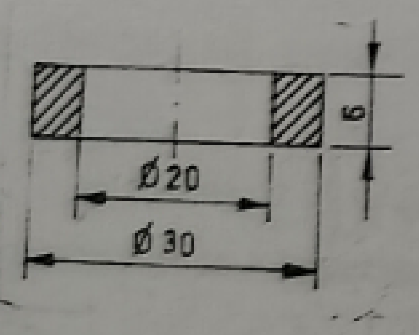


Fig.

$$I = \frac{6}{12}\left[30^3 - 20^3\right] = 9500\ \text{mm}^4$$

$$Z = \frac{I}{C} = \frac{9500}{15} = 633.3\ \text{mm}^3$$

$$I_p = \frac{6}{12}\left[30\left(6^2 + 30^2\right) - 20\left(6^2 + 20^2\right)\right]$$

$$= 9680\ \text{mm}^4.$$

$$Z_p = \frac{I_p}{C} = \frac{9680}{15} = 645.3\ \text{mm}^3$$

Stresses induced in section at B

$$\sigma_b = \frac{M}{Z} = \frac{13 \times 10^4 \times 10^3}{633.3} = 200\ \text{MPa}.$$

$$\tau = \frac{T}{Z_p} = \frac{32.5 \times 10^3 \times 10^3}{645.3} = 50.36\ \text{MPa}.$$

The free body diagram of shaft is shown below.

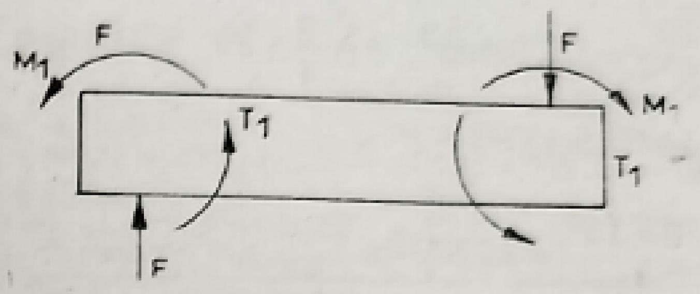


Fig.

At section A it is acted upon by a torque T_1 and bending moment M_1 in addition to the direct shear force F.

$T_1 = 1300 \times 100 = 13 \times 10^4$ N-mm.

$M_1 = 1300\ (125 + 25) = 19.5 \times 10^4$ N-mm.

Stresses at A :

$$\sigma_b = \frac{M_1}{Z} = \frac{32 \times 19.5 \times 10^4 \times 10^3}{\pi \times 20^3} = 248.4\ \text{MPa}.$$

$$\tau = \frac{T_1}{Z_p} = \frac{16 \times 13 \times 10^4 \times 10^3}{\pi \times 20^3} = 82.8\ \text{MPa}.$$

Prob. 1.3 : The stresses are to be calculated at two points A & B on the cantilever shown in the figure. The forces are F = 0.55 kN, P = 8 kN and Torque T = 30 N-m, Calculate the stresses and draw Mohr's circle for both the points.

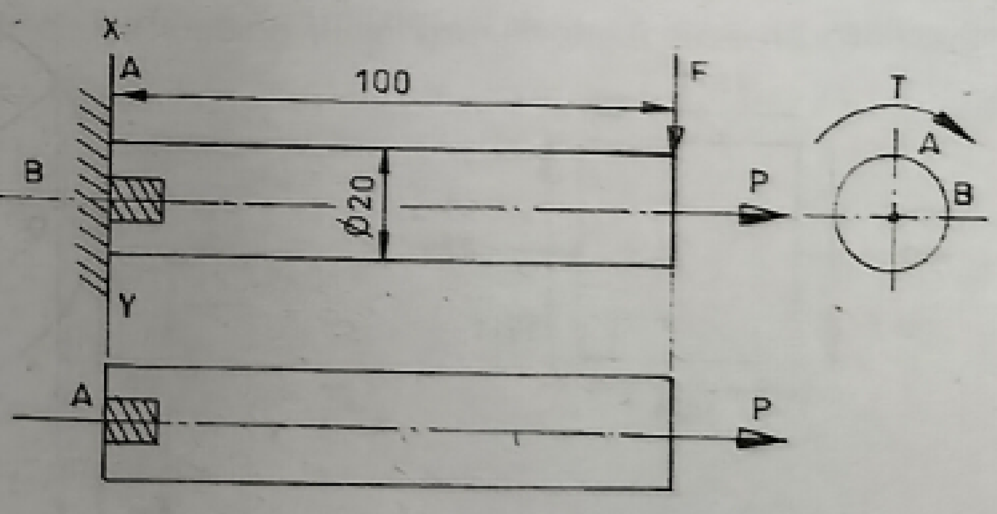


Fig.

Calculations

Ref.

T-I-1 $Z = \frac{\pi D^3}{32} = \frac{\pi \times 20^3}{32} = 785\ mm^3$

$Z_p = \frac{\pi D^3}{16} = \frac{\pi \times 20^3}{16} = 1570\ mm^3,\ A = \frac{\pi}{4} \times 20^2 = 314\ mm^2.$

T-I-4 Direct tensile stress due to P

$$\sigma_t = \frac{8 \times 10^3}{314} = 25.5\ MPa.$$

B.M. at XX, $M = F \times 100 = .55 \times 100 = 55$ N-m.

$\therefore$ Bending stress $\sigma_b = \frac{55 \times 10^3}{785} = 70$ MPa.

Torsional shear stress due to T-,

$$\tau_{1(max.)} = \frac{T}{Z_p} = \frac{30 \times 10^3}{1570} = 19.1\ MPa.$$

T-I-5 Shear stress due to F (bending)

$$\tau_{2(max)} = \frac{4F}{3A} = \frac{16 \times 55 \times 10^3}{3\pi \times 20^2} = 2.34\ MPa$$

Stress at point A :

$\sigma_t = 25.5$ MPa, $\sigma_b(t) = 70$ MPa, $\tau_1 = 19.1$ MPa & $\tau_2 = 0$

$\therefore \sigma_x = 25.5 + 70 = 95.5$ MPa & $\tau = 19.1$ MPa

These are shown in the figure below alongwisth the Mohr's circle.

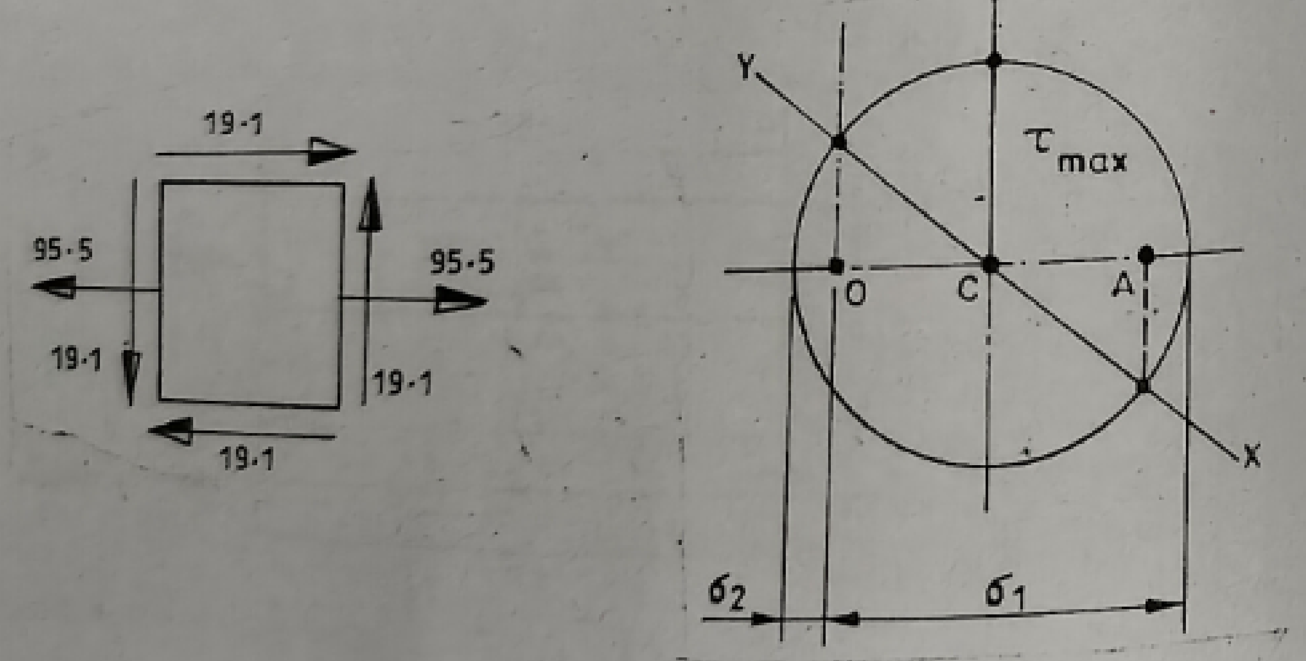


Fig.

$$\sigma_1 = \frac{95.5}{2} + \sqrt{\left(\frac{95.5}{2}\right)^2 + 19.1^2} = 99.17\ MPa$$

$\sigma_2 = 47.75 - 51.43 = -3.67$ MPa.

$\tau_x = 51.43$ MPa.

Stresses at point B :

$\sigma_t = 25.5$ MPa, $\sigma_b = 0$; $\tau_1 = 19.1$ & $\tau_2 = 2.34$

$\therefore$ $\sigma_x = 25.5$ MPa ; $\tau = 19.1 + 2.34 = 21.44$ MPa

The stresses are shown on the element and Mohr's circle is also drawn

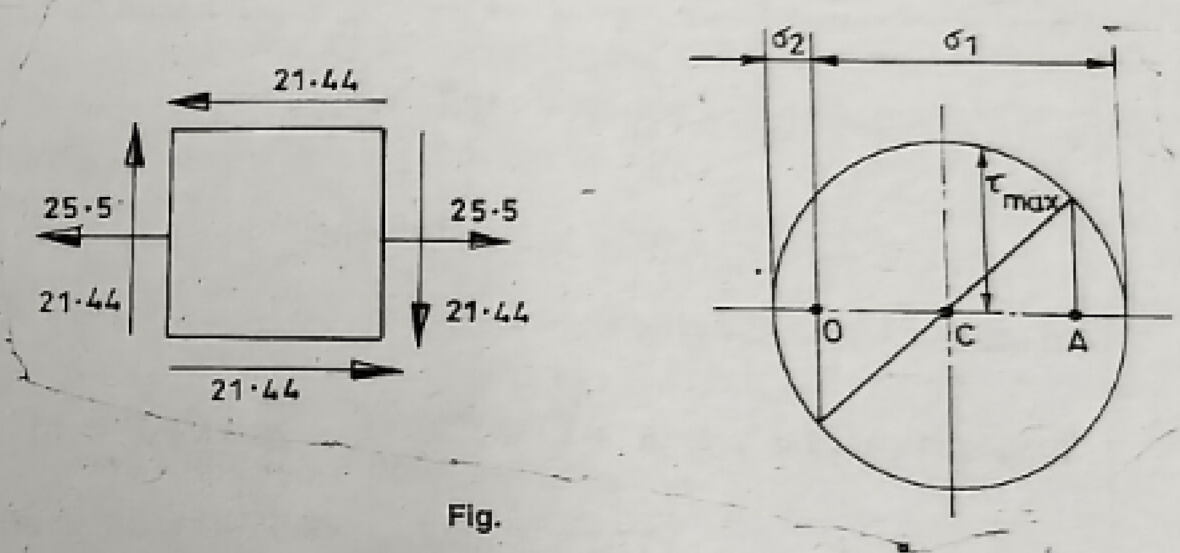


Fig.

$$\sigma_1 = \frac{25.5}{2} + \sqrt{\left(\frac{25.5}{2}\right)^2 + 21.44^2} = 37.69\ MPa$$

$\sigma_2 = 12.75 - 24.94 = -12.19$ MPa.

$\tau_{max.} = 24.94$ MPa.

Prob. 1.4 : Fig. shows an eccentric shaft subjected to a tensile load of 30 kN and torque of 250 N-m. Determine the principal stresses and maximum shear stress at a location where maximum stresses are induced.

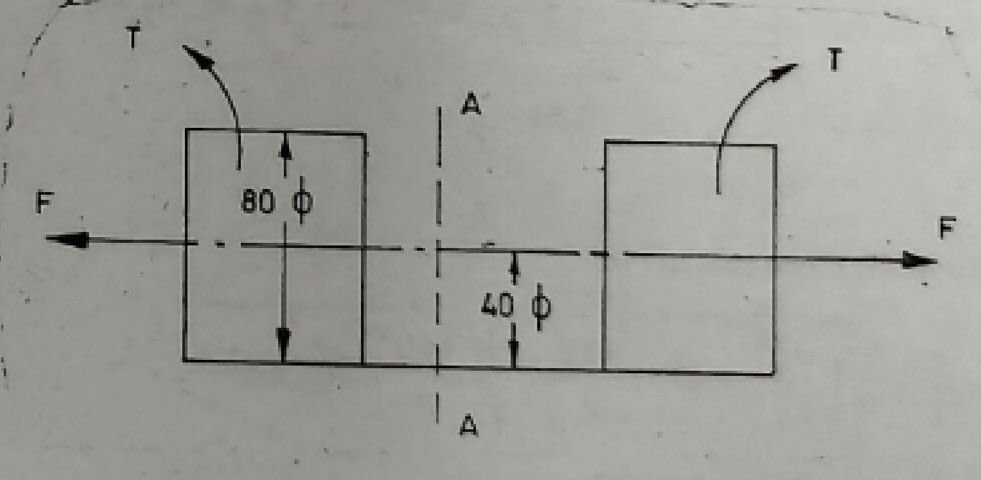


Fig.

Ref.	Calculations

The maximum stresses will be at section AA on the 40φ portion. It will subjected to following loads :

1. Direct tensile force, $F_t = 30$ kN
2. Torque, $T = 250$ N-m.
3. Bending moment, $M = 30 \times 20 = 600$ N-m

Properties of section AA :

Area, $A = \frac{\pi}{4} \times 40^2 = 1256$ mm²

$$Z = \frac{\pi}{32} \times 40 = 6.28 \times 10^3 \text{ mm}^3$$

$$Z_p = \frac{\pi}{16} \times 40^2 = 12.56 \times 10^3 \text{ mm}^3.$$

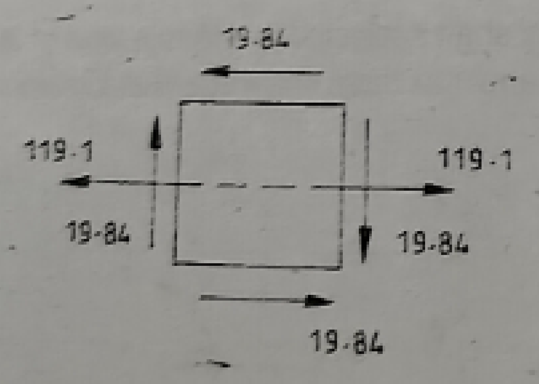
In section AA the stresses are :

Tensile stress due to F, $\sigma_t = \frac{F}{A} = \frac{30 \times 10^3}{1256} = 23.9$ MPa

Shear stress due to T, $\tau = \frac{T}{Z_p} = \frac{250 \times 10^3}{12.6 \times 10^3} = 19.84$ MPa.

Bending stress due to M : $\sigma_b = \frac{M}{Z} = \frac{600 \times 10^3}{6.3 \times 10^3} = 95.2$ MPa

Max. stress will be on element of A :



$$\sigma_x = \sigma_t + \sigma_b = 23.9 + 95.2$$

$$= 119.1 \text{ MPa}$$

$$\tau = 19.84 \text{ MPa.}$$

Fig.

The principle stresses will be :

T-I-B $\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{119}{2} + \sqrt{\left(\frac{119}{2}\right)^2 + 19.84^2}$

$$= 59.5 + \sqrt{3540 + 393}$$

$$= 59.5 + 62.7 = 122.2 \text{ MPa}$$

$$\sigma_2 = 59.5 - 62.7 = -3.2 \text{ MPa}$$

Max. shear stress, $\tau_{(max)} = \sqrt{\left(\frac{119}{2}\right)^2 + 19.84^2} = 62.7$ mPa

Prob. 1.5. : A rectangular section beam has the dimensions b = 20 mm h = 80 mm and is subjected to a pure bending moment of 2200 N-m so as to produce compression of the inner fibre. Find the stresses for the following geometries :

(a) the beam is straight.
(b) the centroidal axis has a radius of curvature of 300 mm.
(c) the centroidal axis has a radius of curvature of 100 mm.

Ref.	Calculations

(a) The beam and the loading is shown below :
M = 2200 N-m.

T-I-4 $\sigma_b = \frac{M}{Z}$; $Z = \frac{bh^2}{6} = \frac{20 \times 80^2}{6} = 21.33 \times 10^3$ mm³

T-I-1 $\therefore \sigma_b = \frac{2200 \times 10^3}{21.33 \times 10^3} = 103.14$ MPa.

(b) The beam is now curved as shown in Fig.

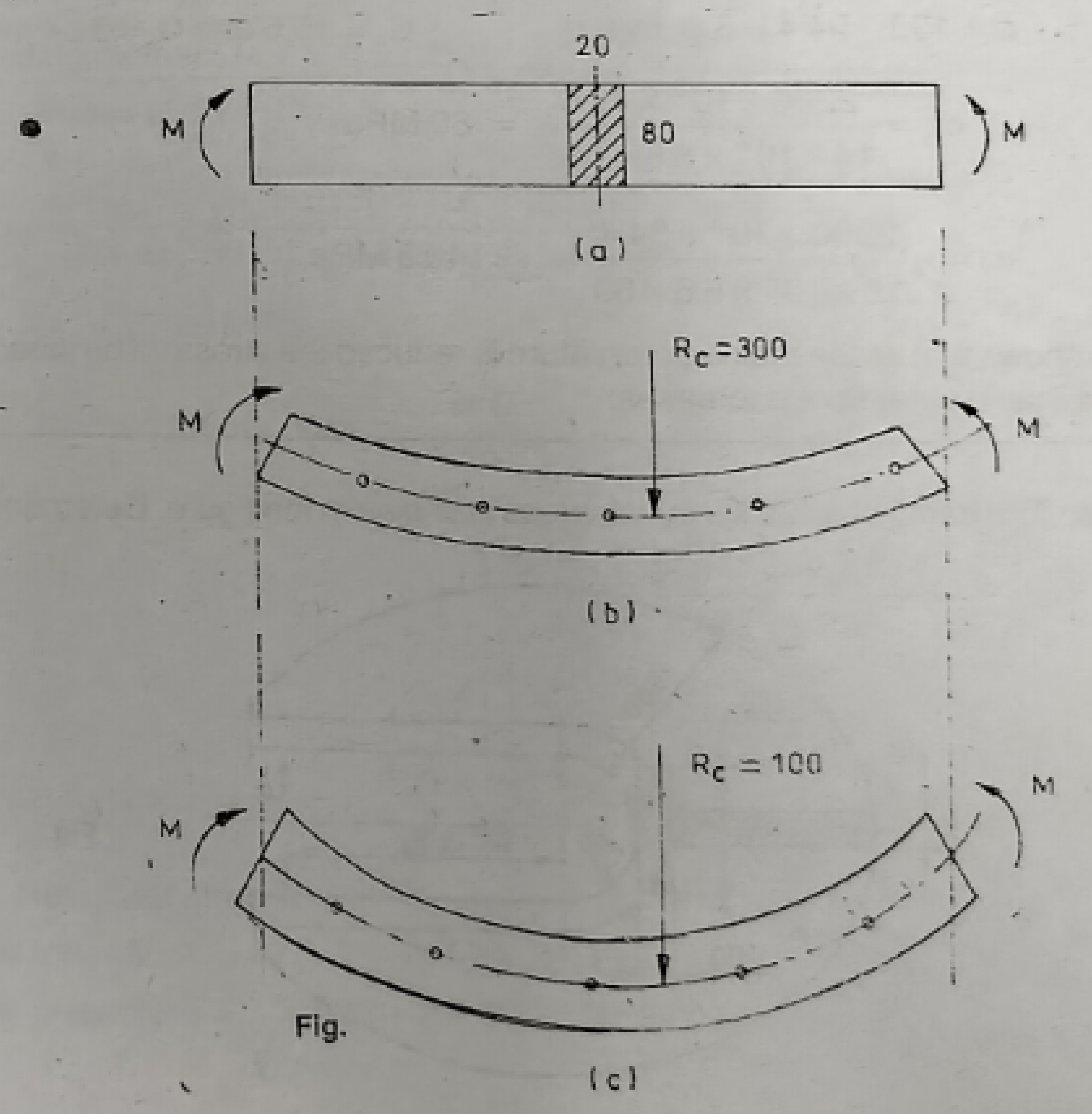


Fig.

$A = 20 \times 80 = 1600\ \text{mm}^2$

$r_o = 340\ ;\quad r_i = 260\ ;\quad r_c = 300$

T-I-25 $r_n = \dfrac{h}{\log_e\left(\dfrac{r_o}{r_i}\right)} = \dfrac{80}{\ln\left(\dfrac{340}{260}\right)} = 298.2$ mm.

$\therefore\ e = r_c - r_n = 300 - 298.2 = 1.8$ mm.

$C_o = 40 + 1.8 = 41.8$ and $C_i = 40 - 1.8 = 38.2$ mm.

$\sigma_o = \dfrac{MC_o}{A e r_o}$ because here M is opposite to that shown in Fig. of the table.

$$-\frac{2200 \times 10^3 \times 41.8}{1.6 \times 10^3 \times 1.8 \times 340} = 93.9\ \text{MPa}$$

$$\sigma_i = -\frac{MC_i}{A e r_i} = -\frac{2200 \times 10^3 \times 38.2}{1.6 \times 10^3 \times 1.8 \times 260} = -112.23\ \text{MPa}$$

(c) Here the only change is in D_{cc}

$r_c = 100\quad r_o = 140\ ;\quad r_i = 60$

$$r_n = \frac{40}{\ln\left(\dfrac{140}{60}\right)} = 94.4\ \text{mm}$$

$e = 100 - 94.4 = 5.6$ mm. $\qquad C_o = 45.6$ and $C_i = 34.4$ mm.

$$\therefore\ \sigma_o = \frac{2200 \times 10^3 \times 45.6}{1.6 \times 10^3 \times 5.6 \times 140} = 80\ \text{MPa}$$

$$\sigma_i = -\frac{2200 \times 10^3 \times 34.4}{1.6 \times 10^3 \times 5.6 \times 60} = -140.8\ \text{MPa}$$

This shows that as the radius of curvature is reduced the stress in the inner fibre increases while that in outer fibre decreases.

Prob. 1.6 : The frame of a 100 kN punch press is shown in the figure. Determine the maximum stress.

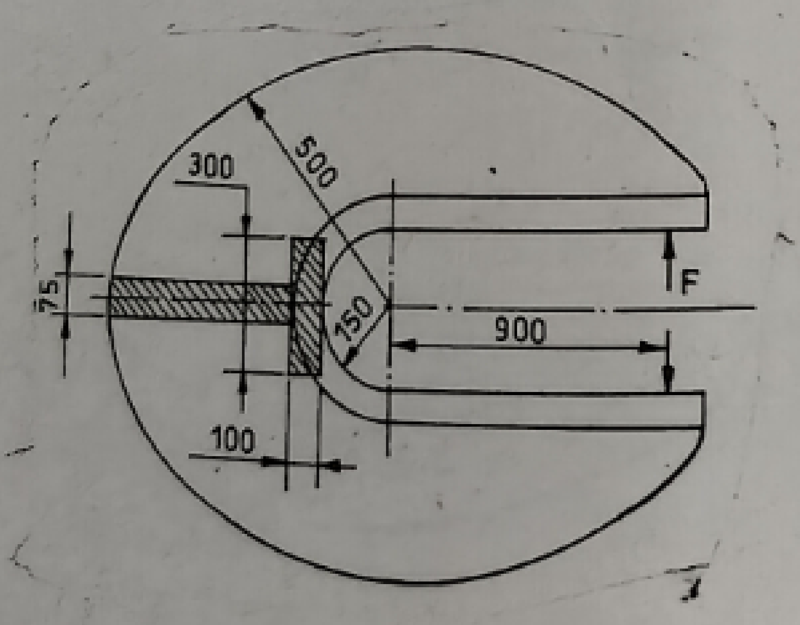


Fig.

Ref.	Calculations

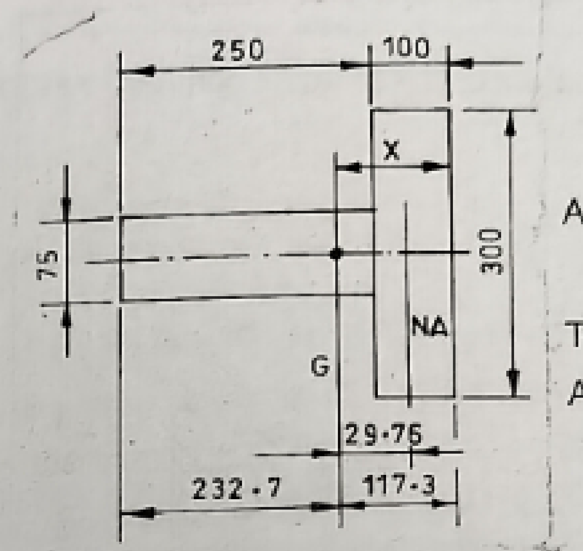


Fig.

$A = 100 \times 300 + 250 \times 75$

$= 48750\ \text{mm}^2.$

To find x, the distance of C.G. from inner face

$A\bar{x} = 300 \times 100 \times 50 + 250 \times 75 \times 225$

$\bar{x} = 117.3$

T-I-25 $r_c = 150 + 117.3 = 267.3$ mm.

$$r_n = \frac{A}{300 \ln \dfrac{150+100}{150} + 75 \ln\left(\dfrac{50}{150+100}\right)} = \frac{48750}{153.24 + 51.98}$$

$= 237.54$ mm.

$\therefore\ e = r_c - r_n = 267.3 - 237.54 = 29.76$ mm.

Bending moment $M = 100 \times (900 + 150 + 117.3) = 116730$ N-m.

$\therefore$ Stress at outer fibre

$$\sigma_o = -\frac{M C_o}{A e r_o} = -\frac{116730 \times 10^3 \times (232.7 + 29.76)}{48750 \times 29.76 \times 500}$$

$= -42.23$ MPa.

$\therefore$ Stress at inner fibre $\sigma_i = \dfrac{M C_i}{A e r_i}$

$$= \frac{116730 \times 10^3\ (117.3 - 29.76)}{48750 \times 29.76 \times 150}$$

$= 47$ MPa

Direct tensile stress $\sigma_t = \dfrac{100 \times 10^3}{48750} = 2$ MPa.

$\therefore$ Resultant stresses

at inner fibre $= 47 + 2 = 49$ MPa tensile

at outer fibre $= -42.23 + 2 = 40.23$ MPa comp.

Prob. 1.7 : The critical section AA of a crane hook is considered for purposes of analysis to be triangular with dimensions as shown. Determine the resultant stresses at points P and Q.

Ref. | **Calculations**

The centroid of section AA and moment of inertia about centroid is as follows :

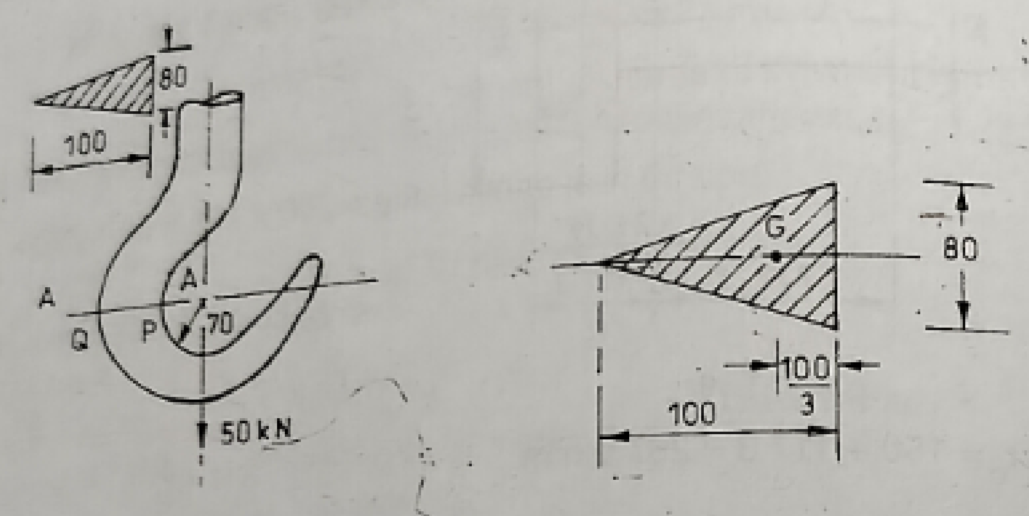


Fig.

$$x = \frac{h}{3} = \frac{100}{3}\text{ mm}$$

$$\bar{I}_{xx} = \frac{bh^3}{36} = \frac{80 \times 100^3}{36} = 2.22 \times 10^6 \text{ mm}^4$$

Now section AA will be subjected to :

1. Direct tensile load, $F = 50$ kN.
2. Bending moment $M = 50 \times \left(70 + \frac{100}{3}\right) = 5166.6$

The FBD at section AA is shown below :

As the bending is on the curved portion the bending stresses will be non-uniformly distributed. Referring to T-I-25.

T-I-25 To find e :

$$r_i = 70,\ r_o = 170,\ r_c = 70 + 33.33 = 103.33$$

$$r_n = \frac{\frac{1}{2} \times 100 \times 80}{\frac{80 \times 170}{100} \ln(170/70) - 80}$$

$$= \frac{4000}{40.67} = 98.34 \text{ mm.}$$

$$e = r_c - r_n = 103.33 - 98.34 = 5 \text{ mm.}$$

$$\sigma_o = -\frac{MC_o}{A\,e\,r_o} = -\frac{5166.6 \times 10^3 \times 200}{4000 \times 5 \times 170 \times 3} = -101.3 \text{ MPa}$$

$$\sigma_i = \frac{MC_i}{A\,e\,r_i} = \frac{5166.6 \times 100 \times 10^3}{4000 \times 5 \times 70 \times 3} = 123 \text{ MPa}$$

Tensile stress due to F : $\sigma_t = \frac{F}{A} = \frac{50 \times 10^3}{4000} = 12.5$ MPa

$\therefore$ Resultant stress at $P = \sigma_o + \sigma_t = -101.3 + 12.5 = -88.8$ MPa.

Resultant stress at $Q = \sigma_i + \sigma_t = 123 + 12.5 = 135.5$ MPa.

Prob. 1.8 : A cylinder has 25 mm ID by 50 mm OD and is subjected to an internal pressure of 150 MPa. Find the maximum shear stress and the radius at which that occurs.

Ref. | **Calucations**

T-I-22 Given $P_i = 150$ MPa, $r_1 = \frac{25}{2} = 12.5$ mm, $r_2 = \frac{50}{2} = 25$ mm.

Wall thickness , $t = r_2 - r_1 = 25 - 12.5 = 12.5$ mm.

$\frac{t}{D_1} = \frac{12.5}{25} = 0.5$ which is greater than .1 hence it is a thick cylinder.The stresses are given by

$$\sigma_R = \frac{R_1^2 P_i}{R_2^2 - R_1^2}\left(1 - \frac{R_2^2}{R^2}\right) = \frac{12.5^2 \times 150}{25^2 - 12.5^2}\left(1 - \frac{R_2^2}{R^2}\right) = 50\left(1 - \frac{R_2^2}{R^2}\right)$$

$$\sigma_t = \frac{R_1^2 P_i}{R_2^2 - R_1^2}\left(1 + \frac{R_2^2}{R^2}\right) = 50\left(1 + \frac{R_2^2}{R^2}\right)$$

T-I-6 Because of this biaxial loading, the maximum shear stress is given by

$$\sigma_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \quad \text{but } \tau = 0$$

$$= \frac{\sigma_x - \sigma_y}{2} = \frac{\sigma_t - \sigma_R}{2} = 50 \times \frac{R_2^2}{R^2}$$

This will be maximum at $R = R_1$

$$\therefore \tau_{max} = 50 \times \left(\frac{25}{12.5}\right)^2 = 200 \text{ MPa.}$$

Prob. 1.9 : A 25 × 25 mm square steel bar 250 mm long is installed with fixed ends so that it is stress free at a room temp. of 30°C. In operation the bar is heated to a uniform temperature of 250°C. Careful measurements indicate that the fixed ends seperate by 0.2 mm. What loads are exerted on the ends of the bar and what are the resultant stresses ?

Ref.	Calculations
T-II-2	For steel $E = 200 \times 10^3$ MPa, $\alpha = 10.7 \times 10^{-6} \dfrac{mm}{mm \times {}^0C}$ $\therefore$ Total expansion if unrestrained $\delta = L \times \alpha \times \Delta T = 250 \times 10.7 \times 10^{-6} \times (250 - 30) = .59$ mm. Expansion allowed = 0.2 mm. $\therefore$ Expansion restrained = .59 − .2 = .39 mm. $\therefore$ Strain suppressed $\varepsilon = \dfrac{.39}{250} = \times 156 \times 10^{-3}$ mm/mm $\therefore$ Stress due to this $\sigma_c = \varepsilon \times E = 1.56 \times 10^{-3} \times 200 \times 10^3$ $= 312$ MPa $\therefore$ Force applied $F_c = 312 \times 25 \times 25$ $= 195 \times 10^3$ N = 195 kN These answers are valid only if the stress is within elastic limit.

Prob. 1.10 : A steel bar of 1m length is fixed between two rigid walls with initial tensile stress of 120 MPa at 28°C. The diameter of bar is 25 mm. At what temp. The stresses in bar will become zero? Take $E = 2 \times 10^5$ MPa and $\alpha = 12 \times 10^{-6}$ /°C.

Ref.	Calculations
	Initial stress = 120 MPa at 28°C $E = 2 \times 10^5$ MPa $\alpha = 12 \times 10^{-6}$ /°C Area of C.S. of bar, $A = \dfrac{\pi}{4} \times 25^2 = 490.625\ mm^2$. Now $E = \dfrac{stress}{strain}$ $\therefore$ strain $= \dfrac{stress}{E} = \dfrac{120}{2 \times 10^5} = 60 \times 10^{-5}$ $\dfrac{\Delta L}{L} = 60 \times 10^{-5}$ $\therefore \Delta L = 60 \times 10^{-5} \times 10^3 = 0.6$ mm.

Ref.	Calculations
	To increase the length of 0.6 mm the increase in temperature required will be : $\alpha \times l \times \Delta\theta = 0.6$ $\therefore \Delta\theta = \dfrac{0.6}{12 \times 10^{-6} \times 10^3} = 50°C$ $\therefore$ Temp. when stress will become zero = 28 + 50 = 78°C.

Prob. 1.11 : The bore of a car engine cylinder is 90 mm and stroke is 100 mm. The thickness of the cast-iron cylinder is 10mm. The maximum pressure in the cylinder is 4 MPa and the heat flow through the cylinder walls is 27×10^6 J/hr. Calculate the resultant stresses at the inner fibre and the outer fibre of the cylinder wall.

Ref.	Calculations
	There will be stress due to pressure and thermal stresses. Stress due to pressure :
T-I-22	Given $R_1 = \dfrac{90}{2} = 45$ mm, $R_2 = 45 + 10 = 55$ mm $\sigma_t = \dfrac{R_1^2 P_i}{R_2^2 - R_1^2}\left[1 + \dfrac{R_2^2}{R}\right] = \dfrac{45^2 \times 4}{55^2 - 45^2}\left[1 + \dfrac{R_2^2}{R^2}\right] = 8.1\left[1 + \dfrac{R_2^2}{R^2}\right]$ σ_t at $R = R_1$; $\sigma_{ti} = 8.1\left[1 + \left(\dfrac{55}{45}\right)^2\right] = 20.2$ MPa. σ_t at $R = R_2$; $\sigma_{to} = 8.1\ [1 + 1] = 16.2$ MPa
T-I-9	For thermal stress : $\sigma_o = \dfrac{\alpha E (\Delta\theta)\left(1 - \dfrac{M}{3}\right)}{2(1-\mu)}$; $\sigma_i - \dfrac{\alpha E (\Delta\theta)\left(1 + \dfrac{M}{3}\right)}{2(1-\mu)}$ Where $M = \dfrac{R_2}{R_1} - 1 = \dfrac{55}{45} - 1 = .222$
T-II-2	For cast iron $\alpha = 10.6 \times 10^{-6}$ mm/mm, $E = 100 \times 10^3$ MPa. $\mu = .271$, $K = 166 \dfrac{kJ}{m\ hr\ {}^0C}$ $\Delta\theta = \dfrac{qt}{K} \times 10^{-3}$ where q = Heat flow $\dfrac{J}{m^2 hr}$ Area of flow $= \pi DL = \pi \times 90 \times 100 \times 10^{-6} = 28.26 \times 10^{-3} m^2$.

$$\therefore\ q = \frac{27\times10^{6}}{28.26\times10^{-3}} = .955\times10^{9}$$

$$\therefore\ \Delta\theta = \frac{.955\times10^{9}\times10}{166\times10^{3}}\times10^{-3} = 57.55^{0}C$$

$$\therefore\ \sigma_o\,(th) = \frac{10.6\times10^{-6}\times100\times10^{3}\times57.55\left(1-\frac{.222}{3}\right)}{2\,(1-.271)}$$

$$= 41.84\,(1-.074) = 38.74 \text{ MPa}$$

$$\sigma_i\,(th) = -41.84\,(1+.074) = -44.94 \text{ MPa}$$

$\therefore$ Resultant stresses :

At inner fibre $\sigma_i = \sigma_{ti} + \sigma_i\,(th) = 20.2 - 44.94 = -24.74$ MPa

At outer fibre $\sigma_o = \sigma_{to} + \sigma_o\,(th) = 16.2 + 38.74 = 54.94$ MPa.

Prob. 1.12 : The free end of a steel cantilever beam is directly over and in contact with a vertical coil spring as show in Fig. The width of the beam is 600 mm and thickness 12 mm. The coil spring has a stiffness of 40 N/mm. Take E = 200 GPa and G = 83 GPa.

(a) What weight should be placed at the free end to produce a deflection of 40 mm ?

(b) If this weight is dropped from a height of 10 mm what will be the deflection and the bending stress at the fixed end of the cantilever ?

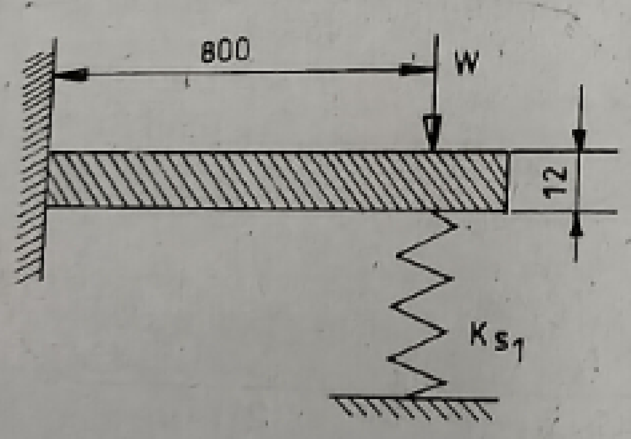


Fig.

Ref.	Calculations
T-I-2	For the steel beam, the stiffness $K_{s2} = \frac{F}{Y} = \frac{3EI}{L^3}$
T-I-1	$I = \frac{1}{12}bh^3 = \frac{1}{12}\times600\times12^3 = 86400\ mm^4, L = 800, E = 200\times10^3$
	$\therefore\ K_{s2} = \frac{3\times200\times10^3\times86400}{800^3} = 101.25$ N/mm

---

As the two springs are acting in parallel, the total stiffness

$K_{se} = K_{s1} + K_{s2} = 40 + 101.25 = 141.25$ N/mm.

$\therefore$ Weight required to produce a deflection of 40 mm.

W = 141.25 × 40 = 5650 N.

(b) If this falls through a height of 10mm and the new deflection is δ, we get

$$W\,(h+\delta) = \frac{1}{2}K_{se}\,\delta^2 \quad \text{or} \quad \delta^2 - \frac{2W}{K_{se}}\delta - \frac{2W}{K_{se}}h = 0.$$

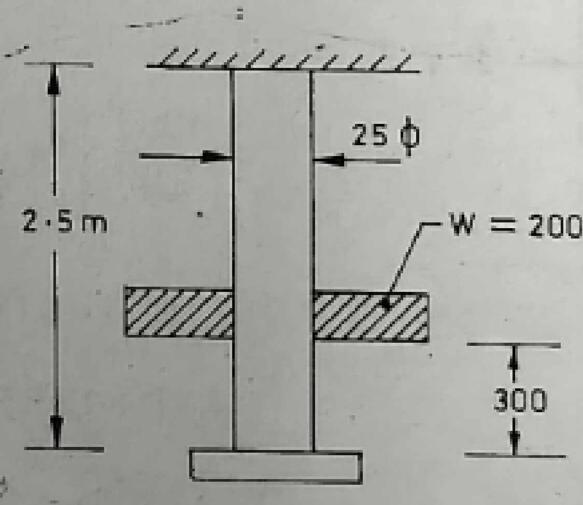
$$\delta = \frac{W}{K_{se}} + \sqrt{\left(\frac{W}{K_{se}}\right)^2 + \frac{2Wh}{K_{se}}} = \frac{5650}{141.25} + \sqrt{1600+800} = 82 \text{ mm.}$$

$\therefore$ Force acting on the beam : $K_{s2}\times\delta = 101.25\times82 = 8302.5$ N

$\therefore$ B. M. at the fixed end = 8302.5 × 800 = 6642 × 10^3 N-mm = 6642 N-m

$$\sigma_b = \frac{M}{Z} = \frac{6642\times10^3\times6}{86400} = 461.25 \text{ MPa.}$$

Prob. 1.13 : A bar of diameter 25 mm and length 2.5 m is fixed at top end and hanges vertically with a rigid collar at its lower end. Find the maximum instantaneous stress developed in the rod and deformation if a weight of 200 kN fall on collar through a height of 300 mm.

Ref.	Calculations	
	 Fig.	
T-I-7	$\sigma = \frac{W}{A}\left[1+\sqrt{1+\frac{2hAE}{WL}}\right]$	$A = \frac{\pi}{4}\times25^2 = 490.6$
	$= \frac{2\times10^5}{490.6}\left[1+\sqrt{1+\frac{2\times300\times490.6\times2\times10^5}{2\times10^5\times2.5\times10^3}}\right]$	h = 300 mm

$$= \frac{2\times10^5}{490.6}\left[1+\sqrt{1+117.6}\right]$$

L = 2500 mm

W = 200 × 10^3 N

$$= \frac{2\times10^5}{490.6}\left[1+10.89\right]$$

Assume E = 2 × 10^5 MPa

$= 4847$ MPa.

$$\text{Elongation} = \frac{4847\times2.5\times10^3}{2\times10^5} = 60.6 \text{ mm.}$$

Prob. 1.14 : A weight W of 1350 N falls through a height h of 1.5 mm before the load strikes the square head of the round steel bar 20 mm in diameter and 450 mm long. (a) determine the stress induced, assuming E = 200 MPa ; (b) Compare it with the stress from the same load acting statically. (c) Find how much the rod length can be changed before the impact stress exceeds 130 MPa. (d) Determine how much the height h can be changed before the stress exceeds 130 MPa.

Ref. **Calculations**

T-I-7

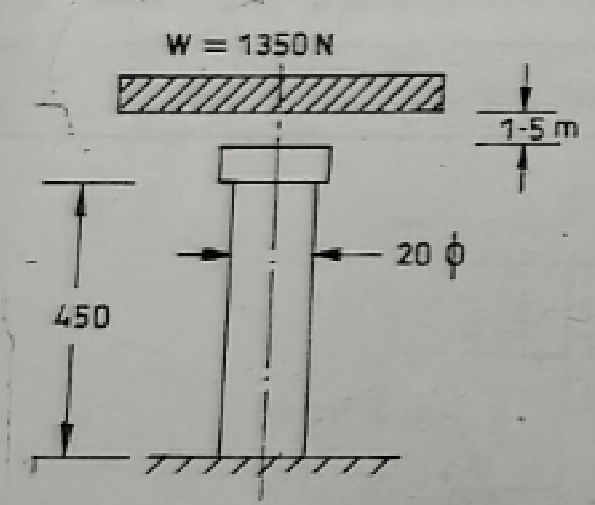


Fig.

$$A = \frac{\pi}{4}\times20^2 = 314 \text{ mm}^2$$

$$\sigma = \frac{W}{A}\left[1+\sqrt{1+2h\frac{AE}{WL}}\right]$$

$$= \frac{1350}{314}\left[1+\sqrt{1+\frac{2\times1.5\times314\times200\times10^3}{1350\times450}}\right]$$

$$= 4.3\left[1+\sqrt{1+310}\right]$$

$$= 4.3\,[1+17.635] = 83.86 \text{ MPa.}$$

(b) For static load, stress $\sigma = \frac{W}{A} = \frac{1350}{314} = 4.3$ MPa

(c) For σ = 130 MPa with changed length L_1 :

$$130 = 4.3\left[1+\sqrt{1+\frac{2\times1.5\times314\times200\times10^3}{1350\,L_1}}\right]$$

$$= 4.3\left[1+\sqrt{1+\frac{139.5\times10^3}{L_1}}\right]$$

$$1+\frac{139.5\times10^3}{L_1} = (29.23)^2 = 854.4$$

$$\therefore\ L_1 = \frac{139.5\times10^3}{853.4} = 163.3 \text{ mm}$$

(d) For σ = 130 MPa with changed height h_1 :

$$130 = 4.3\left[1+\sqrt{1+\frac{2h_1\times314\times200\times10^3}{1350\times450}}\right]$$

$$\sqrt{1+.2067\times10^3 h_1} = 29.23$$

$$\therefore\ h_1 = \frac{853.4}{206.7} = 4.128 \text{ mm}$$

Prob. 1.15 : A 50 kN elevator is supported by a wire rope of 1560 mm^2 cross section and an effective modulus of elasticity of 84 × 10^3 MPa. As the elevator is descending at a constant velocity of 2 m/sec. an accident causes the top of the cable, 18m above the elevator, to suddenly stop. Estimate the maximum elongation and maximum tensile stress developed in the cable.

Ref. **Calculations**

For a round bar, the spring constant,

$K_s = \frac{AE}{L}$ Where A = C.S. area, L = length and E = modulus of elasticity.

$$\therefore\ K_s = \frac{1560\times84\times10^3}{18\times10^3} = 7280 \text{ N/mm}$$

Now the equivalent h for a weight moving with velocity v

$$h = \frac{v^2}{2g} = \frac{2^2}{2\times9.81} = .204 \text{ m} = 204 \text{ mm.}$$

$$\therefore\ W(h+\delta) = \frac{1}{2}K_s\,\delta^2$$

$$\delta = \frac{W}{K_s} + \sqrt{\left(\frac{W}{K_s}\right)^2 + \frac{2\,Wh}{K_s}} = \frac{50\times10^3}{7280} + \sqrt{47.61+2815.2} = 60.4 \text{ mm.}$$

$\therefore$ Equivalent load, $F_e = K_s\delta = 7280\times60.4 = 44$ kN

$$\therefore\ \text{Stress} = \frac{44\times10^3}{1560} = 28.2 \text{ MPa.}$$

Prob. 1.16 : A vertical drive shaft is 20 mm in diameter, 750 mm long and made of steel. The motor to which it is attached at the top is equivalent to a steel fly wheel 300 mm in diameter and 25 mm thick. When the vertical shaft is rotating at 3000 rpm the propeller strikes a heavy obstruction bringing it to a virtually instanteneous stop. Calculate the maximum torsional shear stress. Assume G = 80 GPa for steel and sp. gr. = 7.8.

Ref.	Calculations
	This is a problem of torsional impact. So our basic equation becomes, in this case
	$\frac{1}{2} I w^2 = \frac{1}{2} K_t \theta^2$ Where I = mass moment of inertia of fly wheel = mK^2.
	K_t = spring stiffness in torsion.
	$w = \frac{2\pi \times 3000}{60} = 314$ rad/sec.
T-I-4	For torsion $K_t = \frac{T}{\theta} = \frac{GI_p}{L}$
T-I-1	I_p for shaft $= \frac{\pi d^4}{32} = \frac{\pi \times 20^4}{32} = 1.57 \times 10^4$ mm^4.
	$\therefore K_t = \frac{80 \times 10^3 \times 1.57 \times 10^4}{750}$
	$= 1674 \times 10^3 \frac{\text{N-mm}}{\text{rad}} = 1674 \frac{\text{N-m}}{\text{rad}}$
	Mass of fly wheel $m = \frac{\pi}{4} \times 300^2 \times 25 \times 7800 \times 10^{-9} = 13.8$ Kg.
	$K = \frac{D}{4} = \frac{300}{4} = 75$ mm $= .75$ m.
	$\therefore$ I of fly wheel $= 13.8 \times 0.75^2 = 0.776$ Kg m^2.
	$\frac{1}{2} K_t \theta^2 = \frac{1}{2} I w^2$; $\frac{1}{2} 1674 \times \theta^2 = \frac{1}{2} .0776 \times 314^2$
	$\therefore \theta = \sqrt{\frac{.0776 \times 314^2}{1674}} = 2.13$ rad.
	$\therefore$ Torque $= 1674 \times 2.13 = 3565$ N-m.
	$\therefore$ Stress $\tau = \frac{16 \times T}{\pi d^3} = \frac{16 \times 3565.62 \times 10^3}{\pi \times 20^3} = 2271.09$ MPa.
	As this is very high, shear pins are provided for the safety of the propeller shaft.

Prob. 1.17 : Figure shows two round bars subjected to tensile impact. How do their elastic energy absorbing capacity compare ?

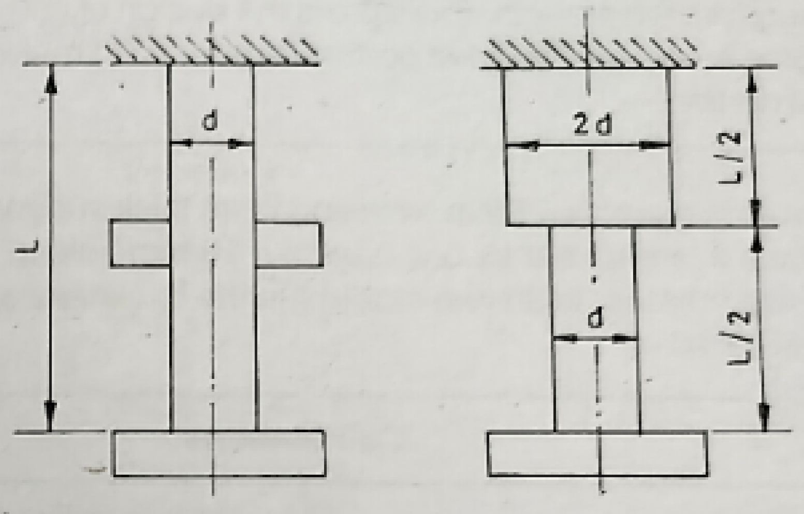


Fig.

Ref.	Calculations
	Let the elastic limit of the material by Sy MPa
T-I-8	Modulus of resilience for tension $= \frac{\sigma^2}{2E}$ $\therefore$ If s = Sy and volume = V
	Energy absorbing capacity $U = \frac{Sy^2 V}{2E}$
	In the first case $V = \frac{\pi}{4} d^2 \times L$ $\therefore U = \frac{\pi d^2 Sy^2 L}{8E}$
	In the second case if the lower portions is stressed upto Sy the stress in the upper portion will be $\frac{Sy}{4}$. The volume of the two portions will be ;
	$V_1 = \frac{\pi}{4} d^2 \frac{L}{2} = \frac{\pi}{8} d^2 L$; $V_2 = \frac{\pi}{4} (2d)^2 \times \frac{L}{2} = \frac{\pi}{2} d^2 L$
	$\therefore$ Energy absorbing capacity of the whole rod
	$U = U_1 + U_2 = \frac{Sy^2 \times \pi \times d^2 \times L}{16E} + \frac{Sy^2 \times \pi d^2 L}{8E}$
	$= \frac{Sy^2 \pi d^2 L}{8E} \left(\frac{1}{2} + \frac{1}{8} \right) = \frac{5}{8} \times \frac{Sy^2 \pi d^2 L}{8E}$
	$\therefore$ Energy absorbing capacity of second rod is only 5/8 of the first rod.
	$\frac{\text{Volume of second rod}}{\text{Volume of first rod}} = \frac{\pi d^2 L \left(\frac{1}{8} + \frac{1}{2} \right)}{\pi d^2 L \left(\frac{1}{4} \right)} = 2.5$

	∴ Energy absorbing capacity per unit volume is only one fourth in the second rod. Therefore the members which are subjected to impact loads should have a uniform cross section throughout their length. Example is the design of connecting rod bolts where the cross-sectional area of unthreaded portion is reduced to make it equal to the core area of the threaded portion.

Prob. 1.18 : A tension member 25mm wide and 6mm thick supports an axial load equal to 'F' N. It is necessary to replace this member by one having a 16 mm hole at the centre. Assuming that the width of the replacement member is 50 mm determine the thickness so that the maximum stress will not exceed the original value.

Ref.	Calculations
	Stress in the first member $\sigma_1 = \frac{F}{25 \times 6} = \frac{F}{150}$ MPa
	Net area in second member $= (50 - 16)\,t = 34t$
	∴ Nominal stress, $\sigma_2 = \frac{F}{34t}$
F-I-6	$\frac{a}{w} = \frac{16}{50} = .32$, for $\frac{a}{w} = .32$, $K_t = 2.35$
	∴ Maximum stres σ_2 (max.) $= K_t \times \sigma_2 = \frac{2.35 \times F}{34\,t}$
	Actual stress will be, $\sigma = \sigma_2 \times K_t$
	To find K_t in second case : For σ_2 (max.) $= \sigma_1$
	$\frac{2.35 \times F}{34\,t} = \frac{F}{150}$ ∴ $t = 10.36$ mm.

Prob. 1.19 : *A shaft transmitting 40 kW at 900 rpm rests in a bearing as shown in the sketch and has attached to it a pinion 75 mm in pitch diameter. The shaft fillet radius r = 3mm. Assuming that the shaft is rigidly supported by the bearing, determine the following :*

(a) Maximum shear stress due to torsion and (b) Maximum bending stress in the shaft.

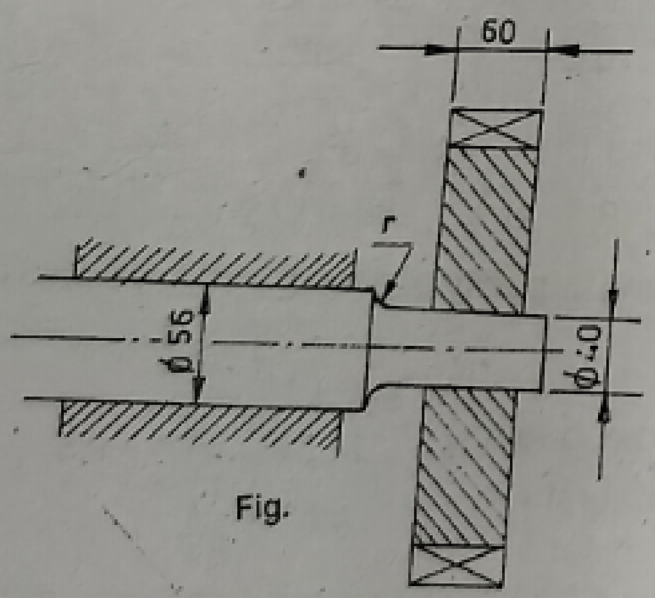


Fig.

Ref.	Calculations
	P = 40 kW at 900 rpm
	∴ Torque $T = \frac{40 \times 10^3 \times 60}{2\pi \times 900} = 424.6\ \text{N-m}$
	Tangential force $F_t = \frac{T}{r} = \frac{424.6 \times 10^3}{37.5} = 11.3$ kN
T-I-12	$M = F_t \times L = 11.3 \times 33 = 372.9$ N-m.
T-I-4	∴ Nominal shear stress $\tau = \frac{16T}{\pi d^3} = \frac{16 \times 424.6 \times 10^3}{\pi \times 40^3} = 33.8$ MPa.
	Nominal bending stress $\sigma_b = \frac{32M}{\pi d^3} = \frac{32 \times 372.9 \times 10^3}{\pi \times 40^3} = 59.4$ MPa
F-1.1	K_t for bending : $\frac{r}{d} = \frac{3}{40} = 0.75$ and $\frac{D}{d} = \frac{56}{40} = 1.4$, $K_t = 1.75$
F-1.2	K_{ts} for torsion : $\frac{r}{d} = 0.75$ and $\frac{D}{d} = 1.4$, $K_{ts} = 1.52$
	∴ Maximum shear stress $\tau_{(max.)} = 33.8 \times 1.52 = 51.4$ MPa
	Maximum bending stress $\sigma_{b(max.)} = 59.4 \times 1.75 = 103.95$ MPa.

Prob. 1.20 : Two mating steel spur gears are 20 mm wide and the tooth profiles have radii of curvature at the line of contact of 10 mm and 15 mm. A force of 200 N is transmitted between them. Compute the maximum contact pressure and the width of contact.

Ref.	Calculations
	This is case of cylinder on cylinder.
	$1 = 20$ mm, $R_1 = 10$mm, $R_2 = 15$mm,
	$E_1 = E_2 = 200 \times 10^3$ MPa.
T-I-10	∴ $a = 1.076 \left[\frac{F\left(\frac{1}{E_1} + \frac{1}{E_2}\right)}{L\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \right]^{\frac{1}{2}} = 1.076 \left[\frac{200}{20} \times \frac{\left(\frac{2}{200 \times 10^3}\right)}{\left(\frac{1}{10} + \frac{1}{15}\right)} \right]^{\frac{1}{2}}$
	$= .026$ mm

	$\therefore$ Width of contact = 2a = .052 mm $\sigma_c\,(\max) = \frac{2F}{\pi\, a\, L} = \frac{2 \times 200}{\pi \times .026 \times 20} = 245$ MPa

Prob. 1.21 : The ball and socket point at the end of a rocker arm involves a hardened steel spherical surface of 10mm dia fitting in a phosphor bronze bearing spherical seat of 10.1 mm diameter. What maximum contact stress will result from a load of 1800 N ?

Ref.	**Calculations**
	This is a case of sphere in spherical cavity. $\therefore R_1 = \frac{10}{2} = 5\text{mm},\ R_2 = \frac{10.1}{2} = 5.05$ mm. R_2 should be given a negative sign.
T-II-2	For steel E = 200×10^3 MPa, $v_1 = 0.3$ For phosp. bronze E = 113×10^3 MPa, $v_2 = 0.35$ $a = \left[\frac{3F}{4} \frac{\left(\frac{1-v_1^2}{E_1}\right) + \left(\frac{1-v_2^2}{E_2}\right)}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} \right]$ $= \left[\frac{3 \times 1800}{4} \frac{\left(\frac{1-.09}{200 \times 10^3}\right) + \left(\frac{1-.1225}{113 \times 10^3}\right)}{0.2 - .198} \right]^{\frac{1}{3}}$ = 2.02 mm. $\sigma_c(\max.) = \frac{3F}{2\pi\, a^2} = \frac{1.5 \times 1800}{\pi \times 2.02^2} = 210.7$ MPa.

* * *

Chapter 2

STRAIN & DEFLECTION ANALYSIS

2.1 Introduction :

Elastic strain, deflection, stiffness and stability are considerations of basic importance to the engineer. Deflection or stiffness, rather than stress, is often the controlling factor in the design of a part. Deflection analysis enters into design situations in many ways. Machine tool frames for example, must be extremely rigid to maintain manufacturing accuracy. In a transmission, the gears must be supported by a rigid shaft. If it bends too much, the teeth will not mesh properly. When the parts are made rigid enough to satisfy rigidity requirements, the stresses may be quite low. Parts may require great stiffness in order to eliminate vibration problems.

Another important aspect of elastic strains is their involvement in experimental techniques for measuring stresses. Stress is not, in general, a measurable quantity, strain is. When the elastic constants of a material are known, experimentally determined strain values can be transposed into corresponding stress values by means of the elastic stress-strain relationships.

Other major problem of importance in design of members subjected to compressive loads is elastic stability. For elastically stable system, a small disturbance of the equilibrium results in corrective elastic restoring forces and / or moments. Such may not be the case in the slender compression member called column. If the column is slender enough, the elastic modulus low, then the column will be elastically unstable and is likely to buckle and collapse. Hence the columns are to be designed for elastic stability.

2.2 Stress-Strain Relationships & Elastic Constants :

Whenever a part is loaded, a stress is induced and there is a change in the dimensions of the part which is called deformation. The ratio of deformation or change in dimension to the original value of dimension is called strain. Generally it is denoted by ε Thus

$$\varepsilon = \frac{\Delta L}{L}$$

The strain in the direction parallel to the force is called linear strain while that in a direction perpendicular to the force is called lateral strain. Within elastic limits the ratio of lateral to linear strains is constant for a given material and is called Poisson's Ratio, μ . Upto certain limiting stress called the elastic limit, the stress-strain relationship is nearly linear for most of the materials and the ratio is called the modulus of elasticity, E and is given by

$$E = \frac{\text{stress}}{\text{strain}}$$

Generally we use the symbol E for normal stresses and the name Young's modulus is used for this quantity. When the loading causes shear stress it is called Transverse modulus of elasticity or Modulus of rigidity and the symbol G is used for this. The relationship between stress and strain for the common type of loading are given in (T-I-4) of data book.

Every linear strain is associated with lateral strain which results in the change of volume of the part. The ratio of the change of volume to the original volume is called volumetric strain, ε_v

$$\varepsilon_V = \frac{dV}{V}$$

When a body is subjected to three mutually perpendicular stresses of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as Bulk Modulus and denoted by K.

$$K = \frac{\sigma}{\varepsilon_V}$$

All the modulii are related to each other through the Poisson's ratio μ. The relationships are as follows

$$G = \frac{E}{2(1+\mu)}; \quad K = \frac{E}{3(1-2\mu)}$$

2.3 Spring Rates :

Elasticity is that property of a material which enables it to regain its original configuration after having been deformed. If we designate the general relationship between force and deflection by the equation.

$$F = F(y)$$

Then spring rate, K is defined as

$$K(y) \lim_{\Delta y - 0} \frac{\Delta F}{\Delta y} = \frac{dF}{dy}$$

Where y must be measured in the direction and at the point of application of F. The three commonly occurring conditions are given below.

(a) Tension or Compression : For a bar of length L and cross section area A subjected to an axial load F the total deformation is given by

$$\delta = \frac{FL}{AE}$$

$$\text{spring rate } K = \frac{F}{\delta} = \frac{AE}{L}$$

(b) Bending Loads : For beams subjected to different loads with various support conditions, the deflection can be found out by the basic relation.

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \qquad \therefore \frac{dy}{dx} = \theta = \int \frac{M}{EI} \text{ and } y = \iint \frac{M}{EI}$$

There are various methods used for finding y. They are given in (T-I-2) of data book. Other methods will be discussed in the next article.

The spring rate for the beams will be given by

$$K = \frac{F}{y}$$

(c) Torsional Load : When a member of length 'L' is subjected to torque, T, the member experienced an angular deflection θ. For a bar the relation is (T-I-4).

$$\theta = \frac{TL}{GI_p} \quad \text{where } I_p = \text{Polar moment of inertia.}$$

Therefore the torsional spring rate is given by

$$K_t = \frac{T}{\theta} = \frac{GI_p}{L}$$

In many applications the springs are combined. To get the equivalent spring rate, we have two types of combination :

1. Series Combination : This is shown schematically in Fig. (2.1). The equivalent spring rate is given by $\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$

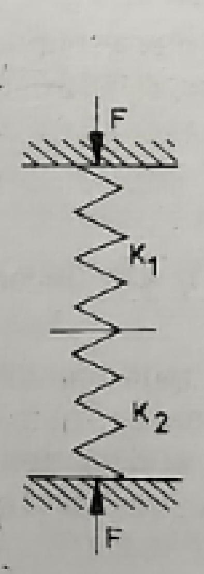


Fig. 2.1 Series combination

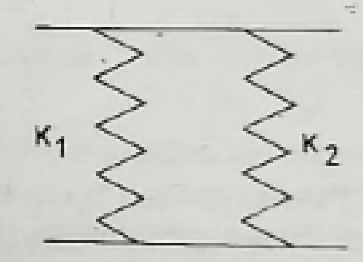


Fig. 2.2 Parallel combination

2. Parallel combination : This is shown schematically in Fig. (2.2)

The equivalent spring rate, Ke is given by $K_e = K_1 + K_2$.

2.4 Shear Force & Bending Moment Diagrams For Beams :

Beams are structural members subjected to transverse loads as discussed in chapter 1. Examples include machinery shafts, leaf springs etc. The types of beams based on different criteria of classification are as follows :

(a) **Statically Determinate :** The beams in which the external forces and moments that support the beam can be found by the equations of statics alone. These beams are further subdivided on the type of supports. These are of three types. (Fig. 2.3)

i) Simple Beam : The beam that rests on two supports in a manner which offers at most one restraint along the beam axis.

ii) Cantilever Beam : The beam held at one end only by a support which is capable of sustaining both moments and forces.

iii) Overhanging Beams : The beam is one that extends beyond its supports at either or both ends.

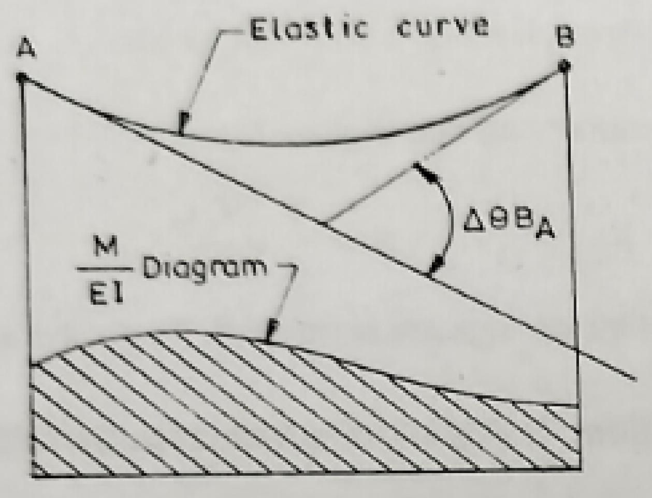


Fig. 2.3 Type of supports and loads

(b) Statically Indeterminate : The beams that have more supports than are necessary to maintain equilibrium. The examples are propped, continually supported or built in beams.

Majority of beams used in machines are statically determinate. Hence we will limit our discussion for this type only. The beams are generally subjected to four types of loads. (Fig. 2.3).

i) Concentrated load, ii) Uniformly distributed load, iii) Non-uniformly distributed load and iv) Pure moment.

Stresses and deflections in beams are functions of support reactions, external forces and moments and for this reason it is convenient to 'map' these quantities and form diagrams that give a complete picture of the magnitudes and directions of the forces and moments that act throughout the beam. These graphs are called *shear and moment diagrams.* A uniform and consistent sign convection must be used in plotting these diagrams which is as follows : (Fig. 2.4).

External forces and reactions that act upward on the left hand freebody diagram cause positive shear.

The moments caused by forces and moments that tend to bend the beam in a way to cause its radius of curvature to be upward are treated as positive moments.

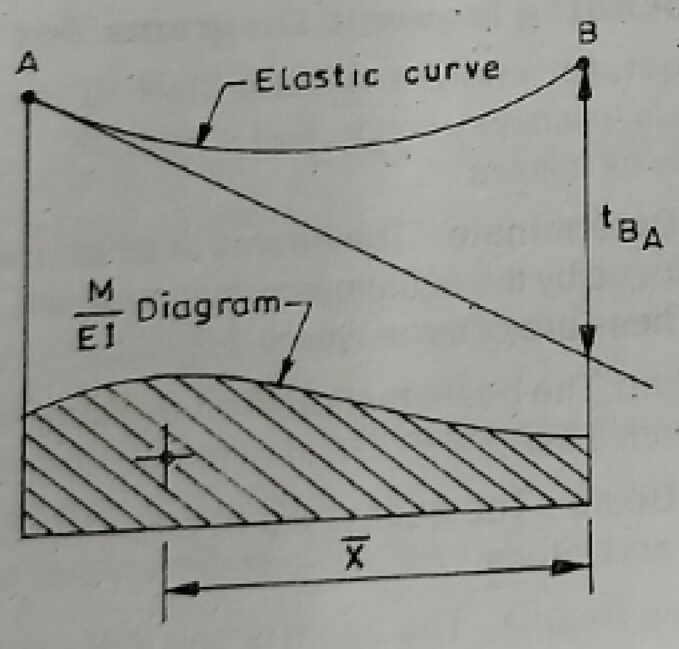


Fig. 2.4 Sign convection

In plotting these diagrams we first determine the reactions by applying the equations of statics. After determining the reactions we can use the following three methods for plotting the S.F. and B.M. diagrams.

1. Method of sections : In this method we take various sections along the beam and write down the equations for S.F. and B.M. for various sections. The number of sections depend on the change in loading along the beam. After determining the magnitudes of S.F. and B.M. at various change points, we get the diagrams by joining various points in the shape given by the equations. The equations by this method are given in T-I-2 of databook.

2. In this method, the relations existing between the loads, shear forces, and bending moments in any beam provide a method of constructing S.F. and B.M. diagrams without writing shear and moment equations. This method is faster if we know the shapes of diagrams for various types of loading. We use the following two relationships in this method.

i) Change in shear force = Area under the loading diagram

$$V_2 - V_1 = \Delta V = (\text{Area})_{1-2} \text{ of loading diagram}$$

ii) Change in bending moment = Area under the S.F.D.

$$M_2 - M_1 = \Delta M = (\text{Area})_{1-2} \text{ of S.F.D.}$$

3. In this method the S.F.D. is constructed by method 2 but the B.M.D. is constructed by cantilever parts. The beam is supposed to be clamped at any point along its length and all the loads including reactions are supposed to act on cantilever clamped at reference point. The resultant B.M.D. is plotted by adding the individual values at various points.

However we will use method 2 in solved problems as that is the fastest.

2.5 Beam Deflections :

The application of various loads to the beams induces stresses in beams as shown in chapter 1 and also causes deflection from its initial position. The amount of deflection depends upon its cross section and bending moment. In many applications we have to limit the deflection of the beam at various points. Therefore it is necessary to determine the deflection of beams for various types of loading. We will therefore discuss the methods used for determining the beam deflections. There are four methods as discussed below :

1. **Method of superposition :** This method states that the deflection at any point on the beam is the sum of the individual deflections produced at that point by each of the loads acting singly. The equations for deflection pertaining to beams of uniform cross section with commonly encountered loads are given in (T – I – 2) of data book.

2. **Macaulay's method :** This method is based on double integration of the relationship.

$$EI \frac{d^2y}{dx^2} = M$$

So a single equation is written for the bending moment for all loadings on a beam, the equation is constructed in such a way that the constants of integration apply to all portions of the beam. This method is convinient for determining the deflection of a beam subjected to point loads. But it can be used for distributed loads also with some ingenuity as shown in solved problems. This method can be only used for beams of uniform cross section.

3. **Area moment method :** For various reasons, many beams do not have a uniform cross section. For example, rotating machine shafts are usually stepped in order to accomodate and locate the bearings and other associated parts. The calculation of deflections in such cases is done by either the area moment method or numerical integration method. The area moment method will now be discussed.

This method is based on two theorems which relate the elastic curve to the $\left(\frac{M}{EI}\right)$ diagram. These theorems are given below.

Theorem 1 : It states that the change in angle measured in radians between the two tangents at any two points A and B on the elastic curve is equal to the $\left(\frac{M}{EI}\right)$ area bounded by the ordinate through A and B. In performing this summation the areas are associated with signs of the moments. If the sum is positive, the tangent on the right rotates in the anticlockwise direction, if negative it rotates in a clock wise direction, if zero, the tangents are parallel Fig. (2.3 (a)).

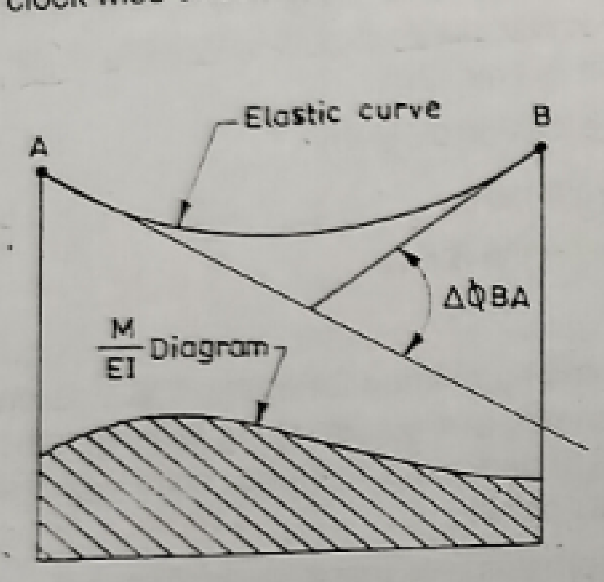


Fig. 2.3 (a)

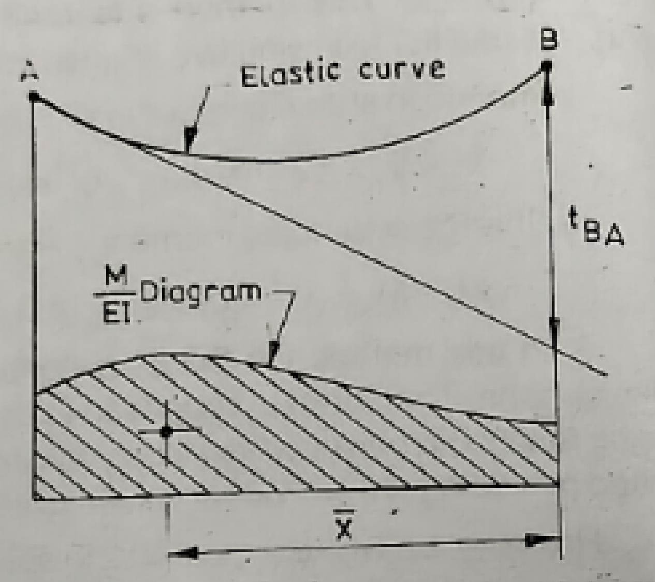


Fig. 2.4 (a)

Theorem 2 : It states that the tangential deviation of a point B on the elastic curve from a tangent through another point A also on the elastic curve is equal to the moment of the bounded section of the $\left(\frac{M}{EI}\right)$ diagram around a vertical line through B. It should be remembered that in most cases, the tangential deviation is not in itself the desired deflection of a beam. The moment will have the sign of the bending moment. A positive value for tangential deviation indicates that a given point lies above a tangent to the elastic curve drawn through the other point Fig. (2.4 (a)).

The sequence of steps for obtaining the deflection of beams by the area moment method is definite and uniform and is given below :

1. Make a good freebody diagram of the beam showing the loading and reaction at the supports.
2. Directly below the beam, construct the bending moment diagram.
3. Below the B.M.D. construct the $\left(\frac{M}{EI}\right)$ diagram.
4. Below the $\left(\frac{M}{EI}\right)$ diagram sketch a definite elastic curve which exaggerates the deflections. The curvature of the elastic curve must conform to the sign of the $\left(\frac{M}{EI}\right)$ diagram. If this diagram is positive the elastic curve is concave upwards and vice versa. No abrupt changes in the direction of the tangents to the curve are permissible. Moreover the elastic curve so drawn must pass over the supports and for a fixed end, its must also be tangent to the intiial direction of the unloaded axis of the beam.

5. On the sketch of the elastic curve, indicate the desired displacement of the beam.
6. Formulate algebraically the quantity sought in (5) in terms of the geometrical conditions of the elastic curve and tangential deviations.
7. Clearly indicate the tangential deviations required in (6) on the elastic curve prepared in (4).
8. By the moment area theorem compute the required deviations. The areas of the $\left(\frac{M}{EI}\right)$ diagram must be considered with the algebraic sign.
9. Solve the relation formulated in (6) for the required deflection.

4. The theorem of Castigliano : This theorem states that when forces operate on elastic systems, the displacement corresponding to any force may be found by obtaining the partial derivative of the total strain energy with respect to that force. The terms force and displacement should be boradly interpreted, since they apply equally to moments and to angular displacements. Mathematically the theorem states :

$$\delta_i = \frac{\partial u}{\partial F_i}$$

Where δ_i is the displacement of the point of application of the ith force F_i in the direction of F_i.

Sometimes the deflection of a structure is required at a point where no force or moment is acting. In this case we can place an imaginary force Q_i at the point, develop the expression for δ_i and then set Q_i equal to zero.

This theorem can also be used to determine the reactions in indeterminate structures. The deflection is zero at these reactions and so we merely solve the equation.

$\frac{\partial u}{\partial R_i} = 0$ to obtain the reaction R_i.

The strain-energies for the commonly occuring cases are given below :

Axial load	$U = \frac{F^2 L}{2EA}$
Bending	$U = \frac{M^2 L}{2EI}$
Torsion	$U = \frac{T^2 L}{2GJ}$

In all these relations E, A, F, M, T, G and J are constant throughout.

2.6 Columns :

A short bar loaded in pure compression by a force F acting along the centroidal axis will shorten, in accordance with Hooke's law, until the stress reaches the elastic limit of the material. If F is increased still more the material bulges and is either squeezed into a flat disc or fractures. Instead of short bar, if we take a long thin straight bar such as a yard stick and apply a compressive load F, the member will shorten according to the Hook's law. However if a member is sufficiently long and F sufficiently large, the bar is under unstable equilibrium. At this point any little crookedness of the member or slight movement of the load, or support will cause the member to collapse by buckling.

If the compression member is long enough to fail by buckling, it is called a *column*. For any column the load at which the unstable equilibrium begins is called the critical load, F_{cr} and the designer tries to keep the $F_{cr} = F \times$ Factor of safety, where F is the operating load. The critical load is influenced by the following factors.

1. Yield strength of the material, Sy
2. Modulus of elasticity, E
3. The minimum radius of gyration, K
4. The end conditions (amount of fixity)

The ratio of length to the radius of gyration $\frac{1}{k}$ is called slenderness ratio. Formulae for finding the critical load F_{cr} are given in (T-I-24) of data book. Accordingly the columns are classified into two types.

1. Euler Column when $\frac{1}{k}$ is greater than 90. In these columns the effect of Sy is negligible. These are also called long columns.

2. Johnson or Rankine Column when $\frac{1}{k}$ lies between 10 & 90. Here the F_{cr} is affected by Sy. These are called short columns.

Regarding the end conditions, it must be realized that, in practice we can never get the ends completely fixed. Hence the value of constant 'n' should always be selected between 1 and 4. However the value 0.25 should always be used for one end fixed and one end free. The use of these formulae will be illustrated with the help of solved problems. In determining the radius of gyration, it is necessary to consider the plane in which failure occurs. The plane of failure will be that for which the products (nl) is the least.

2.7 Beam Columns :

Members required to with-stand axial loads and bending loads simultaneously are called *beam columns.* Examples of beam columns are an eccentrically loaded strut shown in Fig. (2.5).

For eccentrically loaded strut shown in the Fig. it is apparent that the moments at the ends, F × e, will cause transverse deflection of the strut. The axial compressive load F will then have a line of action eccentric to the strut by an amount δ at the centre. This force and distance in turn induce an additional bending moment on the beam. The magnitude of the moment therefore depends on the deflection of the structure, which in turn is related to the loading. This inter relation makes the determination of the moment difficult. Tabulations of some of these bending moments are available. One case of particular interest is the eccentrically loaded strut shown in the Fig. (2.5).

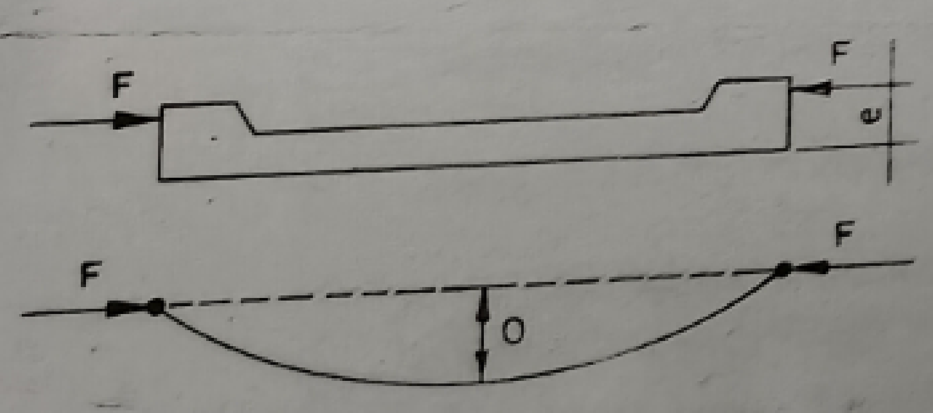


Fig. 2.5

For this strut, the moment is given by

$$M = F.\,e.\,\sec.\ \frac{1}{2K}\sqrt{\frac{F}{EA}}$$

Where F = axial compressive load.

e = eccentricity of the load from centroid of section.

L = Equivalent length of strut (considering end conditions).

K = radius of gyration of cross section.

A = area of cross section.

E = Modulus of elasticity of the material.

The stress at the centre of the beam will be

$$\sigma = \frac{F}{A} + \frac{M}{Z} \quad \text{Where} \quad Z = \text{section modulus.}$$

The value of this stress can be calculated with the help of formula given in (T-I-24).

Alternately we can combine the two stresses :

1. due to column action and
2. due to beam, to give the resultant stresses as :

$$\sigma = \frac{F}{A}\left[1 + \frac{1}{an}\left(\frac{L}{K}\right)^2\right] + \frac{F.\,e.\,C}{I}$$

This approach is simpler and easier to comprehend.

SOLVED PROBLEMS

Prob. 2.1 : A part is subjected to loads as shown in Fig. Calculate the force P_3 for equilibrium if $P_1 = 120$ kN, $P_2 = 220$ kN and $P_4 = 160$ kN. Also determine the net change in length of the member. Assume E = 200 GPa.

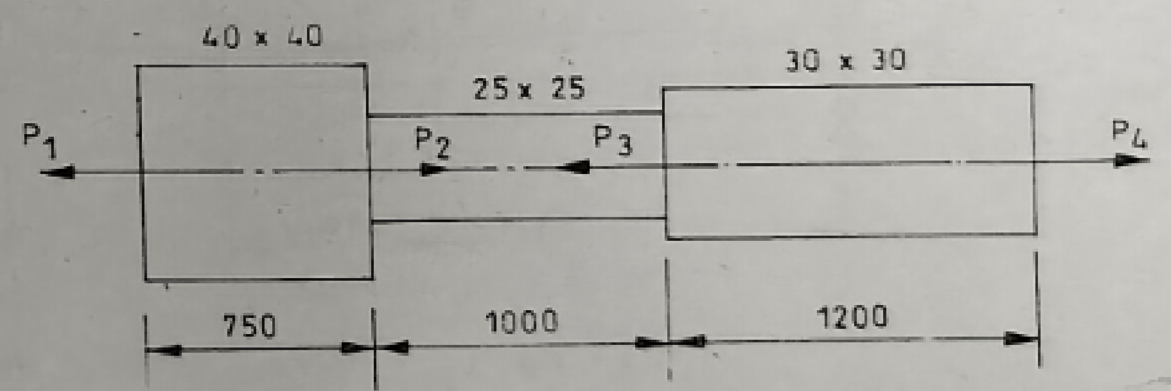
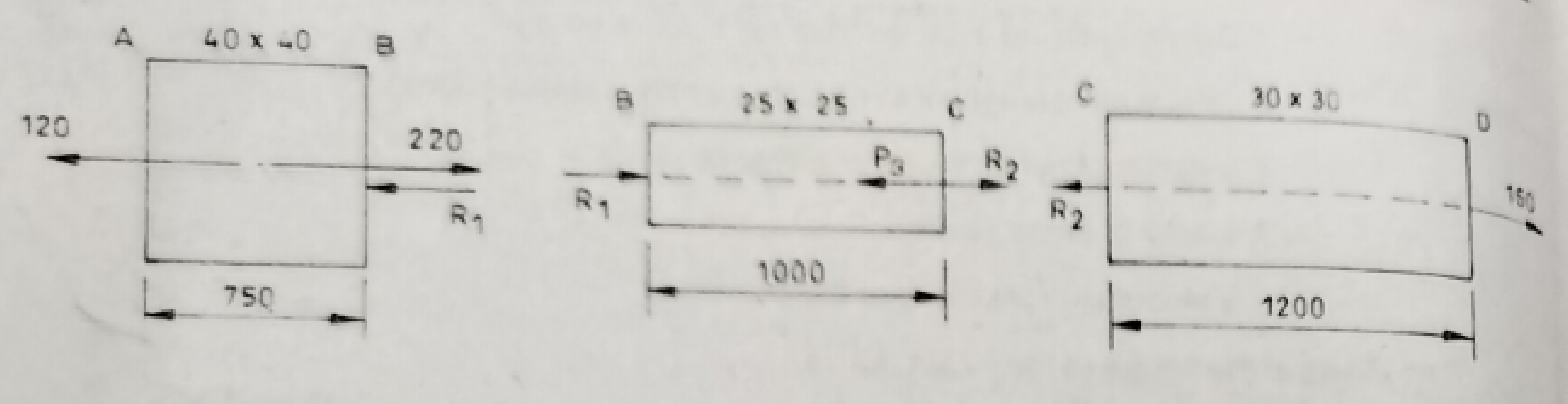


Fig.

Solution : First draw the FBD of the three parts as shown below :



$\Sigma F : 220 - 120 - R_1 = 0$ $\qquad$ $R_2 - P_3 + R_1 = 0$ $\qquad$ $160 - R_2 = 0$

$\therefore R_1 = 100$ $\qquad$ $P_3 = R_2 + 100$ $\qquad$ $R_2 = 160$

$\therefore P_3 = 260$ kN

Check : $- 120 + 220 - 260 + 160 = 0$ Do o.k.

Now to find deformations :

$$\Delta L_{AB} = \frac{120 \times 10^3}{40 \times 40 \times 200 \times 10^3} = .28 \text{ mm (elong)}$$

$$\Delta L_{BC} = -\frac{100 \times 10^3}{25 \times 25 \times 200 \times 10^3} = -0.8 \text{ mm (contr.)}$$

$$\Delta L_{CD} = \frac{160 \times 10^3}{30 \times 30 \times 200 \times 10^3} = 1.066 \text{ mm (elong)}$$

$\therefore$ Net change in length : $.28 - 0.8 + 1.066 = .5466$ mm

Prob. 2.2 : A horizontal bar ABC is fixed between two walls. The right side wall yields by 0.5 mm. Determine the stresses in part AB and BC ; reaction at the walls and displacement of point B. $A_{AB} = 100\ \text{mm}^2$, $A_{BC} = 200\ \text{mm}^2$, $E_{AB} = 200$ GPa, $E_{BC} = 80$ GPa.

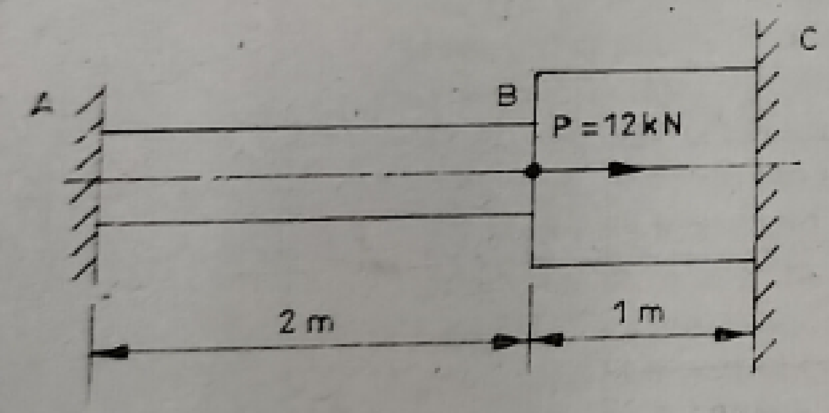


Fig.

Solution : We have FBD's of AB and BC.

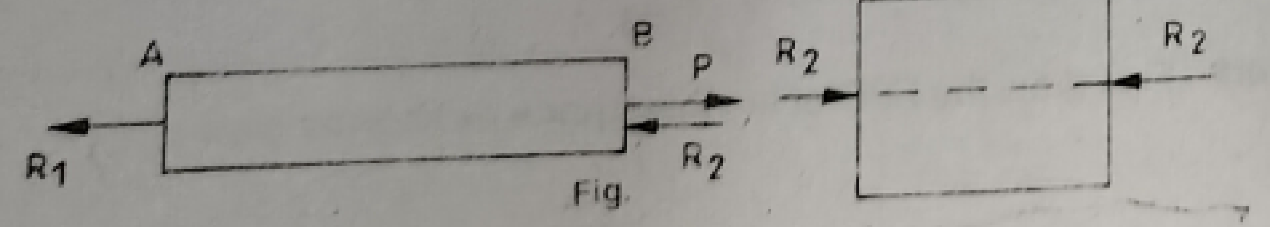


Fig.

$\therefore \Sigma F : P - R_1 - R_2 = 0$

or $\quad R_1 + R_2 = P$ $\qquad$ ... i)

Now we find the deformations :

$$\Delta L_{AB} = \frac{R_1 \times 2 \times 10^3}{100 \times 200 \times 10^3} = R_1 \times 10^{-4} \text{ (elong)}$$

$$\Delta L_{BC} = -\frac{R_2 \times 1 \times 10^3}{200 \times 80 \times 10^3} = -\frac{R_2}{16} \times 10^{-3} \text{ (comp)}$$

If the walls were rigid, $\Delta L_{AB} + \Delta L_{BC} = 0$ But the right wall yields by 0.5 mm.

$\therefore \Delta L_{BC} + \Delta L_{AB} = 0.5$

$-\frac{R_2}{16} \times 10^{-3} + R_1 \times 10^{-4} = 0.5$ $\quad$ or $\quad$ $R_2 = 1.6\,R_1 - 8 \times 10^3$

From (i) : $R_1 + 1.6\,R_1 - 8 \times 10^3 = 12 \times 10^3$

$\therefore R_1 = 7.7 \times 10^3$ N

$R_2 = 4.3 \times 10^3$ N

$\therefore \Delta L_{AB} = 7.7 \times 10^3 \times 10^{-4} = .77$ mm. This is displacement of point B.

$$\sigma_{AB} = \frac{7.7 \times 10^3}{100} = 77 \text{ MPa} \quad ; \quad \sigma_{BC} = \frac{4.3 \times 10^3}{200} = 21.5 \text{ MPa}$$

Prob. 2.3 : A rigid bar of negligible mass, pinned at one end is supported by a steel rod and bronze rod as shown in fig. What max load P can be applied without exceeding a stress of 120 MPa in steel and 70 MPa in bronze.

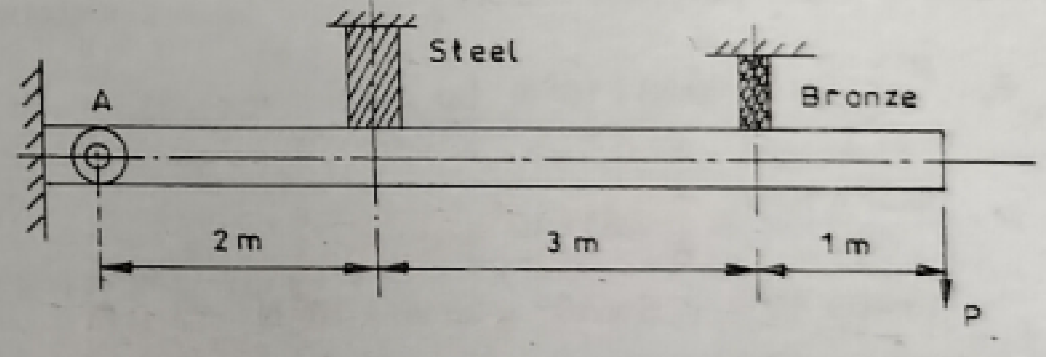


Fig.

Solution : The data for the two rods is as follows :

Item	Steel	Bronze
C.s. area, A, mm²	900	300
E, Gpa	200	83
L, m	3	2

Due to P, there will be tensile forces on the two bars and elongations. The elongations will be proportional to the distances from the pinned end as shown in the Fig.

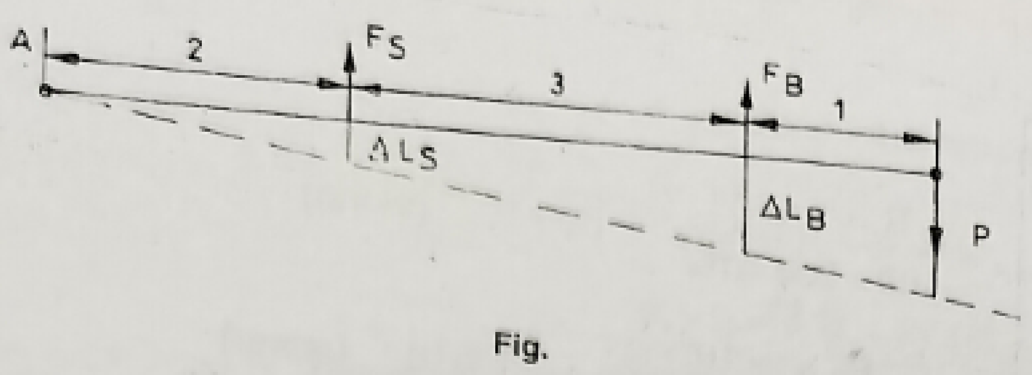


Fig.

$\therefore$ From similar Δ^s : $\frac{\Delta L_s}{2} = \frac{\Delta L_B}{5}$ Or $\Delta L_B = 2.5\,\Delta L_s$ ... 1)

Now taking moments about A :

$$+\circlearrowright \Sigma M_A : -F_s \times 2 - F_B \times 5 + P \times 6 = 0$$

$$\frac{F_s}{3} + \frac{5F_B}{6} = P \qquad \text{... 2)}$$

Now the deformations in the two rods :

$$\Delta L_S = \frac{F_s \times 3 \times 10^3}{900 \times 200 \times 10^3} = 1.66 \times 10^{-5}\,F_s$$

$$\Delta L_B = \frac{F_B \times 2 \times 10^3}{300 \times 83 \times 10^3} = 8 \times 10^{-5}\,F_B$$

From (1) : $8 \times 10^{-5}\,F_B = 2.5 \times 1.66 \times 10^{-5}\,F_s$

$$F_B = .52\,F_s \qquad \text{... 3)}$$

Max. value of $F_s = 900 \times 120 = 108 \times 10^3$ N

$F_B = 300 \times 70 = 21 \times 10^3$ N

Therefore we have to take the limiting value of F_B.

From (3) : $F_s = \frac{21 \times 10^3}{.52} = 40.38 \times 10^3$ N So o.k.

From (2) : $P = \frac{40.38 \times 10^3}{3} + \frac{5}{6} \times 21 \times 10^3$

$= 13.46 \times 10^3 + 17.5 \times 10^3 = 30.96 \times 10^3$ N

Prob. 2.4 : A steel bar 250 mm long is acted upon by a simple tensile load of 18 kN. The diameter of the bar is 8mm for the first 100mm length and 6mm for the remaining portion. Find the spring rate and the elongation. Take E = 200 GPa. Also find the strain energy stored in the bar.

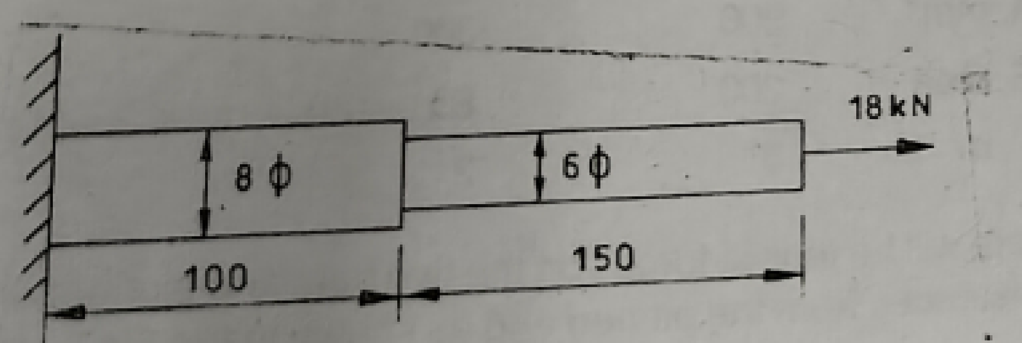


Fig.

Ref.	Calculations
T-I-14	As the area and lengths of the two portions are different, they will have different spring rates or constants. The combination of the two springs is a series combination.
	Spring rate $K = \frac{AE}{L}$
	$A_1 = \frac{\pi}{4} \times 8^2 = 50.24\ \text{mm}^2$ $L_1 = 100$ mm.
	$A_2 = \frac{\pi}{4} \times 6^2 = 28.26\ \text{mm}^2$ $L_2 = 150$ mm.
	$K_1 = \frac{A_1E}{L_1} = \frac{50.24 \times 200 \times 10^3}{100} = 100.48 \times 10^3$ N/mm
	$K_2 = \frac{A_2E}{L_2} = \frac{28.26 \times 200 \times 10^3}{150} = 37.68 \times 10^3$ N/mm
	$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} \qquad \therefore = \frac{1}{100.48 \times 10^3} + \frac{1}{37.68 \times 10^3} = .0365 \times 10^{-3}$
	$\therefore K_e = 27.4 \times 10^3$ N/mm
	$\therefore$ Total elongation $\delta = \frac{F}{K_e} = \frac{18 \times 10^3}{27.4 \times 10^3} = 0.66$ mm.
	To find strain energy
	$\delta_1 = \frac{F}{K_1} = \frac{18 \times 10^3}{100.48 \times 10^3} = .18$ mm. ; $\delta_2 = \frac{F}{K_2} = \frac{18 \times 10^3}{37.68 \times 10^3} = .48$ mm
	$\therefore$ strain energy $U = \frac{1}{2}K_1\delta_1^2 + \frac{1}{2}K_2\delta_2^2$
	$= \frac{1}{2} \times 100.48 \times 10^{-3} \times .18^2 + \frac{1}{2} \times 37.68 \times 10^3 \times .48^2$
	= 5.97 N-m or J.
	Alternately, strain energy $U = \frac{1}{2}K_e\,\delta^2 = \frac{1}{2} \times 27.4 \times 10^3 \times .66^2 = 5.97$ N – m or J.

Prob. 2.5 : A cantilever steel spring of rectangular cross section should have a spring rate of 25 kN/m. The length of the spring is 200 mm. The width should be four times the thickness. Suggest suitable dimensions. What is the maximum stress if the operating range is 6mm ? E = 200 GPa.

Ref.	Calculations
T-I-2	For a cantilever $y_{max} = \frac{FL^3}{3EI}$ or $K = \frac{F}{y} = \frac{3EI}{L^3} = \frac{25\ kN}{m}$ $= \frac{25N}{mm}$ $b = 4h$
T-I-1	$I = \frac{1}{12} bh^3 = \frac{h^4}{3}$ $\quad \therefore 25 = \frac{3 \times 200 \times 10^3 \times h^4}{3 \times 200^3}$ $h = 5.62$ mm. Assume, $h = 5.62$ mm & $b = 22.5$ mm. $\therefore$ For $y = 6$mm, $F = 25 \times 6 = 150$ N. $\therefore$ B.M. at the support $= 150 \times 200 = 30 \times 10^3$ N-mm.
T-I-4	$Z = \frac{bh^2}{6} = \frac{22.5 \times 5.62^2}{6} = 118.3\ mm^3$ $\sigma = \frac{M}{Z} = \frac{30 \times 10^3}{118.3} = 253.6$ MPa.

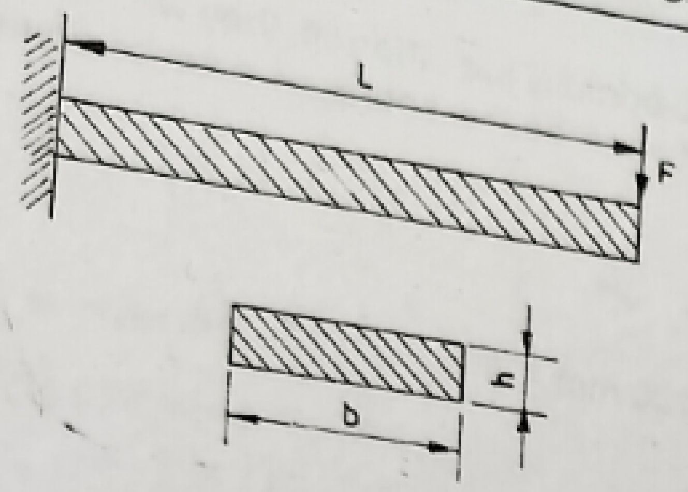


Fig.

Prob. 2.6 : A steel torsion spring is made from a round bar of steel 15 mm in diameter 2.5m long. The maximum torque applied to the spring is 300 N-m. Find the spring rate, deflectio degrees and the torsional stress. G = 80 GPa. Also find the strain energy.

Ref.	Calculations
T-I-4	For a round bar subjected to torque T
T-I-1	$\frac{T}{I_p} = \frac{G\theta}{L}$ or $K_t = \frac{T}{\theta} = \frac{GI_p}{L}$
	$I_p = \frac{\pi}{32} d^4 = \frac{\pi}{32} \times 15^4 = 4967.6\ mm^4$
	$\therefore$ spring rate $K_t = \frac{80 \times 10^3 \times 4967.6}{2.5 \times 10^3} = 159 \times 10^3 \frac{N-mm}{rad} = 159 \frac{N-m}{rad}$

	Angular deflection $\theta = \frac{T}{K_t} = \frac{300}{159} = 1.887\ rad = 108^0$
	$Z_p = \frac{\pi}{16} d^3 = \frac{\pi}{16} 15^3 = 662.3\ mm^3$; $\tau = \frac{T}{Z_p} = \frac{300 \times 10^3}{662.3} = 453$ MPa
	strain energy $U = \frac{1}{2} K_t \theta^2 = \frac{1}{2} \times 159 \times 1.887^2 = 283$ J
T-I-8	Alternatively, $U = R \times V_d = \frac{\tau^2}{4G} \times \frac{\pi}{d} d^2 \times L$ $= \frac{453^2}{4 \times 80 \times 10^3} \times \frac{\pi}{4} \times 15^2 \times 2.5 \times 10^3$ $= 283.16 \times 10^3$ N-mm $= 283.16$ J

Prob. 2.7 : Find the diameter of a hollow round steel shaft to transmit 15 kW at 60 Hz if the angular deflection is not to exceed 1° in a length of 30 OD. Compute the corresponding torsional stress. Assume that I.D. $= \frac{3}{4}$ O.D., G = 80 GPa

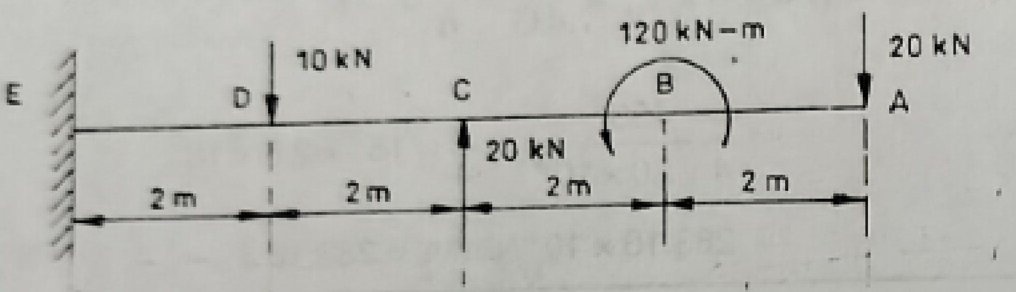
Ref.	Calculations
	Power = 15 kW at 60 Hz = 60 rev/sec. $\therefore$ Torque $= \frac{15 \times 10^3}{2\pi \times 60} = 39.8\ N-m.$
T-I-4	$\frac{T}{I_p} = \frac{G\theta}{L}$ or $\frac{\theta}{L} = \frac{T}{I_pG}$ Given $\theta = 1°$ in 30 D_o. $= \frac{\pi}{180}$ rad in 30 D_o or $\frac{\pi}{180 \times 30\ D_o} = \frac{39.8 \times 10^3}{.067\ D_o^4 \times 80 \times 10^3}$ $\quad I_p = \frac{\pi}{32}\left[D_o^4 - D_i^4\right] = .067\ D_o^4$ $D_o^3 = \frac{39.8 \times 5.4 \times 10^3}{\pi \times .067 \times 80}$; $D_o = 23.35$ mm, $D_i = 17.5$ mm. $Z_p = \frac{\pi}{16}\left[\frac{D_o^4 - D_i^4}{D_o}\right] = \frac{\pi D_o^3}{16}\left[t - \left(\frac{D_i}{D_o}\right)^4\right]$

$$= \frac{\pi \times 23.35^3}{16} \times .684 = 1708 \text{ mm}^3$$

$$\tau = \frac{T}{Z_p} - \frac{39.8 \times 10^3}{1708} = 23.3 \text{ MPa}$$

It is very low because the shaft is designed from the rigidity considerations.

Prob. 2.8 : Draw shear force and bending moment diagram for the cantilever beam shown in Fig.



Solution : We draw the S.F.D. and B.M.D. below the loading diagram by using the area method explained earlier.

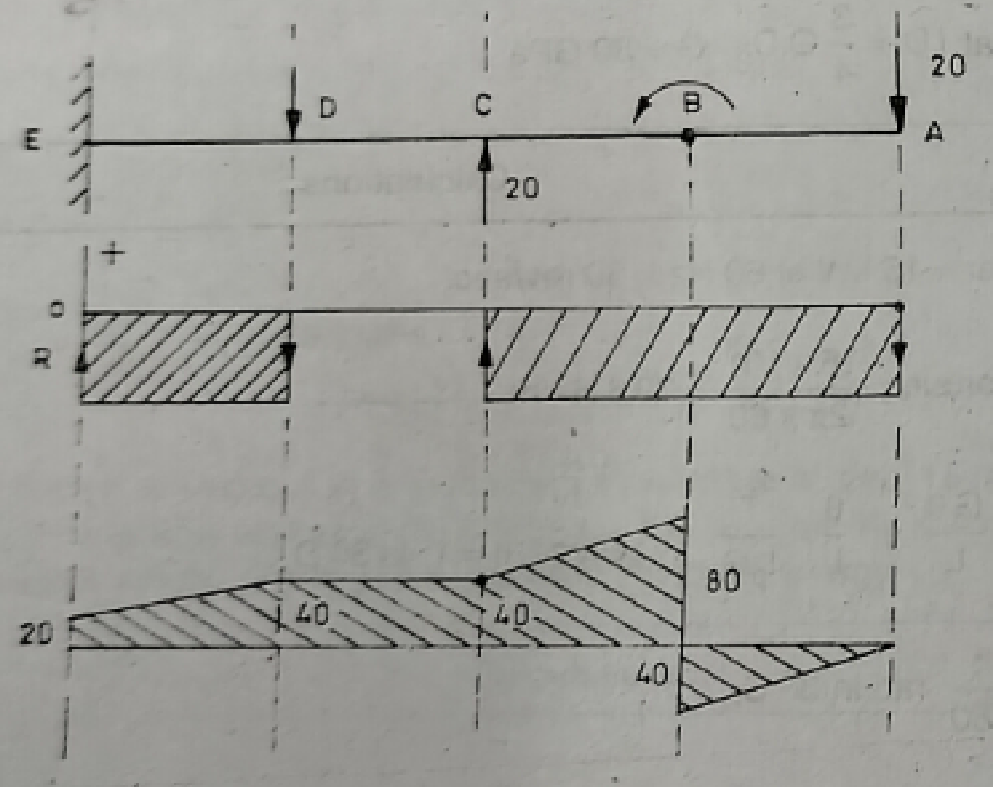


Fig.

As there are point loads, S.F.D. can be drawn by inspection.

$V_A = -20$ $\quad V_C = 20 - 20 = 0$

$V_D = -10$ $\quad V_E = -10$

$\therefore$ Support reaction will be, $R = +10$ KN

For B.M.D. : $\quad M_A = 0$

$$M_B - M_A = (ASFD)_{AB} = -20 \times 2 = -40$$

$$\therefore M_B = -40$$

After B, $\quad M_{B+} = -40 + 120 = 80$

$$M_C - M_{B+} = (ASFD)_{BC} = -20 \times 2 = -40$$

$$\therefore M_C = -40 + 80 = 40$$

$$M_D - M_C = (ASFD)_{CD} = 0$$

$$\therefore M_D = 40$$

$$M_E - M_D = (ASFD)_{DE} = -10 \times 2 = -20$$

$$M_E = -20 + 40 = 20$$

$\therefore$ Fixing moment at the support will be –20 kN-m.

Prob. 2.9 : A beam ABCDE is simply supported at B and E. The loading and dimensions are shown in Fig. Find the position and magnitude of max. bending moment.

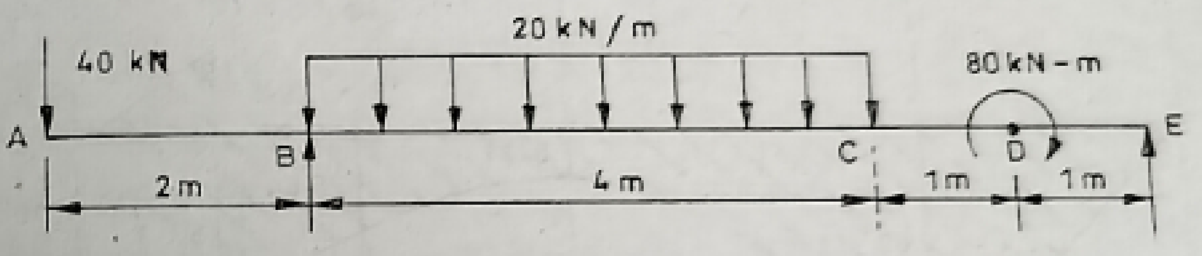


Fig.

Solution : To draw the S.F.D. and B.M.D. we have to first the reactions at B and E. We replace the UDL by its resulant acting at midpoint of BC.

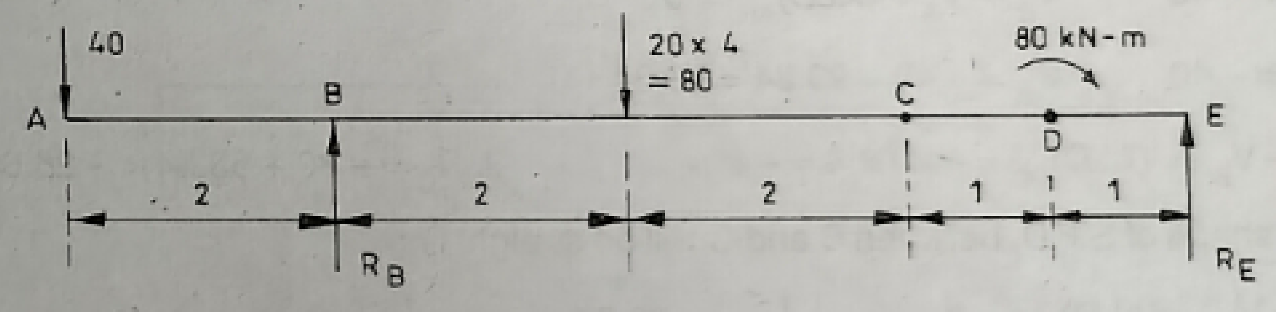


Fig.

$$+\circlearrowright \Sigma M_B : -40 \times 2 + 80 \times 2 + 80 - R_E \times 6 = 0$$

$$\therefore R_E = \frac{160}{6} = 26.66 \text{ kN}$$

$$+\circlearrowright \Sigma M_E : -48 \times 8 + R_B \times 6 - 80 \times 4 + 80 = 0$$

$$\therefore R_B = \frac{560}{6} = 93.34 \text{ KN}$$

Check : – 40 + 93.34 – 80 + 26.66 = 0 So o.k.

To draw the S.F.D. and B.M.D. we will use the area method.

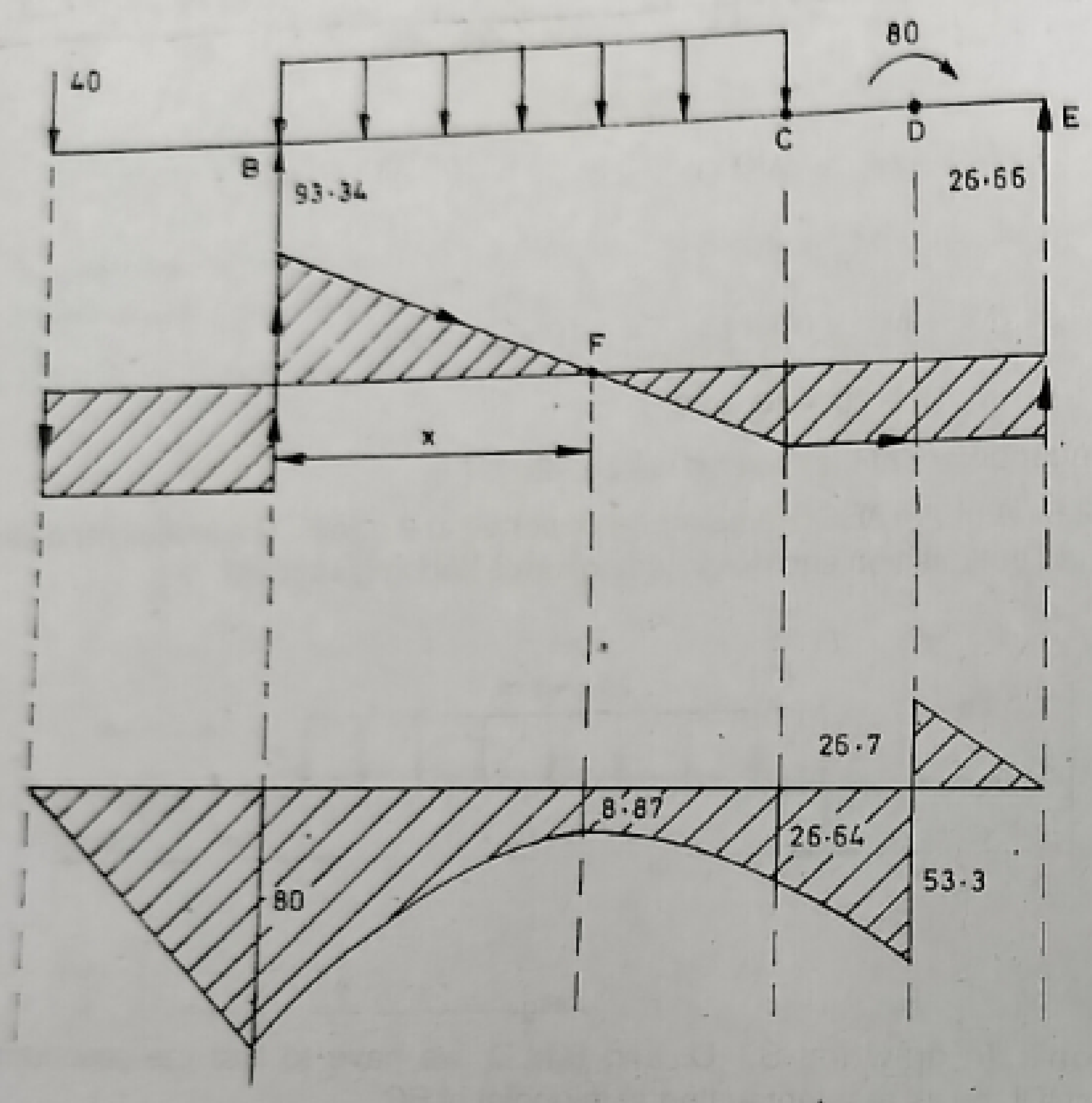


Fig.

$V_A = -40 \qquad V_B - V_A = (ALD)_{AB} = 0$

$V_B = -40 \qquad V_{Bt} = -40 + 93.34 = 53.34$

$V_C - V_{Bt} = (ALD)_{BC} = -20 \times 4 = -80 \qquad \therefore V_C = -80 + 53.34 = -26.66$

The shape of S.F.D. between B and C will be straight line.

$V_E - V_C = (ALD)_{CE} = 0 \qquad V_E = -26.66$

$V_{Et} = -26.66 + 26.66 = 0$

To find the position of zero S.F.

From similar Δ^s : $\frac{x}{53.34} = \frac{4-x}{26.66}$

$$x = \frac{8}{3} = 2.66 \text{ m}$$

To draw B.M.D. : $M_A = 0$

$M_B - M_A = (ASFD)_{AB} = -40 \times 2 = -80$

$M_B = -80$

It will be straight line from A to B,

$M_F - M_B = (ASFD)_{BF} = +\frac{1}{2} \times 53.34 \times 2.66$

$= 71.13$

$\therefore \ M_F = 71.13 - 80 = -8.87$

This will be a parabola.

$M_C - M_F = (ASFD)_{FC} = -\frac{1}{2} \times 26.66 \times 1.33 = -17.77$

$\therefore \ M_C = -17.77 - 8.87 = -26.64$

$M_D - M_C = (ASFD)_{CD} = -26.66 \times 1 = 26.66$

$\therefore \ M_D = -26.66 - 26.64 = -53.30$

This will be a straight line.

$M_{D+} = -53.30 + 80 = 26.70$

$M_E - M_{D+} = (ASFD)_{DE} = -26.66 \times 1 = -26.66$

$\therefore \ M_E = -26.66 + 26.70 \approx 0$

This will be a straight line.

The maximum B.M. is at B = – 80 kN-m.

Prob. 2.10 : The figure shows a member loaded in bending by the forces F_1 and F_2. The deflection at B must not be greater than 200 μm. Using the method of superposition, determine the diameter of a steel bar to support these loads. E = 200 GPa.

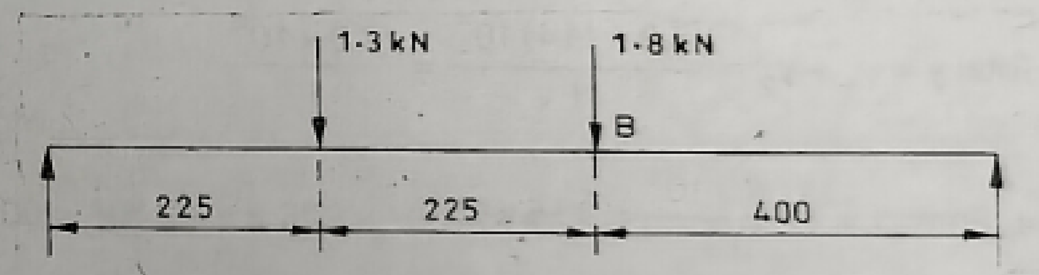
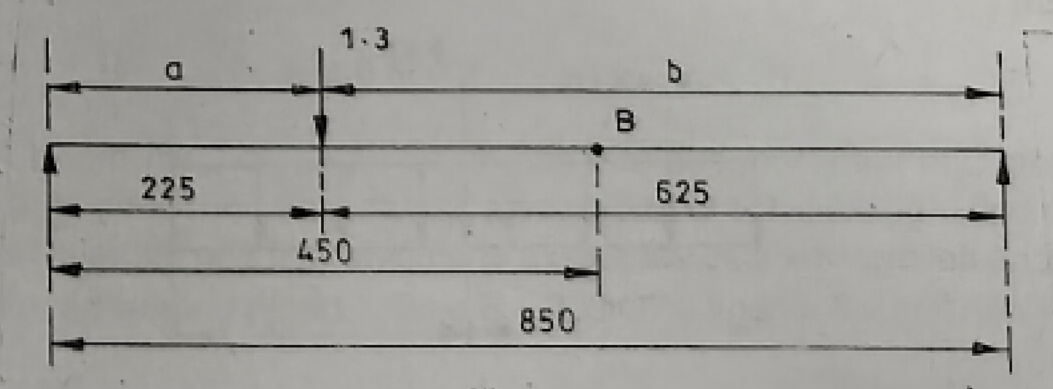


Fig.

Ref.	Calculations
	We have to find the deflection at B due to F_1 and F_2 separately and then add then. For F_1 only  Fig.

T-I-2

For simply supported beam with intermediate load

$$y_1 = \frac{Wa(L-x)}{6EIL}\left[2Lb - b^2 - (L-x)^2\right]$$

$$= \frac{1.3 \times 10^3 \times 225(850-450)}{6 \times 200 \times 10^3 \times I \times 850}\left[2 \times 850 \times 625 - 625^2 - (850-450)^2\right]$$

$$= \frac{5.9 \times 10^4}{I}$$

For F_2 only

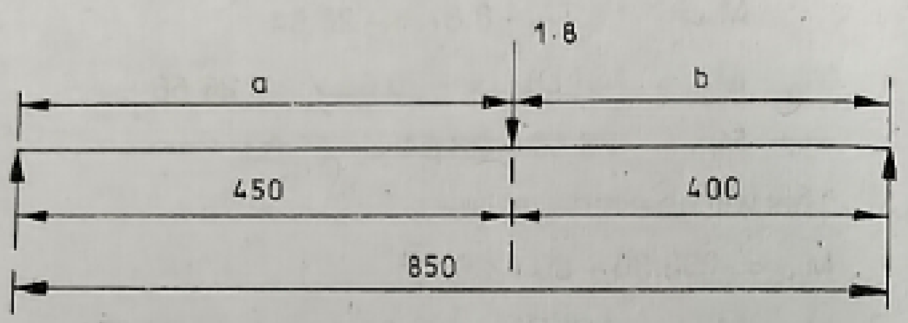


Fig.

$$y_2 = \frac{1.8 \times 10^3 \times 450 \times 400}{6 \times 200 \times 10^3 \times I \times 850}\left[2 \times 850 \times 400 - 400^2 - 400^2\right]$$

$$= \frac{11.4 \times 10^4}{I}$$

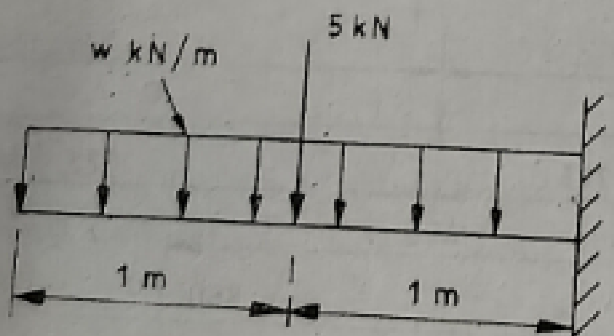
$$\therefore \text{ Total } y = y_1 + y_2 = \frac{(5.9 + 11.4)10^4}{I} = \frac{17.3 \times 10^4}{I}$$

if $y = .2\text{mm}$, $I = \frac{17.3 \times 10^4}{0.2} = 86.5 \times 10^4$

$$I = \frac{\pi}{64} d^4 = 86.5 \times 10^4 \text{ mm}^4 \quad ; \quad d = 64.8 \text{ mm.}$$

Prob. 2.11 : A cantilever beam of 2m length carries a U.D.L. of ω kN/m over its entire span and a concentrated load of 5 kN acting downwards at midspan. The beam is rectangular in section, width = 70 mm and depth = 120 mm. Modulus of elasticity, E = 200 GPa. Determine the maximum intensity of U.D.L. so that the deflection of free end does not exceed 10 mm.

Solution :



Using Macaulay's method :

$$EI\frac{d^2y}{dx^2} = -\frac{wx^2}{2}\Big| -5(x-1)$$

$$EI\frac{dy}{dx} = -\frac{w}{6}x^3 + C_1\Big| -\frac{5}{2}(x-1)^2$$

$$EIy = -\frac{w}{24}x^4 + C_1x + C_2 -\Big| -\frac{5}{6}(x-1)^3$$

The end condition are : $y = 0$ at $x = 2$, $y = \frac{10}{1000}$ m at $x = 0$

$\frac{dy}{dx} = 0$ at $x = 2$

$$\therefore 0 = -\frac{w \times 2^4}{24} + 2C_1 + C_2 - \frac{5(2-1)^2}{6} \qquad \text{... i)}$$

$$2C_1 + C_2 = \frac{5}{6} + \frac{2}{3}w$$

$$-EI \times .01 = C_2 \qquad \text{... ii)}$$

$$0 = -\frac{w \times 2^3}{6} + C_1 \frac{-5(2-1)}{2}$$

$$C_1 = \frac{5}{2} + \frac{4}{3}w \qquad \text{... iii)}$$

Given $E = 200 \times 10^3$ MPa $= 200 \times 10^6$ KN/m²

$$I = \frac{70 \times 120^3}{12}\text{ mm}^4 = 1.008 \times 10^{-5}\text{ m}^4$$

$$\therefore EI = 200 \times 10^6 \times 1.008 \times 10^{-5} = 2016 \text{ kN-m}^2$$

$\therefore$ From (ii) $C_2 = -2016 \times .01 = -20.16$

From (i) and (iii) : $2\left(\frac{5}{2} + \frac{4}{3}w\right) - 20.16 = \frac{5}{6} + \frac{2}{3}w$

$2w = 15.16 + \frac{5}{6} = 16 \qquad \therefore w = 8$ kN/m

Prob. 2.12 : A beam ABC of length 7.5 m has one support at left end other support at a distance of 2.5 m from right end. The beam carries a point load of 1.2 kN at right end and also carries a u.D.L. of 2.5 kN/m over a length of 2.5 m which is at a distance of 2.5 m from left end. Determine the slope and deflection at overhanging point C. Take E = 200 GPa and $I = 4 \times 10^8$ mm⁴

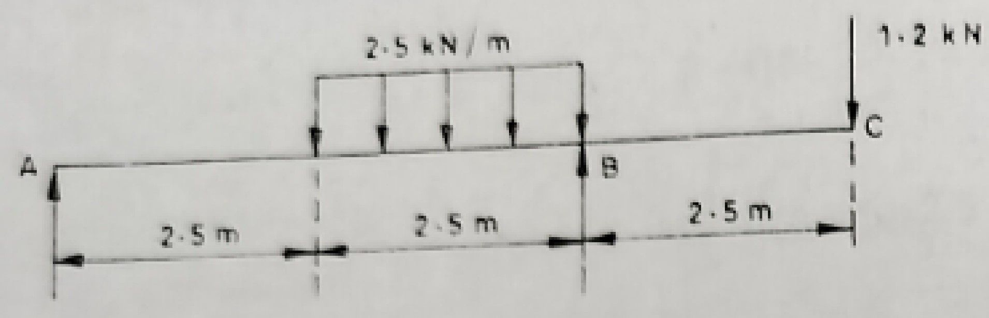


Fig.

Solution : We will use Macaulay's method as the I is constants. To use this method we will have to find the reactions first. We replace u.D.L. with its resultant.

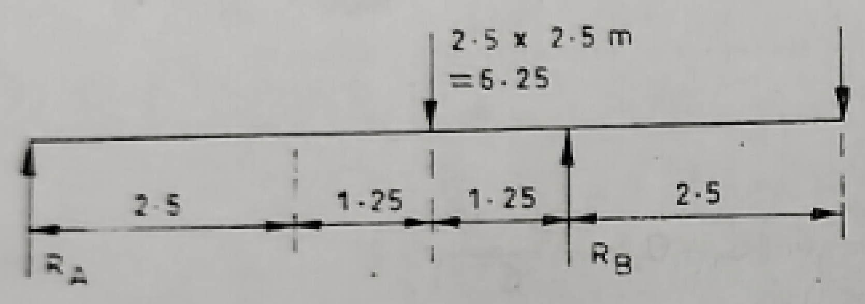


Fig.

$$+\circlearrowright \Sigma M_A : 1.2 \times 7.5 - R_B \times 5 + 6.25 \times 3.75 = 0$$

$$R_B = 6.4875$$

$$+\circlearrowright \Sigma M_B : R_A \times 5 - 6.25 \times 1.25 + 1.2 \times 2.5 = 0$$

$$R_A = .9625$$

Check : $.9625 - 6.25 + 6.4875 - 1.2 = 0$ so o.k.

Now to formulate the equations : As the u.D.L. is not acting upto point C we will have to introduce imaginary loads to formulate the equations properly. This loading is shown in Fig. below :

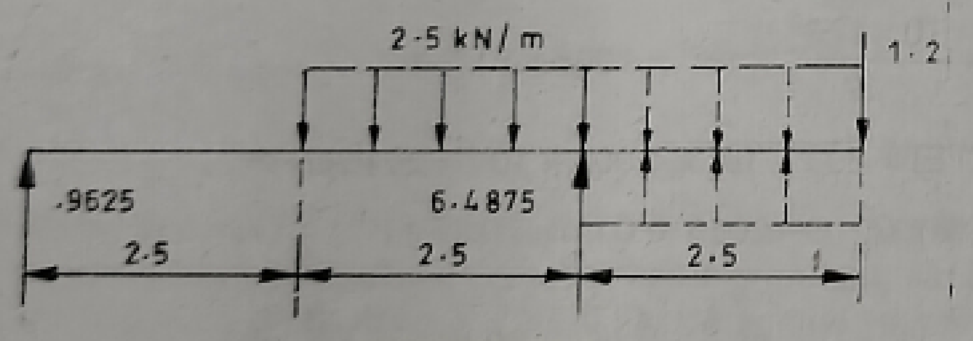


Fig.

$$E = 200 \times 10^6 \text{ kN/m}^2$$

$$I = \frac{4 \times 10^8}{10^{12}} = 4 \times 10^{-4} \text{ m}^4$$

$$EI = 200 \times 10^6 \times 4 \times 10^{-4} = 8 \times 10^4$$

$$\therefore EI \frac{d^2y}{dx^2} = .9625\, x \Big| -\frac{2.5}{2}(x-2.5)^2 \Big| 6.4875\,(x-5) + \frac{2.5}{2}(x-5)^2$$

$$EI \frac{dy}{dx} = \frac{.9625\, x^2}{2} + C_1 \Big| -\frac{1.25}{3}(x-2.5)^3 \Big| \frac{6.4875}{2}(x-5)^2 + \frac{1.25}{3}(x-5)^3$$

$$EI = \frac{.9625}{6} x^2 + C_1\, x + C_2 \Big| -\frac{1.25}{12}(x-2.5)^4 \Big| \frac{6.4875}{6}(x-5)^3 + \frac{1.25}{12}(x-5)^4$$

Now $y = 0$ at $x = 0$, $\therefore C_2 = 0$

$$y = 0 \text{ at } x = 5, \quad 0 = \frac{.9625}{6} \times 5^3 + 5C_1 - \frac{1.25}{12} \times 2.5^4$$

$\therefore C_1 = -3.196$

$\therefore$ Slope equation :

$$EI \frac{dy}{dx} = \frac{.9625}{2} x^2 - 3.196 \Big| \frac{-125}{3}(x-2.5)^3 \Big| \frac{6.4875}{2}(x-5)^2 + \frac{1.25}{3}(x-5)^3$$

$$\therefore \theta_C \text{ at } x = 7.5, \; EI\theta_C = \frac{.9625}{2} \times 7.5^2 - 3.196 - \frac{1.25}{3} \times 5^3 + \frac{6.4875}{2} \times 2.5^2 + \frac{1.25}{3} \times 2.5^3$$

$$= -1.426$$

$$\therefore \theta_C = -\frac{1.426}{8 \times 10^4} = -17.82 \times 10^{-6} \text{ rad.}$$

y_C at $x = 7.5$,

$$EIy_C = \frac{.9625}{6} \times 7.5^3 - 3.196 \times 7.5 - \frac{1.25}{12} \times 5^4 + \frac{6.4875}{6} \times 2.5^3 + \frac{1.25}{12} \times 2.5^4$$

$$= -.435$$

$$\therefore \quad y_C = -\frac{.435}{8 \times 10^4} = -0.054 \times 10^{-4} \text{ m} = -.0054 \text{ mm}$$

Prob. 2.13 : An aluminium cantilever beam 400 mm long is loaded as shown in the figure. For a distance of 150 mm from the fixed end, the beam has moment of inertia $I_1 = 20 \times 10^5 \text{ mm}^4$ and for the remainder length $I_2 = 4 \times 10^5 \text{ mm}^4$. Find the deflection and the angular rotation of the free end. Neglect the weight of the beam.

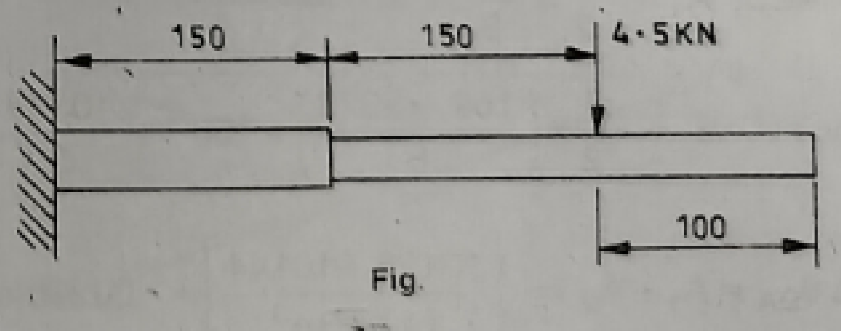


Fig.

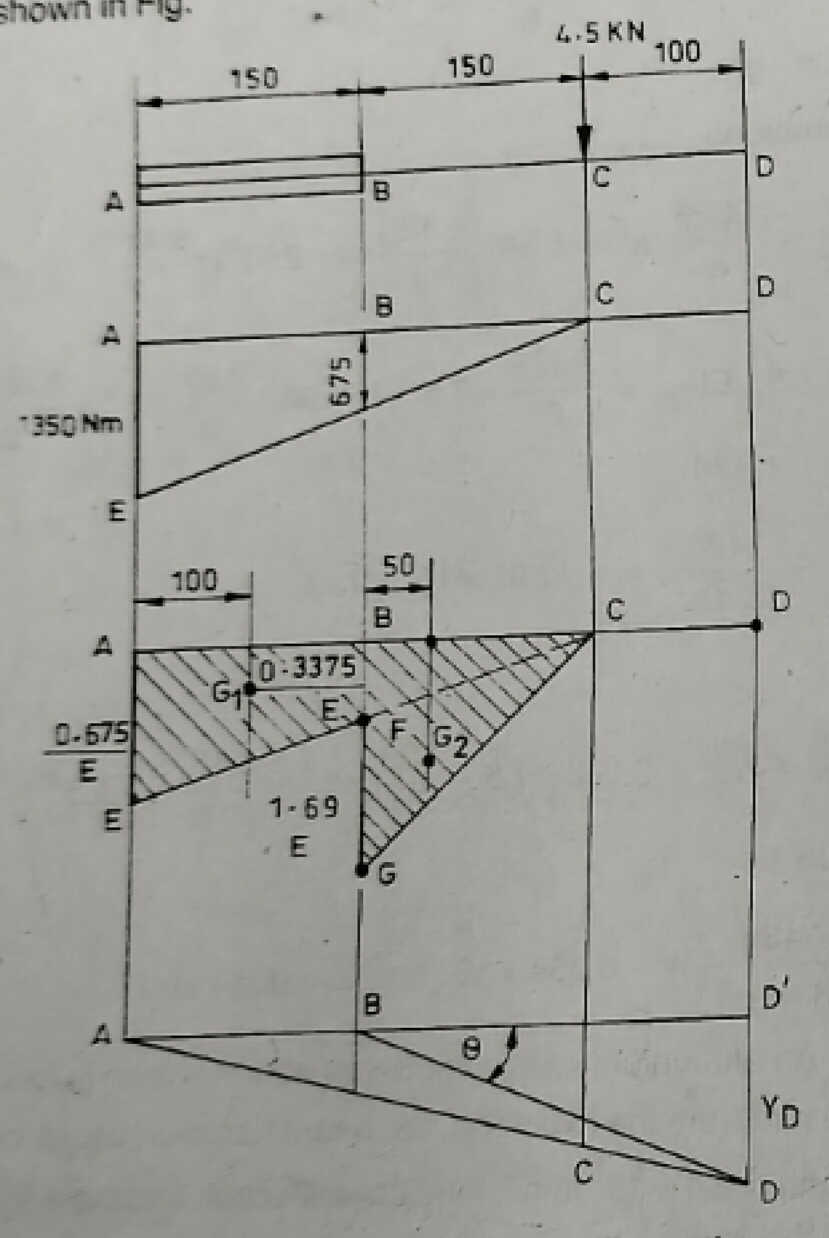
Ref. Calculations

I-II-2

E for aluminium = 73.5 GPa

We first draw the B.M. diagram and then convert 'it' into $\frac{M}{I}$ diagram.

From the elastic curve it may be seen that DD′ represents the desired deflection of the free end. However DD′ is also the tangential deviation of the point D from the tangent at A, t_{DA}. The $\frac{M}{EI}$ diagram is AEFGCD. For convenience we extend EF to C to complete the triangle AEC. So B.M. diagram has two triangles AEC and CFG with centroids G_1 and G_2 as shown in Fig.



4.5 KN
150
150
100
A
B
C
D
A
B
C
D
350 Nm
675
E
100
50
A
B
C
D
0.3375
G1
E
F
G2
0.675 / E
E
1.69 / E
G
A
B
θ
D′
YD
C
D

Fig.

$$\text{Area of } \Delta AEC,\ A_1 = -\frac{1}{2} \times \frac{.675}{E} \times 300 = -\frac{101.25}{E}$$

$$\text{Area of } \Delta CFG,\ A_2 = -\frac{1}{2} \times \left(\frac{1.69 - .3375}{E}\right) \times 150 = -\frac{101.44}{E}$$

$$\therefore\ \theta_D = \Delta\theta_{DA} = A_1 + A_2 = -\left[\frac{101.25 + 101.44}{73.5 \times 10^3}\right] = -.00275 \text{ rad}$$

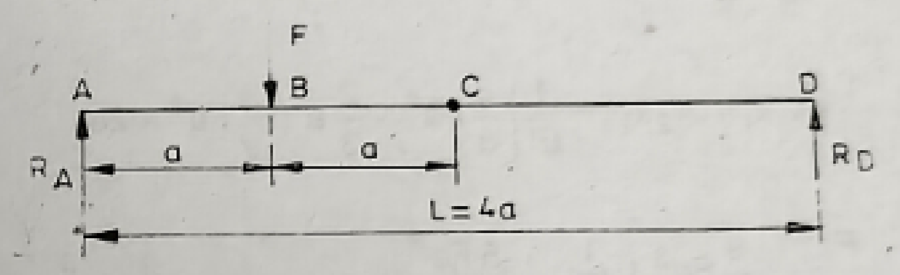
The negative sign indicates clock wise rotation of the tangent at D in relation to the tangent at A.

$$y_D = t_{DA} = A\bar{x} = A_1\bar{x}_1 + A_2\bar{x}_2$$

$$= -\left[\frac{101.25 \times 300}{73.5 \times 10^3} + \frac{101.44 \times 200}{73.5 \times 10^3}\right] = -.689 \text{ mm.}$$

The negative sign of t_{DA} means that point D is below the tangent through A.

Prob. 2.14 : Find the deflection at the centre of a simply supported beam due to a concentrated load F applied as shown in the figure. The flexural rigidity EI is constant.



F
A
B
C
D
RA
a
a
RD
L = 4a

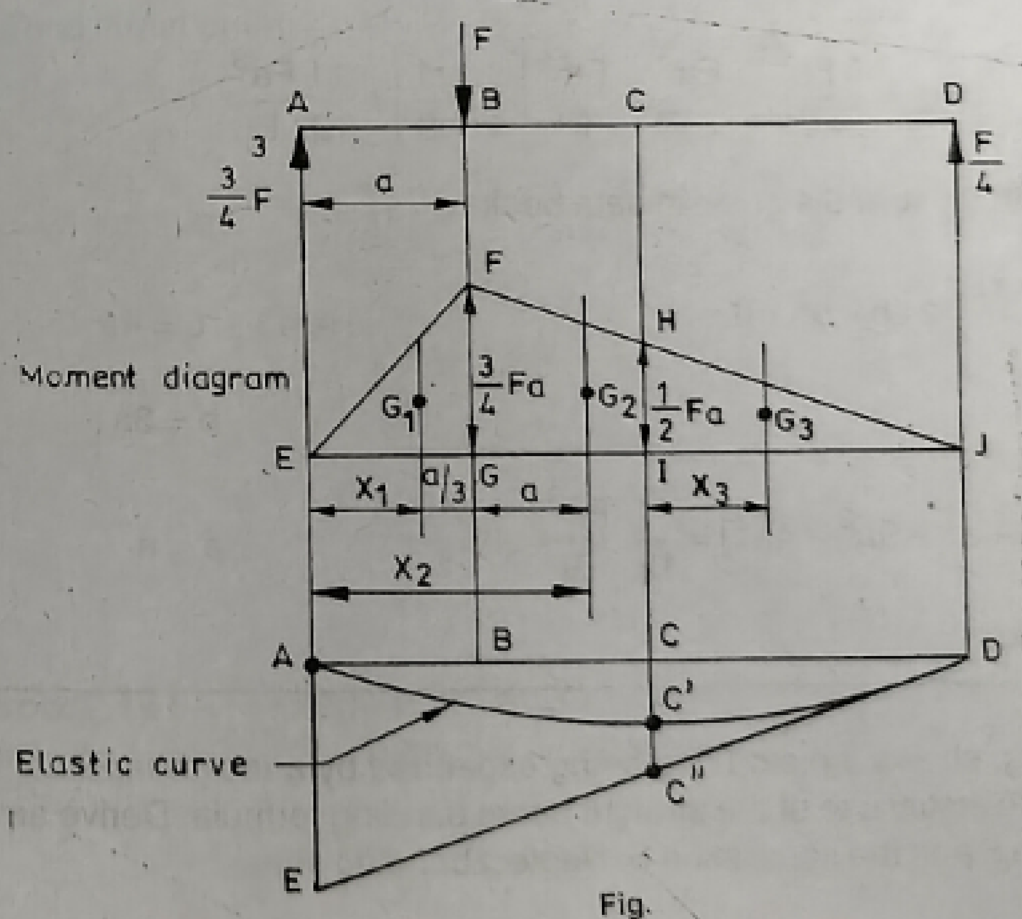
Fig.

Ref. Calculations

By taking moments about D

$R_A \times 4a = F \times 3a \therefore R_A = \frac{3}{4}F;\ R_D = \frac{1}{4}F$

The B.M. diagram and the elastic curve is shown in Fig.



F
A
B
C
D
3/4 F
a
F/4
F
H
Moment diagram
G1
3/4 Fa
G2
1/2 Fa
G3
E
J
X1
a/3
G
a
I
X3
X2
A
B
C
D
C′
Elastic curve
C″
E

Fig.

As EI is constant $\frac{M}{EI}$ diagram need not to drawn.

Deflection at $C: y_c = CC'' - C'C''$

$$= \frac{t_{AD}}{2} - t_{CD}$$

To find the tangential deviations we divide the B.M.D. into two triangles EFG and JFG.

$$\Delta\, EFG,\ A_1 = \frac{1}{2} \times \frac{3}{4} Fa^2 = \frac{3}{8} Fa^2$$

$$\Delta\, JFG,\ A_2 = \frac{1}{2} \times \frac{3}{4} F \times 3a^2 = \frac{9}{8} Fa^2$$

$$\therefore\ t_{AD} = \frac{1}{EI}\left[A_1\bar{x}_1 + A_2\bar{x}_2\right] = \frac{1}{EI}\left[\frac{3}{8} Fa^2 \times \frac{2}{3} a + \frac{9}{8} Fa^2 \times 2a\right]$$

$$= \left[\frac{Fa^3}{4} + \frac{9}{4} Fa^3\right] \frac{1}{EI} = \frac{5\, Fa^3}{2\, EI}$$

For t_{CD} we must consider ΔJHI only.

$$\text{Area of } \Delta JHI,\ A_3 = \frac{1}{2} \times \frac{Fa}{2} \times 2a = \frac{Fa^2}{2.}$$

$$\therefore t_{CD} = \frac{A_3\, \bar{x}_3}{EI} = \left[\frac{Fa^2}{2} \times \frac{2a}{3}\right] \frac{1}{EI} = \frac{Fa^3}{3\, EI}$$

$$\therefore\ y_c = \frac{5\, Fa^3}{4\, EI} - \frac{Fa^3}{3\, EI} = \frac{Fa^3}{EI}\left[\frac{5}{4} - \frac{1}{3}\right] = \frac{11}{12} \frac{Fa^3}{EI}$$

To check with the formula given in data book.

T-I-2 $$y = \frac{Wa\,(L - x)}{6\, EIL}\left[2\, Lb - b^2 - (L - x)^2\right]$$ Here $L = 4a$

$b = 3a$

$$= \frac{2\, Fa^2}{24\, EIa}\left[24\, a^2 - 9a^2 - 4a^2\right] = \frac{11}{12} \frac{Fa^3}{EI}$$ $a = a$

so it is checked.

Prob. 2.15 : The fig. shows a piston ring being expanded by a tool to facilitate installation. The ring is suficiently 'thin' to justify use of the straight beam bending formula. Derive an expression relating the seperating force F and the seperation S. Neglect bending shear.

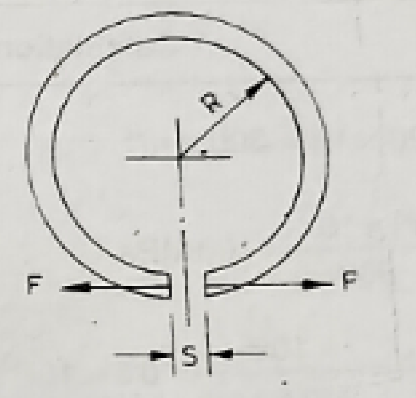


Fig.

Solution : We draw FBD of a typical segment of ring bounded by angle θ.

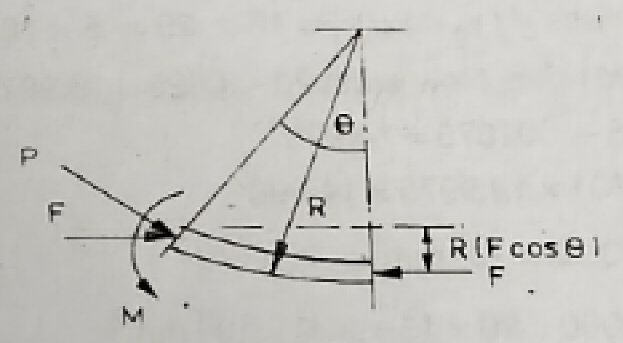


Fig.

∴ The segment is subjected to two loads :

1. Axial load $= P = F \cos\theta$
2. Bending load $= M = FR\,(1 - \cos\theta)$

$$\therefore\ \text{Total strain energy : } U = \int_0^{2\pi} \frac{P^2 R\, d\theta}{2EA} + \int_0^{2\pi} \frac{M^2 R\, d\theta}{2EI}$$

$$\therefore\ \text{Deflection at F, } \delta = \frac{\partial U}{\partial F} = \int_0^{2\pi} \frac{Fc^2\, R\, d\theta}{EA} + \int_0^{2\pi} \frac{FR^3}{EI}\,(1 - \cos\theta)^2\, d\theta$$

$$= \frac{\pi FR}{EA} + \frac{FR^3}{EI}\,(2\pi - 0 + \pi)$$

$$= \frac{\pi FR}{EA} + \frac{FR^3}{EI} \times 3\pi$$

Prob. 2.16 : A rectangular bar of 2m length 20 mm width and 15 mm thickness is subjected to a tensile load of 30 kN. Find the final volume of the bar, if Poisson's ratio is 0.25 and E = 200 GPa.

Ref.	Calculations
	Area of C.S. $A = 20 \times 15 = 300\ \text{mm}^2$.
T-I-4	$\therefore\ \sigma_t = \dfrac{30 \times 10^3}{300} = 100\ \text{MPa}$
	$\varepsilon_1 = \dfrac{\sigma_1}{E} = \dfrac{100}{200 \times 10^3} = 0.5 \times 10^{-3}\ \text{mm/mm}$
	$\therefore$ Elongation $= L\,\varepsilon_1 = 2 \times 10^3 \times 0.5 \times 10^{-3} = 1$ mm.
	Reduction in width $= b\,\varepsilon_2 = b\,\mu\,\varepsilon_1 = 20 \times .25 \times 0.5 \times 10^{-3} = 2.5 \times 10^{-3}$ mm.
	Reduction in thickness $= t\,\varepsilon_3 = t\,\mu\,\varepsilon_1 = 15 \times .25 \times .5 \times 10^{-3} = 1.885 \times 10^{-3}$ mm.
	$\therefore$ New length $= 2001$ mm New width $20 - .0025 = 19.9975$
	New thickness $= 15 - .001875 = 14.998$
	$\therefore$ New volume $= 2001 \times 19.9975 \times 14.998$.
	$= 600.15 \times 10^3\ \text{mm}^3$.
	Original volume $= 2000 \times 20 \times 15 = 600 \times 10^3\ \text{mm}^3$.
	We can also get it by considering the volumetric strain in a rectangular body.(Prob.2.17)
	$\varepsilon_v = \dfrac{\delta v}{V} = \varepsilon_1 (1 - 2\mu) = 0.5 \times 10^{-3} (1 - 2 \times .25) = .25 \times 10^{-3}$
	$\therefore\ \delta v = \varepsilon \times V = .25 \times 10^{-3} \times 600 \times 10^3 = 150\ \text{mm}^3$
	$\therefore$ New volume $= V + \delta v = 600 \times 10^3 + .15 \times 10^3 = 600.15 \times 10^3$.

Prob. 2.17 : On a M.S. plate a circle of dia. 50 mm is drawn before the plate is stressed as shown in Fig. Find the length of major and minor axis of an ellipse formed as a result of the deformation of the circle marked. Take $E = 200$ GPa and $\mu = 0.25$

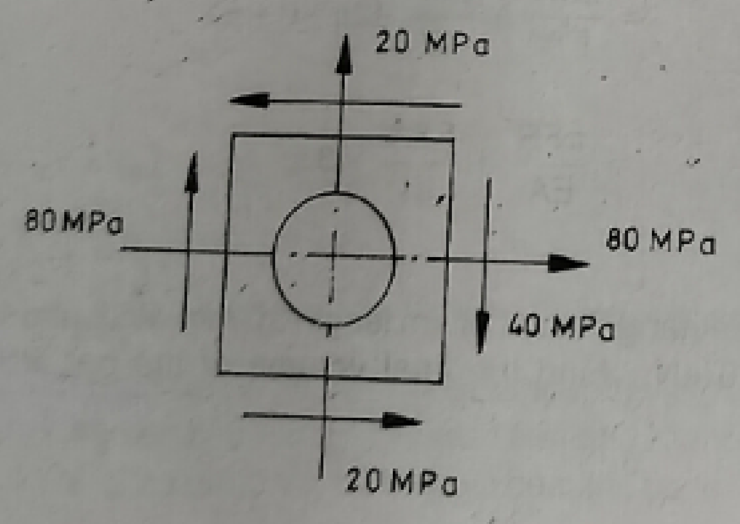
Ref.	Calculations
	

Fig.

Ref.	Calculations
	The principle stresses for this loading will be :
	Given $\sigma_x = 80,\ \ \sigma_y = 20,\ \ \tau_{xy} = 40$
T-I-6	$\sigma_1 = \dfrac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
	$= \dfrac{80 + 20}{2} + \sqrt{\left(\dfrac{80 - 20}{2}\right)^2 + 40^2} = 50 + 50 = 100\ \text{MPa}$
	$\sigma_2 = \dfrac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 50 - 50 = 0\ \text{MPa}$
	$\therefore$ Deformation along σ_1 axis : $\Delta L_1 = \left(\dfrac{\sigma_1}{E} - \mu\,\dfrac{\sigma_2}{E}\right) L_1$
	$= \dfrac{100}{200 \times 10^3} \times 50 = 25 \times 10^{-3}$ mm
	Deformation along σ_2 axis : $\Delta L_2 = \left(\dfrac{\sigma_2}{E} - \mu\,\dfrac{\sigma_1}{E}\right) L_2$
	$= \dfrac{-0.25 \times 100}{200 \times 10^3} \times 50 = -6.25 \times 10^{-3}$ mm
	$\therefore$ Major axis $= 50 + 0.25 = 50.025$ mm
	Minor axis $= 50 - .00625 = 49.99375$ mm.

Prob. 2.18 : A solid is loaded as shown in Fig. Find the change in volume of the solid. What change must be made in the 4 kN load in order that there shall be no change in volume of the soild. Assume $E = 200$ GPa and $\mu = 0.3$. Derive the formula used.

Solution :

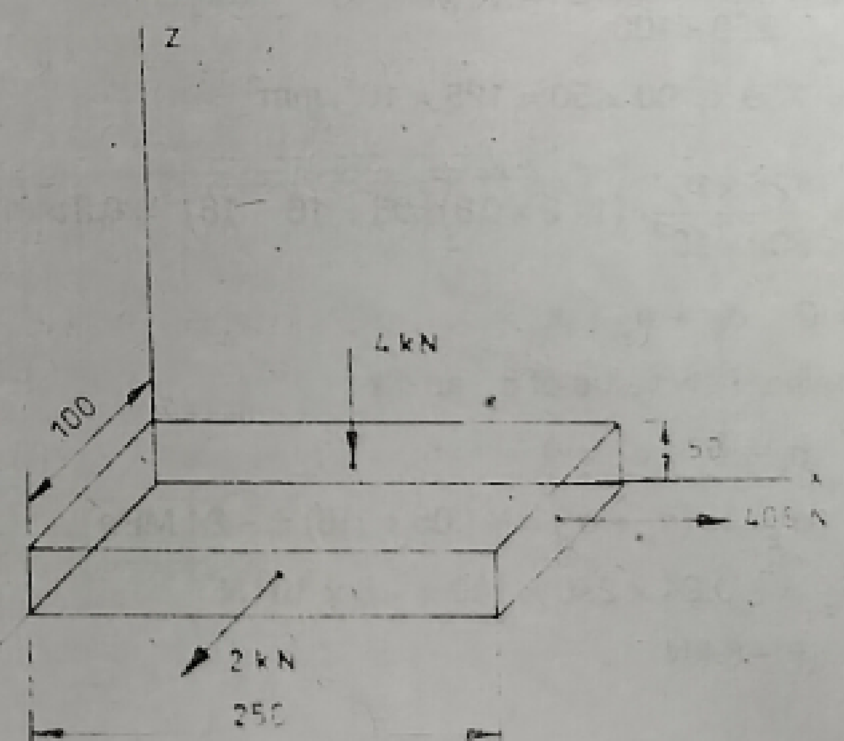


Fig.

The solid will be subjected to three stresses, σ_x, σ_y and σ_z.

Let the dimensions of the solid along x, y and z axes be X, Y, and Z respectively.

∴ Original volume, $V = X \,.\, Y \,.\, Z$

∴ Deformations along the three axes will be :

$$\Delta X = \left[\frac{\sigma_x}{E} - \mu\left(\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)\right] X = \frac{X}{E}\left[\sigma_x - \mu\left(\sigma_y + \sigma_z\right)\right]$$

$$\Delta Y = \left[\frac{\sigma_y}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_z}{E}\right)\right] Y = \frac{Y}{E}\left[\sigma_y - \mu\left(\sigma_x + \sigma_z\right)\right]$$

$$\Delta Z = \left[\frac{\sigma_z}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E}\right)\right] Z = \frac{Z}{E}\left[\sigma_z - \mu\left(\sigma_y + \sigma_x\right)\right]$$

∴ New volume, (neglecting small terms) $V' = (X + \Delta X)(Y + \Delta Y)(Z + \Delta Z)$

$$= (XY + X\Delta Y + Y\Delta X)(Z + \Delta Z)$$

$$= XYZ + XZ\Delta Y + YZ\Delta X + XY\Delta Z$$

∴ Change in volume, $\Delta V = V' - V = XZ\,\Delta Y + YZ\Delta X + XY\Delta Z$

Putting volumes of ΔX, ΔY and ΔZ :

$$\Delta V = \frac{XYZ}{E}\left[\sigma_z - \mu\left(\sigma_y + \sigma_x\right) + \sigma_x - \mu\left(\sigma_y + \sigma_z\right) + \sigma_y - \mu\left(\sigma_x + \sigma_z\right)\right]$$

$$= \frac{V}{E}(1 - 2\mu)\left(\sigma_x + \sigma_y + \sigma_z\right)$$

Now to use the formula :

$$\sigma_x = \frac{400}{50 \times 100} = .08 \text{ MPa}, \quad ; \quad \sigma_y = \frac{2 \times 10^3}{250 \times 50} = .16 \text{ MPa}$$

$$\sigma_z = -\frac{4 \times 10^3}{250 \times 100} = -.16 \text{ MPa}$$

$$V = 250 \times 100 \times 50 = 125 \times 10^4 \text{ mm}^3$$

$$\therefore \Delta V = \frac{125 \times 10^4}{200 \times 10^3}(1 - 2 \times 0.3)(.08 + .16 - .16) = 0.2 \text{ mm}^3$$

For $\Delta V = 0$, $\sigma_x + \sigma_y + \sigma_z = 0$

∴ Assume the new value of σ_z as $\sigma' z$:

$$\sigma_x + \sigma_y + \sigma'_z = 0$$

or $\sigma'_z = -(\sigma_x + \sigma_y) = -(.08 + .16) = -24 \text{ MPa}$

$$\therefore F'_z = -0.24 \times 250 \times 100 = -6 \times 10^3 \text{ N}$$

$$= -6 \text{ kN}$$

Prob. 2.19 : A 20 mm diameter steel rod of $S_y = 350$ MPa is loaded as a column with pinned ends. If sufficiently short it can carry a limiting load of $S_y A = 110$ kN. How long can the rod be and still carry 50 percent of this load ? E = 200 GPa.

Ref.	Calculations
	Area of rod, $A = \frac{\pi}{4} 20^2 = 314 \text{ mm}^2$
T-I-1	Radius of gyration $K = \frac{D}{4} = \frac{20}{4} = 5\text{mm}.$
	For carrying 50 percent load, $F_{cr} = 110 \times .5 = 55$ kN.
T-I-24	Treating it as a short column and using Rankine's formula
	$S_y = \frac{F_{cr}}{A}\left[1 + \frac{1}{an}\left(\frac{L}{k}\right)^2\right]$ n = 1 for both ends hinged
	$a = \frac{\pi^2 E}{S_y} = \frac{\pi^2 \times 200 \times 10^3}{350} \doteq 5634$
	$\therefore 350 = \frac{55 \times 10^3}{314}\left[1 + \frac{1}{5634}\left(\frac{L}{k}\right)^2\right]$
	$\therefore \frac{L}{k} = \sqrt{6534} = 75$; $\therefore L = 75 \times 5 = 375$ mm.

Prob. 2.20 : The figure shows a boom and the rod arrangement supporting a load of 6 kN. The tie rod is made of steel having a tensile yield strength of 400 MPa.

(a) What is the safety factor of the tie rod with respect to static yielding ?

(b) What is the safety factor of the tie rod if the 6 kN load acts upward ?

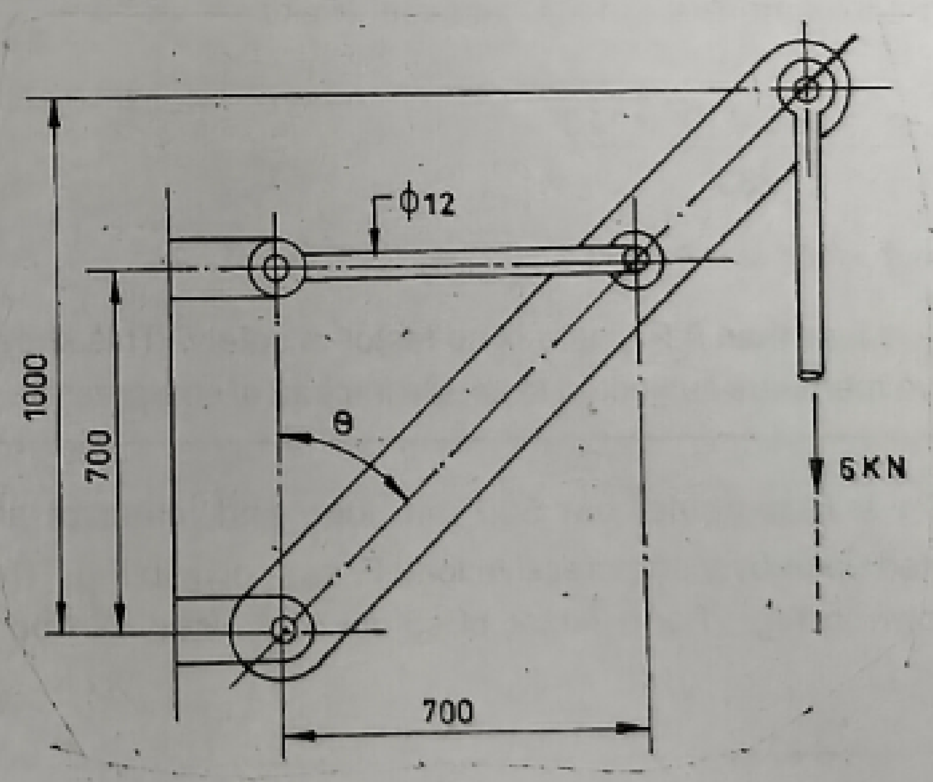
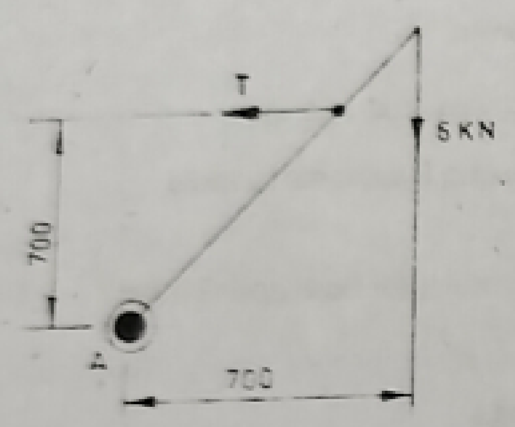


Fig.

Ref. Calculations

$\tan\theta = \frac{700}{700} = 1$ $\quad\therefore\ \theta = 45^0$

The forces acting on the boom are shown below.



Taking moments about A

$T \times 700 = 6 \times 1000$

$\therefore\ T = 8.57$ kN

$\therefore\ \sigma_t = \frac{T}{A}$ $\qquad A = \frac{\pi}{4} 12^2 = 113\ \text{mm}^2$

$= \frac{8.75 \times 10^3}{113} = 75.84$ MPa

$\therefore$ Factor of safety, $N = \frac{400}{75.84} = 5.27$

Fig.

If the load acts upward, force T will be compressive. Hence we have to find F_{cr}.

$K = \frac{D}{4} = \frac{12}{4} = 3$mm.

$\therefore\ \frac{1}{k} = \frac{700}{3} = 233$ so it is a Euler column as $\frac{L}{k} > 90$

T-I-24 $\quad F_{cr} = \frac{n\pi^2 EA}{\left(\frac{L}{k}\right)^2}$ $\qquad n = 1$

$= \frac{\pi^2 \times 200 \times 10^3 \times 113}{233^2}$

$= 4.1 \times 10^3\ \text{N} = 4.1$ kN.

As F_{cr} is less than 8.57 there is no factor of safety. This shows that it is always safer to have members subjected to tensile instead of compressive loads.

Prob. 2.21 : A rectangular bar 500 mm long and made of aluminium alloy, having $S_{yt} = 200$ MPa is acted upon by a compressive load F, as shown in Fig. The end conditions in two planes are also shown in Fig. For a factor of safety of 3, what can be maximum value of F? Take E = 73.5 GPa.

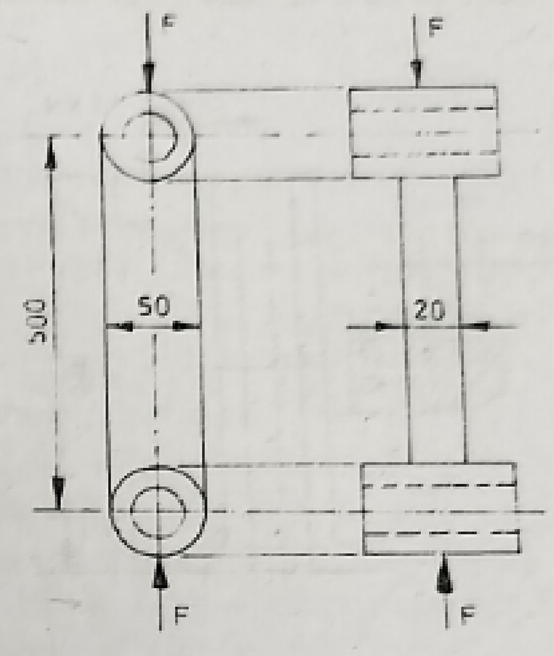


Fig.

Ref. Calculations

Here we have two planes for buckling. Hence we will have to decide the plane of failure on the basis of the product (nk^2).

T-I-24 For plane 1 : Both ends hinged

T-I-1 $n = 1,\ k = .289\ h = .289 \times 50 = 14.45$ mm ; $nk^2 = 1 \times 14.45^2 = 209.7$

For plane 2 : Both ends fixed

$n = 4,\ k = .289\ h = .289 \times 20 = 5.78$; $nk^2 = 4 \times 5.78^2 = 133.6$

So the plane of failure is plane 2. In this plane

$\frac{L}{k} = \frac{500}{5.78} = 86.5$. As this is less than 90, it is a short column.

Using Rankine formula :

$S_y = \frac{F_{cr}}{A}\left[1 + \frac{1}{an}\left(\frac{L}{k}\right)^2\right]$ $\qquad F_{cr} = 3 \times F$

$A = 50 \times 20 = 1000\ \text{mm}^2$.

$200 = \frac{3F}{1000}\left[1 + \frac{86.5^2}{3623 \times 4}\right]$ $\qquad a = \frac{\pi^2 E}{S_y} = \frac{\pi^2 \times 73.5 \times 10^3}{200}$

$\therefore\ F = 43.96 \times 10^3$ N $\qquad = 3623$

$= 44$ kN $\qquad n = 4$

Prob. 2.22 : Calculate the factor of safety for the column made of steel pipe for the loading shown in the Fig. Assume the material as SAE 1020 steel. Consider one end fixed and other end guided.

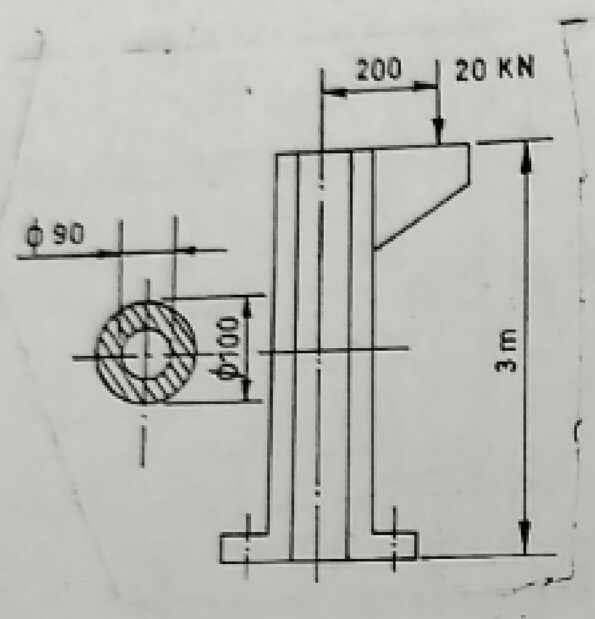


Fig.

Ref.	Calculations
T-I-24	This is beam column. Hence we have to use the secant formula.
	$\sigma = \frac{F}{A}\left[1 + \frac{ec}{k^2}\sec\theta\right]$ where $\theta = \frac{L}{2k}\sqrt{\frac{F}{EA}}$ rad
	$L =$ Equivalent length $= \frac{L}{\sqrt{n}} = \frac{3000}{\sqrt{2}} = 2121.3,$
	$A = \frac{\pi}{4}\left(100^2 - 90^2\right) = 1491.4\ \text{mm}^2$
	$k^2 = \frac{I}{A} = \frac{\pi\left(100^4 - 90^4\right)}{64 \times 1491.5} = .113 \times 10^4 \quad \therefore k = 33.6\ \text{mm.}$
	$\frac{L}{k} = \frac{3000}{33.6} = 89.28$
T-II-7	So secant formula can be used $S_{yt} = 246$, $E = 200$ GPa
	$\theta = \frac{21213}{2 \times 33.6}\sqrt{\frac{20 \times 10^3}{200 \times 10^3 \times 1491.5}} = 0.258\ \text{rad.} = \frac{.258 \times 180}{\pi} = 14.8^0$
	$\therefore \sigma = \frac{20 \times 10^3}{1491.5}\left[1 + \frac{200 \times 50}{33.6^2}\sec 14.8^0\right] = 13.4\left[1 + 9.16\right] = 136.2\ \text{MPa.}$
	$\therefore FS = \frac{246}{136.2} = 1.80$

T-I-24	Alternately :
	$\sigma = \frac{F}{A}\left[1 + \frac{1}{an}\left(\frac{L}{k}\right)^2\right] + \frac{F \times e \times c}{I}$
	$I = Ak^2 = 1491.5 \times 33.6^2.$
	$= 168.5 \times 10^4\ \text{mm}^4.$
	$= \frac{20 \times 10^3}{1491.5}\left[1 + \frac{89.28^2}{8016 \times 2}\right] + \frac{20 \times 10^3 \times 200 \times 50}{168.5 \times 10^4}$
	$a = \frac{\pi^2 E}{S_y} = \frac{\pi^2 \times 200 \times 10^3}{246}$
	$= 8016$
	$n = 2$
	$= 13.4\ [1 + .497] + 118.6$
	$= 20.06 + 118.6 = 138.7$ MPa
	$\therefore FS = \frac{246}{138.7} = 1.77$
	Both methods give comparable results.

* * *

Chapter 3

ENGINEERING MATERIALS & MANUFACTURING PROCESSES

3.1 Introduction :

The selection of a material for a machine or structural member is one of the most important decisions, the designer is called upon to make. This decision has to be usually made before the dimensions of the part are determined. After choosing the material and process (the two cannot be separated), the designer can then proportion the member so that the internal stresses and strains have reasonable and satisfactory values compared with the properties associated with failure of the material.

As important as the stress and deflection of mechanical parts are, the selection of a material is not always based upon these factors. So many other factors besides stress and strain may govern the design of parts, that a versatile background in materials and processes is necessary.

3.2 Technical Properties :

The technical properties of materials used by the designer may be classified as :

(a) **Physical Properties :** These include - composition, structure, homogeneity, specific gravity, thermal conductivity, specific heat, expendability, resistance to corrosion etc. Some of these for common engineering materials are listed in (T-II-2) of data book.

(b) **Technological Properties :** These are the properties related to manufacturing such as weldability, forgeability, machinability, formability etc.

(c) **Mechanical Properties :** These are the properties established by tests and used in machine design. The basic mechanical properties are elastic limit, yield strength, ultimate strength, endurance limit, percentage elongation and hardness. These are listed in various tables of chapter-II of the data bok. For machine design the mechanical properties of a material that are of prime importance are strength, stiffness, hardness and ductility.

i) **Strength :** The *static strength* is measured by the yield strength, S_y or the ultimate strength, S_u. These are determined by the standard tensile test. For ductile materials the yield strength is the basis for design while ultimate strength is used for brittle materials because they do not have a distinct yield strength. The materials having a percentage elongation less than 5 percent are treated as brittle materials while materials having a percentage elongation more than 5 percent are treated as ductile materials.

For fatigue loading the design criteria is endurance strength S_e or the fatigue limit S_f. This is determined by the standard reversed bending test and is given in tables as S_{-1b}. The use of these strength in fatigue design is given in chapter - 4. For impact loading the strength is measured by the corresponding modulus of resilience, u. This is given in (T-I-8) of data book.

ii) **Stiffness or Rigidity :** This is measured by the modulus of elasticity, E being used for tension and compression while G being used for shear.

iii) **Hardness :** This property is required to know the wear resisting capacity of the material. This is a relative characteristic. There are various methods of measuring hardness : Brinell Hardness, Rockwell Hardness, Vickers Hardness and Shore Hardness. Generally for metals - Brinell Hardness and Rockwell Hardness on B and C scales are used.

iv) **Ductility :** A material is said to be ductile if it is capable of undergoing a large permanent deformation without rupturing. There is no absolute measure of ductility. The percentage elongation, e or the percentage of reduction of area during a tensile test is used as a relative measure. This point is discussed earlier.

All these properties along with the fields of use of various materials are given in chapter-II of data book.

3.3 Ferrous Materials :

All the ferrous metals used in the manufacture of machines can be classified into three groups. Cast Iron; Plain Carbon Steels, Alloy steels and Cast steels.

(A) Cast Iron : It is a mixture of iron and carbon, the carbon percentage is always greater than 1.7 percent. Part of carbon is in solid solution while part of the carbon is in the form of flakes of free graphite in a mechanical mixture. Cast iron for engineering products usually contains 2.5 to 3.5 percent carbon and is used to produce articles of desired shapes. The commonly used cast iron are:

i) Grey Cast Iron : Of all the cast materials, grey cast iron is the most widely used because it is very cheap, is easily cast in large quantities and is easy to machine. The principal disadvantage is its it brittleness with percentage elongation of 1-2 percent and its poor tensile strength. This material is difficult to weld.

ii) White Cast Iron or Chilled Cast Iron : If all the carbon in cast iron is in the form of cementite and pearlite with no graphite present, the resulting structure is white and is known as white cast iron. This may be done by either changing the composition or rapid cooling of grey iron castings called chilling. These are very brittle and hard to machine but very resistant to wear.

iii) Malleable Cast Iron : If white cast iron within a certain composition range is annealed, a product called malleable cast iron is formed. Due to annealing the carbon is freed and is in a nodular form and not in the form of flakes. A good grade of malleable cast iron may have a tensile strength of 350 MPa and percentage elongation of as much as 18 percent.

iv) Nodular Cast Iron : Because of the lengthy heat treatment required (up to 6 days for large and heavy castings) to produce malleable iron, cast iron has long been desired which would combine the ductile properties of malleable cast iron with the ease of casting and machining of grey iron and at the same time would posses these properties in the as cast conditions without any further treatment. A process for producing such castings using cesium with magnesium seems to fulfill these requirements. The castings produced by this process are called S.G. iron castings. They have a high modulus of elasticity 172 GPa almost equal to that of steel.

v) Alloy Cast Iron : Nickel, Chromium and Molybdenum are the most common alloying elements used in cast iron. *Nickel* increases the strength and density, improves the wear resistance and raises the machinability. *Chromium* increases the hardness and wear resistance. When both, nickel and chromium, are added, the hardness and strength are improved without reduction in the machinability rating. *Molybdenum* increases the hardness, stiffness, tensile strength and impact resistance.

(B) Plain carbon steel : Steel is the most extensively used material for machine components. By suitably varying the composition, thermal treatment, and mechanical treatment a tremendous range of mech. properties can be obtained. Three basic principles must be remembered for the appropriate selection of steel.

i) All steels have essentially the same modulii of elasticity. Thus if *rigidity* is the critical requirement of the part, all steels perform equally well and the cheapest steel should be selected.

ii) Carbon content, almost alone, determines the maximum hardness that can be developed in steel. Maximum potential hardness increases with carbon content upto about 0.7 percent. This means that relatively small, regularly shaped parts can be heat treated to give essentially the same hardness and strength with plain carbon steels as with more costly alloy steels.

iii) Alloying elements (manganese, molybdenum, chromium, nickel and others) add hardenability to steel. Thus the potential hardness and strength (which is controlled by carbon content) can be realized with less drastic heat treatments when the alloying elements are used. This means that with an alloy steel (a) parts with large sections can achieve higher hardnesses in the centre or core of the section, and (b) irregularly shaped parts, subject to warpage during a drastic quench, can achieve the desired hardness from a more moderate heat treatment.

Plain carbon steels consist chiefly of iron, carbon and manganese, other elements such as silicon, phosphorous, and sulfur, are always present but are not essential to the formation of the steel. These steels can be subdivided into three main groups : i) low carbon steels, having a carbon content of 0.05 to 0.25 percent and suitable where only moderate strength but considerable ductility is required ; ii) machinery steels, which have a carbon content of 0.30 to 0.55 percent and can be heat treated to develop high strength; and iii) high carbon steels, which contain 0.60 to 1.30 percent and are widely used for tools and springs, mostly with heat treatment. The properties and uses of plain carbon steels and wrought iron are given in (T-II-7) of data book.

(C) Alloy Steel : The steel in which one or more elements other than carbon are introduced in sufficient quantities to modify its properties substantially are called *alloy steels.* As discussed earlier the basic purpose of adding alloying elements to steel is to increase hardenability. The commonly used alloying elements are listed below in terms of their relative effectiveness in the decreasing order : Boron ; Manganese 1.2%; Manganese between 1.2 to 2.0 percent, Molybdenum 1.0 percent; Chromium 2.0 percent; Silicon 2.0 percent and Nickel 2.0 percent. Commonly used alloying elements and their effect on properties is discussed below :

i) Chromium, Cr, (Melting point 1920° C) : It increases the hardness and strength and only minimally reduces the elasticity. It improves the resistance to heat and non scaling properties. With higher chrome content the steels become corrosion resistant and wear resistant. However, the welding properties deteriorate in pure chromium steels with increasing chromium content.

ii) Manganese , Mn (Melting point 1244° C) : It improves the strength while only slightly impairing the elasticity; furthermore it has a favourable influence on the forging and welding properties. A higher content of manganese in the presence of carbon increases the wear resistance very substantially.

iii) Molybdenum, Mo (Melting point 2610° C) : It improves the tensile strength, especially the heat resistance and it also has favourable influence on the welding properties. It is frequently used in conjunction with chromium.

iv) Nickel, Ni (Melting point 1453° C) : It raises the strength less than does silicon or manganese with the elasticity dropping insignificantly. It ensures through hardening, especially so when the steel also contains chromium. Chrome-nickel steels are stainless and resistant to scaling and also heat resistant. It does not impair the welding properties.

v) Silicon, Si (Melting point 1410°C) : It is present in all steels as an impurity. Hence the term silicon steel is used to steels having a silicon content above 0.4 percent. It increases the strength, the resistance to scaling & the density, specially of cast steel. The elasticity is only insignificantly affected. A high silicon content (about 14%) enables steel to resist chemical attacks but it can no longer be forged.

The properties and uses of commonly used alloy steels are given in (T-II-8) of data book. Similarly (T-II-6) gives the commonly used steels for engineering components.

(D) Cast Steels : Casting process has the main advantage that parts having complex shapes can be manufactured at cost much less than the fabrication process by other means. Thus the choice of steel casting is logical when the part is complex and requires high strength which can not be obtained in the cast irons. However, the higher melting temperatures of steels do aggravate the casting problems. The same alloying elements used for the wrought steels can be used for cast steels to improve the properties. Cast steel can be heat treated and they can be easily welded. The commonly used varieties of cast steel and their properties are given in (T-II-2) of data book.

3.4 Specification of Steels :

There are various codes of specification of steels based on chemical composition or Mechanical properties. The commonly used specifications are :

i) **SAE or AISI specifications :** The Society of Automotive Engineers has adopted specifications for various steels based on their chemical composition. Each specification is given a four figure or five figure number. The first two digits indicate the class, while the last two or three indicate the carbon content in hundredths of one percent. (T-II-4) of data book gives details of these specifications. All the properties in data book are as per these specifications.

ii) **ISS or Indian standard specifications.**

iii) **BSS or British standard specification :** The steels used for engineering purposes are called En steels and given a number.

iv) **DIN specifications :** These are German specifications. They specify the chemical composition.

v) **GOST specifications :** These are Russian specifications.

(T-II-5) of data book gives approximately equivalent of commonly used SAE steels in other specifications.

3.5 Non Ferrous Metals & Alloys :

The most commonly used non-ferrous metals are Aluminium, Copper and Magnesium.

(A) Aluminium & Its Alloys : The outstanding characteristics of aluminium and its alloys are their strength weight ratio, (sp. gr. of aluminium is about one third of steel) their resistance to corrosion, and their high thermal and electrical conductivity. Considering the cost and strength of aluminium and its alloys, they are among the most versatile materials from the stand point of fabrication. The alloys can be processed by sand casting, die casting, hot or cold working or extruding. These alloys can be machined, press worked, soldered, brazed or welded. They are commercially available in the from of plate, sheet, foil, bar, rod and tube and structural and extruded shapes. The alloys are classified in two groups : Casting alloys and Wrought alloys. The properties and uses of some of the more common alloys are given in (T-II-10) of data book.

(B) Copper & Its Alloys : Pure copper is not used for the fabrication of machine parts but copper base alloys are extensively used specially as bearing metals. They are divided in two groups : (a) Brasses when it is alloyed with zinc and (b) Bronzes when they are alloyed with other elements.

(a) Brasses : Different kind of brasses are produced by varying the percentage of zinc. As the zinc is cheaper than copper the cost varies according to the percentage of zinc in the alloy. There are three groups : (i) Brass with 5 to 15 percent zinc which are easy to cold work, are ductile but often hard to machine; their corrosion resistance is good. Out of these the *Red brass* with 15 percent zinc is used a great deal in the form of tubing or piping to carry hot water in such applications as radiators or condensers (ii) Brasses with 20 to 36 percent zinc which have better machinability and slightly greater strength but poor corrosion resistance. In this group, *Cartridge brass* (zinc 30 percent) has the best combination of ductility and strength while *Yellow brass* (zinc 35 percent) can be used practically in any fabricating process and is therefore employed in a large variety of products ; *Admiralty metal* (zinc 28 percent) contains 1 percent tin which imparts excellent corrosion resistance, especially to saltwater which makes it suitable to be used in powerplants and chemical equipment, *Al brass* (zinc - percent) contains 2 percent aluminium and is used for the same purpose as admiralty metal in the form of tubing or piping because it has better resistance to erosion caused by high velocity water iii) Brasses with 36 to 40 percent zinc, are less ductile than cartridge brass but cannot be cold worked hence are hot worked and extruded. *Muntz metal* (zinc 40 percent) is low in cost and mildly corrosion resistant while *Naval brass* has the same composition except for the addition 0.75 percent tin which contributes to the corrosion resistance.

*(b) **Bronzes** :* More important in this group are the alloys with silicon, tin & aluminium. *Phosphor Bronze* (11 percent tin) contains small amounts of phosphorous, is specially resistant to fatigue & corrosion, has a high tensile strength & a high capacity to absorb energy & is also resistant to wear. It is widely used as a spring material *Aluminium bronze* (12 percent aluminium) is a heat treatable alloy, has strength and corrosion resistance properties better than brass. If iron is added upto 4 percent, the alloy has a high endurance limit, a high shock and wear resistance. *Silicon bronze* (3 percent silicon & 1 percent manganese) has mechanical properties equal to those of mild steel as well as good corrosion resistance.

(C) Magnesium & Its Alloys : The density of magnesium is about one fourth that of steel and is lightest of all commercial metals. Its greatest use is in aircraft industry. These alloys are used in applications where strength is not an important consideration.

The properties and uses of important copper alloys is given in T-II-9 of databook.

3.6 Plastics :

Plastics constitute a large group of synthetic organic materials. Although solid in the finished state, plastics are liquid in some stage of manufacture, permitting them to be formed in various shapes. The forming usually involves application of heat and/or pressure. The term plastic is really a misnomer as the material is truly plastic only during a stage of its manufacture. The basic chemical units of plastic materials are *monomers* which under appropriate conditions, usually involving heat and/or pressure, combine into *polymers*, the process being called *polymerization*. The plastics are divided into two main groups.

(A) Thermoplastics : This term is used to mean any plastic that is mouldable when heat and/or pressure is applied to it. These plastics can be remolded when heated.

(B) Thermosetting : It is a plastic for which the polymerization process is finished in a hot moulding press where the plastic is liquefied under pressure. These plastics cannot be remolded.

Typical characteristics and uses of common plastics are given in (T-II-12) while their properties are given in (T-II-11) of data book.

3.7 Elastomers :

The most important naturally occuring elastomers are ; 1) Rubber and 2) Leather. Rubber is the most important natural elastomer which resists abrasion, heat, strong alkalis and acids. In machines the most important use of leather and rubber is for power transmission belting. Rubber is also used as a bearing material and vibration absorbing mounting blocks. It is available in different hardnesses in the range of shore hardness between 30 to 90 on the A scale. In addition to these natural elastomers, there are various types of thermosetting and thermoplastic elastomers. These elastomers vary widely in cost and other factors. Unless the designer has had experience in the design and selection of elastomer, it is good practice to consult the manufacturers for specifying the most suitable material.

3.8 Manufacturing Processes :

The various processes used for manufacture of machine parts may be divided into three groups :

(A) Forming Process : These processes are meant for making the parts into different shapes. The dimensions obtained are not so precise. Hence, the formed parts are finished by various machining processes to get the required dimensions.

The important processes in this group are :

i) Casting, ii) Forging,
iii) Extrusion iv) Rolling,
v) Press working, vi) Sintering

We will discuss the important points about these processes in the next articles.

(B) Fabricating or Joining Processes : Mechanical fabrication is intended to mean the permanent joining of structural or machine parts that are made as separate units for manufacturing assembling or transportation reasons. The important processes in this group are :

i) Rivetting, ii) Welding, iii) Soldering,
iv) Brazing & v) Adhesive bonding.

These processes will be discussed in the next articles.

(C) Finishing Processes : These processes are meant for obtaining the required dimensions, surface finish or other characteristics of the core and surfaces required for the proper functioning of parts. The important processes are (i) All machining processes, (ii) Surface treatment processes such as electroplating, anodizing etc. (iii) Heat treatment.

The surface finish obtained by various manufacturing processes is given in (T-III-6) of databook while the tolerance capabilities of various machining operations is given in (T-III-7) of databook. The surface roughness normally used for a variety of engineering components is given in T-III-9 of databook.

3.9 Casting :

The casting processes consist of introducing a molten metal into the cavity of a mould, where it is permitted to solidify and retain the shape of the mould cavity. Because it can produce a definite shape without extensive machining, casting is used in manufacturing such widely different products as rugged machine bases made from cast iron to gas turbine blades made from steels with high alloy content. There are numerous variations, but the casting processes are most easily classified on the basis of moulding method in one of the following groups :

1. Sand 2. Centrifugal
3. Permanent mold 4. Plaster mold
5. Investment and 6. Die.

Out of these the most widely used casting processes for engineering components are :

1. Sand 2. Permanent Mold and 3. Die.

Sand casting requires a separate mold for each component while permanent mold processes employ a metal mold which can be used for a number of components but it can be used for relatively low melting temperature metals. Die casting is used in quantity production of small parts made of low melting temp. alloys. The general requirements and characteristics of various casting processes are given in (T-III-4) of data book. The two main rules that should be observed in the design of the castings are the following.

i) The section thickness should preferably be uniform.
ii) Where section uniformly is not possible, light sections should be blended into heavy ones gradually.

3.10 Forging :

This process is more expensive than most other methods. Nevertheless, the process is used quite extensively because forged parts have great strength and resistance to shock. Forging methods may be divided into the following groups :

i) Hand forging,
ii) Machine forging,
iii) Die forging or drop forging and
iv) Press forging.

In hand or machine forging the material is heated to a plastic state and is then subjected to several of the following operations : bending, slitting, cutting out, drawing down (thinning) swaging, upsetting (thickening) punching etc. Light and small parts are made by hand forging while machine forging is used for large and heavy parts when power hammers are used for performing the basic operations mentioned above.

In die forging the energy is applied to force the metal to assume the form determined by the shape of the dies. Most of the die forging requires a series of dies, usually three sets, for a gradual change of the form of steel into an accurately shaped machine part.

In press forging the piece is shaped by the application of steady pressure rather than by repeated blows. Press forging is better than die forging for pieces with heavy sections. For the shaping of pieces with thin sections die or drop forging is good.

The relative forgeability index of various metals is given in T-III-1. The forging design data is given in (T-III-2) & (T-III-3) of data book.

3.11 Other Forming Processes :

Rolling is to form long members of uniform section, such as bars, rails, structural sections and plates. The material may be hot rolled for large reduction in section or cold rolled for accurate sizing or grain control.

The *press working* operation include shearing, blanking, bending, drawing and squeezing which includes coining and extrusion. *Metal spinning* is a type of drawing used for forming surfaces of revolution including beads and seams. The *extrusion* process is used to form softer metals such as aluminium and magnesium alloys into a variety of shapes.

Sintering is a process that is used to combine compressed powedered metals or a mixture of powdered metals by heat treatment so that a rigid piece having desirable properties is formed. Non metallic substances may or may not be included. Some examples of sintered products are bearing sleeves composed of a mixture of bronze and graphite, porous bronze bearing sleeves impregnated with oil etc.

3.12 Joining Processes :

Rivetting and welding processes are discussed in chapter-6. Soldering is used to join metal parts by the use of a filler metal whose melting temperature is lower than that of the metals to be joined. Brazing uses a harder filler metal than is used in soldering and forms a stronger joint. Brazing in a furnace is very effective and forms strong and neat joints.

* * *

Chapter 4

DESIGN FOR STATIC & FATIGUE LOADING

4.1 Introduction :

Many factors enter when the designer wants to specify the shape, dimensions and materials for an element of a machine. In many cases, a limitation of the material or the manufacturing processes will be the final criteria for some dimensions. This is particularly true when the part is being made as a casting or a weldment or when heat treatment is involved. It is also true that most of the practical considerations do not lend themselves to expression as exact theoretical equations and consequently exact solutions of design problems are seldom found. A primary consideration in designing a part is the necessity of making it strong enough so that it will not fail in service. Most people, commonly associate the word *failure* with the actual breaking of a part. However, a designer must have a broader interpretation and understanding of what really should be considered as failure. A designer must consider a part to have failed when it is no longer capable of performing its required function in a satisfactory manner. This definition includes many types of failures in addition to actual breaking. For example, failure may be related to the effects of wear, level of noise generated elastic and plastic deformation, creep of materials under elevated temp. The designer, in considering all the possible modes of failure, must base his calculations upon one of the following :

1. Allowable stress or strength
2. Elastic stability
3. Wear
4. Manufacturing process requirements.

In almost every case, the calculations will give the minimum permissible dimensions and any further increase will generally make the part stronger, more rigid and more stable.

4.2 Factor of Safety :

The term factor of safety is one of the most misunderstood and misused expressions in the language of machine design. There are numerous ways in which factors of safety have been defined, but in general, the *variations* correspond closely to one of the two equations.

$$\text{F.S.} = \frac{\text{Critical strength}}{\text{Allowable stress}} \quad ; \quad \text{F.S.} = \frac{\text{Maximum safe load}}{\text{Normal service load}}$$

Equation in terms of stress is the more common definition, and most machine parts are designed on the basis of limiting or allowable stress. It is very important to recognize that the rigidity and elastic stability of parts are usually not direct functions of stress and therefore in these cases it is necessary to consider the factor of safety in terms of loads. The selection of proper value of factor of safety is quite difficult and therefore we must keep in mind that these calculations give appropriate values. If possible the results should be checked experimentally.

The selection of factor of safety can be based on the following sources.

(a) Legislated safety factors in various design codes.

(b) Published factors of safety by various institutions.

(c) Factors of safety based on experience.

However, if the equipment is being designed for the first time it is better to select the factor of safety independently. There are basically three sources of uncertainly.

1. Uncertainty in the magnitude of loads.
2. Uncertainty in the properties of materials.
3. Uncertainty in the analysis.

Further to these the factor of safety is meant to take care of the desired reliability, effect of break down etc. The factor of safety F.S. may be considered as a product of three factors :

F.S. = a × b × c where a = type of load
b = manner in which load is applied
c = factor of ignorance.

The criteria for selecting the damaging stress or critical stress depends on the type of load i.e. whether static or variable.

4.3 Factor of Safety with Static Loading :

Static loading does not mean that the load carried by the part is literally static and never varies in any manner; it does mean the load varies relatively few times and that the variations occur slowly enough so that neither fatigue, nor impact need be considered. Before a factor of safety can be related and applied for a design problem, it is necessary to consider the behaviour of the chosen material under the specified operating conditions. For the case of static loading, the basic material property is that of ductility, and it is desirable to classify a material as either *ductile or brittle*. Unfortunately there is no absolute dividing line between the two classes. The usual practice is to consider a material ductile if its percentage elongation is more than 10 percent and brittle if it is less than 5 percent. In between 5 and 10 percent, it is grey area.

The most important attribute of a ductile material under static loading is that the high stresses due to stress concentration at discontinuities will be relieved by local yielding at the region of high stresses. Thus if minor dimensional changes due to the local yielding can be tolerated, stress concentration effects need not be considered when designing a part made of ductile material under static loading. Further, ductile materials have well defined yield points. Hence the critical stress in these cases is the *yield strength* S_y and not the ultimate strength S_u. Therefore the factor of safety for a ductile material under static loading is defined as :

$$\text{F.S.} \frac{\text{Yield strength}}{\text{Design stress}} = \frac{S_y}{\sigma_d} \text{ or } \frac{S_{ys}}{\tau_d}$$

The term design stress or allowable stress have the same meaning.

A part made from brittle material would give no warning in the form of yielding before fracture, nor would there be any local yielding in the region of high stresses due to stress concentrations. Consideration of the behaviour of a brittle material under static loading leads to a definition of factor of safety for this case.

$$\text{F.S.} = \frac{S_u}{K_t\,\sigma_d} \quad \text{Where } \sigma_d = \text{nominal design stress.}$$

K_t = theoretical stress concentration factor.

Brittle materials are generally used for tensile or compressive loads only.

4.4 Failure Theories :

In designing parts to resist failure, we usually assure ourselves that the internal stresses do not exceed the strength of the material. If the material is ductile, then it is the yield strength we are usually interested in. For brittle materials, such as cast iron, which have no yield point the ultimate strength is the criteria of failure. Now when the parts are subjected to combined stresses, we have to decide the particular damaging stress or critical stress to which the factor of safety can be applied. To solve this problem, many theories of failure have been suggested. The three commonly used theories of failure will be discussed below.

(a) The Maximum-Normal-Stress Theory or Rankine Theory : This theory states that failure occurs whenever the largest principal stress equals the strength. Suppose we arrange the three principal stresses for any stress state in the form.

$$\sigma_1 > \sigma_2 > \sigma_3.$$

Then if yielding is the criteria of failure, this theory predicts that failure occurs when

$$\sigma_1 = S_{yt} \text{ or } \sigma_3 = -S_{yc}.$$

If the ultimate strength is used, then failure occurs when

$$\sigma_1 = S_{ut} \text{ or } \sigma_3 = -S_{uc}.$$

For pure torsion $\sigma_1 = \tau = -\sigma_3$ and $\sigma_2 = 0$. Thus this predicts wrong results, hence this theory is only used for tensile and compressive loads and not for pure torsion.

(b) The Maximum Shear Stress Theory or Guest Theory : This theory states that yielding begins whenever the maximum shear stress becomes equal to the maximum shear stress in tensile-test-specimen of the same material when that specimen begins to yield. Thus this theory predict that failure will occur whenever.

$$\tau_{max.} = \frac{S_y}{2} \text{ or } \sigma_1 - \sigma_3 = S_y$$

Note that this theory also states that the yield strength in shear is given by the equation.

$$S_{ys} = 0.50\ S_y.$$

(c) The Distortion-Energy Theory or Von-Mises-Hanckey Theory : it is only slightly more difficult to use than the maximum shear stress theory and it is the best theory for ductile material. Like the maximum-shear-stress theory, it is employed to design only the beginning of yield. This theory states that the beginning of yield will occur whenever.

$$2S_y^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

For biaxial stress state, $\sigma_3 = 0$, this criteria becomes :

$$S_y^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2.$$

This theory predicts that for pure torsion.

$$S_{ys} = .577\ S_y.$$

These theories and failure criteria are given in (T-I-21). Accordingly the Rankine theory is not used for pure torsion. It is generally used for brittle materials. The Guest theory always gives safe results. The distortion energy theory should be used when part is to be designed without actual testing. The use of these theories will be illustrated in the solved problems.

4.5 Design For Fatigue Strength :

Until about the middle of the nineteenth century engineers treated fluctuating or repeated loading the same as static loading, except for the use of larger safety factors. But this often resulted in over designing of parts which resulted in increased costs. Hence the research was continued & today parts can be designed for the time varying load ....fatigue loads ... with the same factors of safety as static loads.

Fatigue fractures (Progressive fractures) begin with a minute crack (usually microscopic) at a critical area of high local stress. This is almost always at a geometric stress raiser. Additionally minute material flaws or preexisting cracks are commonly involved due to numerous factors. Once a crack has developed, the stress concentration effect becomes greater and the crack progresses more rapidly. As the stressed area decreases in size, the stress increases in magnitude until finally, the remaining area fails suddenly. A fatigue failure is, therefore characterized by two distinct areas of failure :

1. The first is that due to progressive development of the crack.
2. The second is due to sudden fracture.

The area of sudden fracture is very similar in appearance to the fracture of a brittle material such as cast-iron.

When machine parts fail statically, they usually develop a very large deflection in ductile materials and the part is replaced before fracture occurs. Thus many static failures are visible ones & give warning in advance. But a fatigue failure gives no warning, it is sudden and total and hence dangerous.

4.6 The S-N Diagram :

To determine the properties of materials under static loading, the standard tension test is carried out and the stress-strain diagram obtained. In a similar fashion to determine the strength of materials under the action of fatigue loads, specimen are subjected to repeated or varying forces of specified magnitudes while the cycles are counted to destruction. The most widely used fatigue testing machines is the K. R. Moore high speed rotating beam machine which subjects the specimen to pure bending (no transverse shear) by means of weights. The results of the test are plotted in the form of S-N diagram on log-log paper. There are two types of S-N diagram obtained in practice Fig. (4.1).

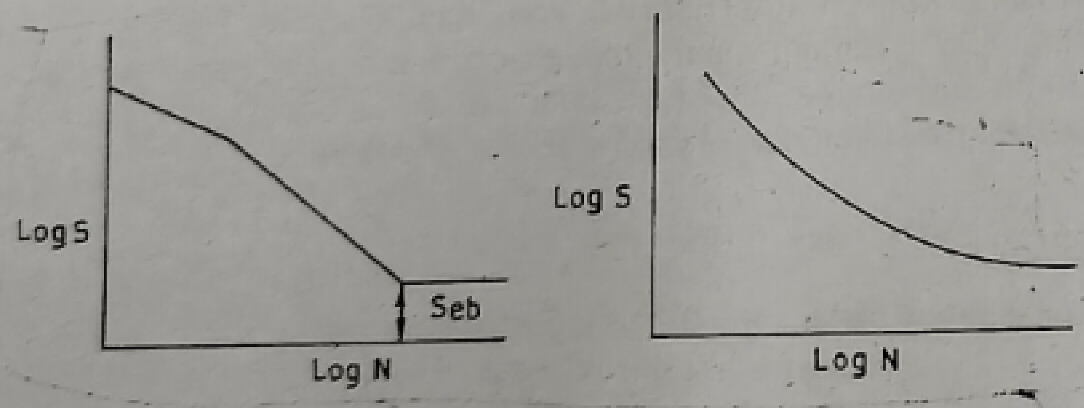


Fig. 4.1

The first diagram is for ferrous metals and alloys for whom the graph becomes horizontal after the material has been stressed for a certain number of cycles. The stress corresponding to the knee is called the *endurance limit or fatigue limit.* The second diagram never becomes horizontal and is for non-ferrous metals and alloys. Hence they have no endurance limit. However we define a fatigue limit which must always correspond to the number of cycles to failure. The endurance limit under bending load called S_{eb} is given in tables of chapter-II of data book. Generally this knee occurs between

10^6 and 10^7 cycles. Therefore it is customary to make the conservative assumption that ferrous materials must not be stressed above the endurance limit if a life of 10^6 or more cycles is required. In the absence of knee or true endurance limit, the fatigue strength at 10^8 or 5×10^8 cycles is often used.

The parts subjected to fatigue loading are generally designed for either finite life i.e. low cycle fatigue or for infinite life i.e. high cycle fatigue. If the stress reversals are less than 10^3 cycles it is called low cycle fatigue whereas stress reversals of more than 10^3 cycles is called high cycle fatigue. If the life is more than 10^7 cycles it is called infinite life. To find the fatigue limit between 10^3 cycles to 10^6 cycles for steels from the results of simple tensile test the following diagram is plotted and used Fig. (4.2).

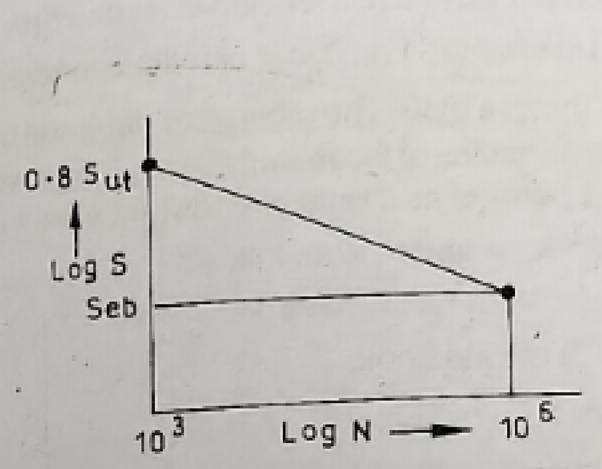


Fig. 4.2

(T-I-19) of data book also gives equations for finding the fatigue limit for finite life, S_{ef}.

4.7 Endurance Limit Modifying Factors :

It can be seen that the rotating beam specimen used in the laboratory to determine endurance limits or fatigue limits is prepared very carefully and tested under closely controlled conditions. It is unrealistic to expect the endurance limit of a mechanical element to match values obtained in the laboratory. Hence we modify the *endurance limit of the material* S_{eb} by some modifying factors to obtain the *endurance limit of a part* S_e.

As per (T-I-12) of data book we have ;

$$S_e = \frac{A . K_R K_\theta K_Z K_S}{K_f} \times S_{eb}$$

Where A, K_R, K_θ, K_Z, K_S are modifying factors and K_f is the stress concentration factor. These factors are discussed below.

(a) Factor for type of loading A : The standard laboratory test is carried out under reversed bending load. For other types of load the modifying factor A is to be used.

(b) Reliability Factor K_R : Generally the machines are designed for a reliability of 50 percent to 99.99 percent depending on the application. Hence we must choose a proper value of K_R as per the requirement.

(c) Temperature Factor K_θ : The temperature changes all the mechanical properties of the material and that the existence of a static or mean load also induces creep in the material. Hence, if the operating environment has a temp. greater than 70°C the endurance limit is modified by the factor K_θ.

(d) Size Factor K_Z : The round rotating beam test gives endurance limit for a specimen usually having a diameter of 7.5 or 12.5 mm. It turns out that the endurance limits of machine elements having a larger size or a different shape, seldom approach the values found in the laboratory tests. This effect due to dimensions or shape is called the size effect. This is given in (T-I-13) and (T-I-14) of data book.

(e) Surface Finish Factor K_S : The surface of the rotating beam specimen is highly polished. Obviously most machine elements do not have such a high quality finish. This modification factor depends upon the quality of the finish and the tensile strength of the material. It is given in (F-I-7) of data book.

(f) Fatigue Stress Concentration Factor K_f : Most mechanical parts have discontinuities which change the stress distribution. The stress concentration effect for static loading is taken care of by the theoretical stress concentration factor K_t as discussed in chapter-1. In cyclic or fatigue loading, the effect of the notch or fillet is usually less than predicted by the use of K_t. The difference depends on the stress gradient in the region of stress concentration and the hardness of the material. This difference is defined by a factor called Notch sensitivity factor q which is defined as a ratio of endurance limit of parts with notch and without notch. The relationship between K_t, q and K_f is as follows :

$$K_f = 1 + q\,(K_t - 1)$$

All the modifying factors are given in (T-I-12) to (T-I-18) of data book.

4.8 Fatigue Strength Under Fluctuating Stresses :

Quite frequently it is necessary to determine the strength of the parts corresponding to stress situations other than complete reversals. Many times in design the stresses fluctuate without passing through zero. These are called Repeating and Alternating loads as discussed in article 1.10 of chapter-1. Any load can be thought of consisting of two portions (1) Mean load F_m which may be treated as static load and (2) variable load F_v which should be treated as Fatigue load. The design principles associated with static loads and fatigue loads may be applied to the respective portions of load. However various diagrams have been suggested for failure of parts under a combination of mean and variable load. (T-I-20).

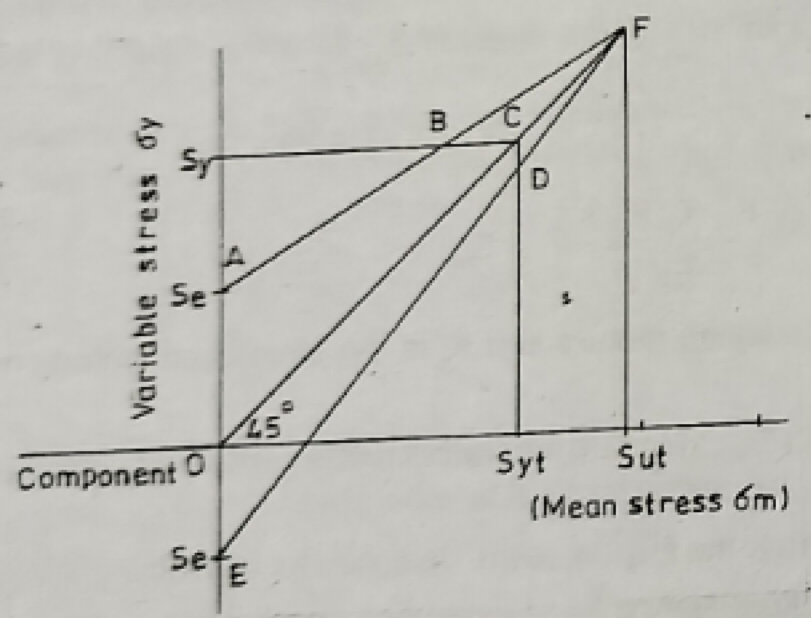


Fig. 4.3

(a) Modified Goodman Diagram : This diagram shows all the strengths and defines the safe area when a combination of mean and variable stress is induced in the component. Fig. (4.3)

(b) Soderberg Diagram : In this diagram which is very conservative the diagram is plotted on only one side of the variable stress. Fig. (4.4).

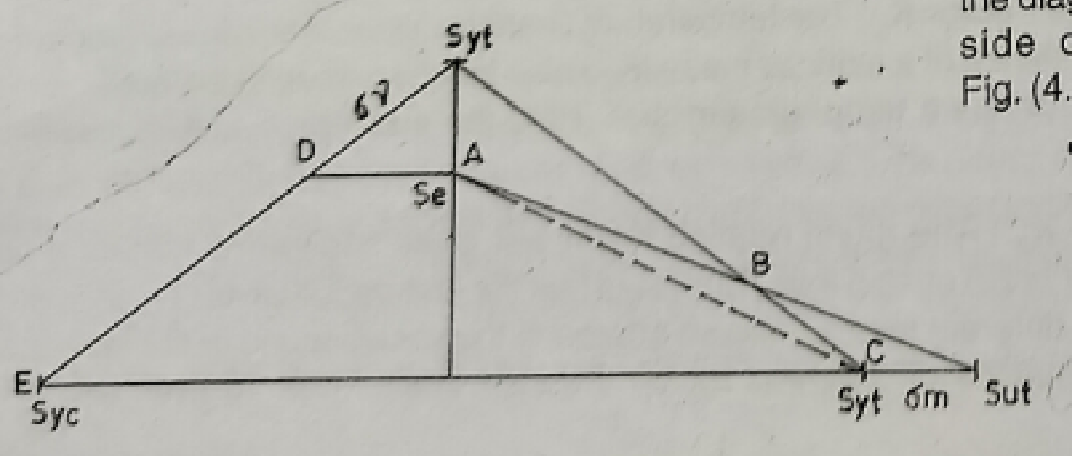


Fig. 4.4

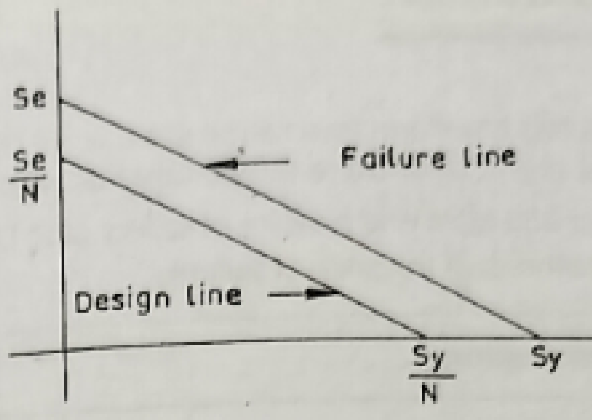


Fig. 4.5

As soderberg diagram is simple, many design equations are based on the failure line AC shown dotted in the diagram. For designing parts a design line can be plotted with a factor of safety N as shown in the diagram. Fig. (4.5)

Therefore the design equation becomes

$$\frac{1}{N} = \frac{\sigma_m}{S_y} + \frac{\sigma_v}{S_e}$$

This equation is further modified for ductile materials, brittle materials and combined stresses. These are given in the next article.

4.9 Design Equation for Fatigue Loading :

In using these equations we must use proper values of stress concentration factors as applicable to the given situation. To summarize.

(a) For mean stresses : Use proper value of K_t for brittle materials only. For ductile materials no K_t value is to be used.

(b) For variable stresses : Use proper value of K_f for both ductile and brittle materials.

For combined stresses, find the principal stresses for the mean components, σ_m and for the variable components, σ_v and then use the proper theory of failure. The design equations according to various theories of failure are given in T-I-20 of data book. They are

1. **Max. Normal stresses theory :** $\frac{1}{N} = \left[\sigma_{1m} + \frac{\sigma_{1v}}{S_e}.S_y\right]\frac{1}{S_y}$

2. **Max. Shear stress theory :** $\frac{1}{N} = \left[\sigma_{1m} + \frac{\sigma_{1v}}{S_e}.S_y\right]\frac{1}{(\sigma_{1m} - \sigma_{2m})}$

3. **Distortion Energy theory :** $\frac{1}{N} = \left[\sigma_{1m} + \frac{\sigma_{1v}}{S_e}.S_y\right]\frac{1}{\sqrt{(\sigma_{1m}^2 - \sigma_{1m}.\sigma_{2m} + \sigma_{2m}^2)}}$

Alternately, for each induced normal and shear stress consisting of mean and variable part, an equivalent static stress can be found by the relations.

$$\sigma_{eq} = \sigma_m + \frac{S_y}{S_e}.\sigma_v \quad \text{and} \quad \tau_{eq} = \tau_m + \frac{S_{ys}}{S_e}.\tau_v$$

The values of S_e for each case should be calculated for the corresponding type of load. These equivalent stresses can then the treated as static stresses and the design completed as for static loads.

The use of these procedures will be illustrated with the help of solved problems.

SOLVED PROBLEMS

Prob. 4.1 : The member shown in the figure has a uniform transverse load, $w = 0.72$ kN/m; an axial preload, $F = 2$kN and a torsional load, $T = 24$ N-m. Determine the suitable diameter d of the member using a material having a yield strength $S_y = 385$ MPa and a factor of safety of 2. Design it as per distortion energy theory and check it as per the other two theories of failure.

Ref.	Calculations

Total bending load, $W = wL = 0.72 \times 0.5 = .36$ kN

$$\therefore R_1 = R_2 = \frac{W}{2} = 1.8 \text{ kN}$$

$\therefore$ Maximum B.M.

$$M_{max.} = R_1 \times \frac{L}{2} - \frac{W}{2} \times \frac{L}{4}$$

$$= .18 \times \frac{500}{2} - \frac{.36}{2} \times \frac{500}{4}$$

$$= 22.5 \text{ N-m.}$$

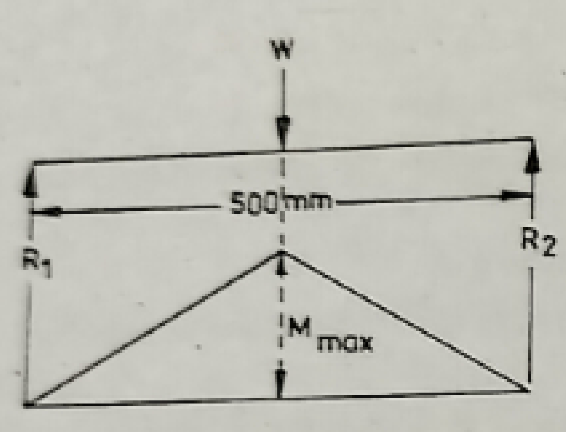


Fig.

$\therefore$ At the critical section i.e. centre of the shaft we have three forces acting and the three stresses are given below :

T-I-4 1. Axial compression = 2 kN, $\sigma_c = \dfrac{4 \times 2 \times 10^3}{\pi d^2} = \dfrac{2.55 \times 10^3}{d^2}$ MPa.

T-I-1 2. B.M. = 22.5 N-m, $\sigma_b = \dfrac{32 \times 22.5 \times 10^3}{\pi d^3} = \dfrac{229.3 \times 10^3}{d^3}$ MPa

3. Torque = 24 N-m. $\tau = \dfrac{16 \times 24 \times 10^3}{\pi d^3} = \dfrac{122.3 \times 10^3}{d^3}$ MPa

Now σ_c and σ_b will be added on the top surface of the shaft.

$$\therefore \text{Total } \sigma_c = \left[\frac{2.55 \times 10^3}{d^2} + \frac{229.3 \times 10^3}{d^3}\right], \; r = \frac{122.3 \times 10^3}{d^3}$$

$$\sigma_1 = -\frac{\sigma_c}{2} + \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + \tau^2}$$

$$= -\frac{114.65}{d^3}(.01d + 1) + \frac{10^3}{d^3}\sqrt{114.65^2(.01d + 1)^2 + 122.3^2}$$

$$\sigma_2 = -\frac{\sigma_c}{2} - \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + r^2}$$

$$= -\frac{114.65 \times 10^3}{d^3}(.01d + 1) - \frac{10^3}{d^3}\sqrt{114.65^2(.01d + 1)^2 + 122.3^3}$$

$$\therefore \tau_{max.} = \frac{\sigma_1 - \sigma_2}{2} = \frac{10^3}{d^3}\sqrt{114.65^2 . (.01d + 1)^2 + 122.3^2}$$

For trial and error, neglect (.01 d) term

$$\sigma_1 = \frac{10^3}{d^3}\left[-114.65 + \sqrt{13144.6 + 14957.3}\right] = \frac{53 \times 10^3}{d^3}$$

$$\sigma_2 = -\frac{10^3}{d^3}[114.65 + 167.63] = \frac{-282.28 \times 10^3}{d^3}$$

$$\therefore \text{Max. normal stress} = \frac{-282.28 \times 10^3}{d^3}$$

$$\text{Max. shear stress} = \frac{\sigma_1 - \sigma_2}{2} = \frac{167.63 \times 10^3}{d^3}$$

For distortion energy theory :

$$\sigma_e = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} = \frac{10^3}{d^3}\sqrt{53^2 + 282.28 \times 53 + 282.28^2} = \frac{312.17 \times 10^3}{d^3}$$

$\therefore$ Based on distortion energy theory, $\sigma_e = \dfrac{S_y}{FS}$ or $\dfrac{312.17 \times 10^3}{d^3} = \dfrac{385}{2}$

or $d = 11.74$ mm.

As we have neglected .01 d we should increase d by a factor $\sqrt[3]{(1 + 0.1d)}$

$d = 11.74 \times \sqrt[3]{(1 + .01 \times 11.74)} = 12.18$ mm. So select $d = 12.5$ mm.

$$\therefore \sigma_1 = \frac{10^3}{12.5^3} - 114.65(1 + .01 \times 12.5) + \sqrt{114.65^2(.01 \times 12.5 + 1)^2 + 122.3}$$

$$= 24.93 \approx 25 \text{ MPa}$$

$$\sigma_2 = -0.512 \times [-129 - 177.7] = -157 \text{ MPa}$$

$$\tau_{max.} = \frac{\sigma_1 - \sigma_2}{2} = \frac{25 + 157}{2} = 91 \text{ MPa}$$

	FS for max. normal stress theory $= \frac{385}{157} = 2.45$ FS for max. shear stress theory $= \frac{385}{91 \times 2} = 2.09$ $\sigma_e = \sqrt{25^2 + 25 \times 157 + 157^2} = 170.87$ MPa FS for distortion energy theory $= \frac{385}{170.87} = 2.253$

Prob. 4.2 : A lever subjected to a downward static force of 1.8 kN is keyed to a 25 mm round bar as shown in Fig.

(a) Find the critical stresses in the round bar

(b) The round bar is made of a material with $S_y = 1020$ MPa. Find the factor of safety using the distortion energy theory.

(c) Find the factor of safety using the max. shear stress theory and compare with (b).

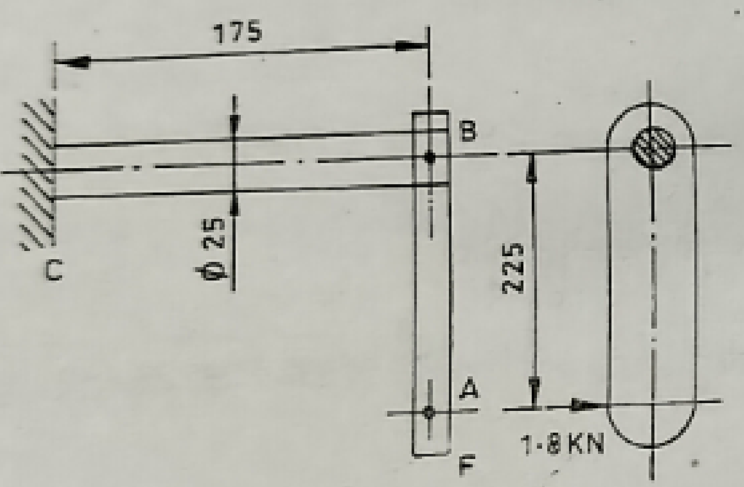


Fig.

Ref.	Calculations
	The critical section is at C. There will be three types of stresses at C.
	C.S. Area of rod $= \frac{\pi}{4} \times 25^2 = 490.6 \text{ mm}^2$
	1. Bending shear stress which is max. at the centre. but zero at the top & bottom where bending stresses are maximum. So not considered for finding principal stresses.
T-I-4	2. Shear stress due to torsion $\tau = \frac{1.8 \times 10 \times 225 \times 16}{\pi \times 25^3} = 132$ MPa.
	3. Bending stress due to B.M. $\sigma_b = \frac{1.8 \times 10^3 \times 175 \times 32}{\pi \times 25^3} = 205.5$ MPa

T-I-6	$\sigma_1 = \frac{205.5}{2} + \sqrt{\left(\frac{205.5}{2}\right)^2 + 132^2} = 270$ MPa $\sigma_2 = 102.75 - 167.27 = -64.52$ MPa
T-I-21	with distortion energy theory : $\sigma_e = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} = \sqrt{270^2 + 270 \times 64.52 + 64.52^2} = 307.4$ MPa $\therefore$ Factor of safety $= \frac{1020}{307.4} = 3.32$ Max. shear stress $= \tau_{max.} = \frac{\sigma_1 - \sigma_2}{2} = 167.27$ MPa $\therefore$ Factory of safety $= \frac{1020}{167.27 \times 2} = 3.05$

Prob. 4.3 : A steel strut as shown in the figure, is rectangular in cross -section and is to be part of the operating mechanism of a machine. The compressive load to be carried is 180 kN. Determine the following :

(a) the relationship between b and w required for equal strengths of the strut in failure by buckling in either plane.

(b) the required dimensions b and w assuming a FS of 4 and SAE 1020 steel.

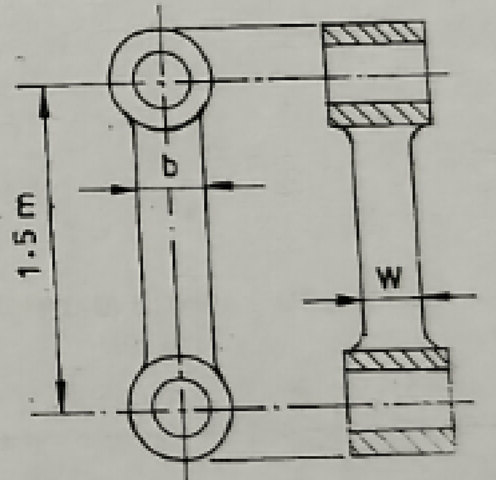


Fig.

Ref.	Calculations
T-I-24	Here the end conditions in two planes and moment of inertia are different.
	In plane 1, the two ends are hinged. Hence $n_1 = 1,\ I_1 = \frac{wb^3}{12}$
	In plane 2, the two ends are fixed Hence $n_2 = 4,\ I_2 = \frac{bw^3}{12}$

$\therefore$ For the strut to be equally strong in the two planes

$$n_1 I_1 = n_2 I_2 \text{ or } \frac{wb^3}{12} = \frac{4\,bw^3}{12} \quad \text{or} \quad b^2 = 4w^2 \text{ or } b = 2w.$$

Hence we employ the definition of factor of safety in terms of load

$\therefore F_{cr} = 4 \times F = 4 \times 180 = 720$ kN.

T-I-7 For SAE 1020 : $S_{yt} = 246 \;\; E = 205 \times 10^3 \;\; \therefore \; S_d = \frac{246}{4} = 61.5$ MPa

As a pure compn. member, $A \times S_d = F$; $2w^2 \times 61.5 = 180 \times 10^3 \quad w = 38.25$ mm.

$$K = \frac{w}{\sqrt{3}} = \frac{38.25}{\sqrt{3}} = 22.08 \text{ mm.} \qquad \therefore \frac{l}{k} = \frac{1500}{22.08} = 67.92$$

So it will be a short column. Use Rankine's formula.

T-I-24

$$S_y = \frac{F_{cr}}{A}\left[1 + \frac{1}{an}\left(\frac{l}{k}\right)^2\right] \qquad A = 2w^2 \,;\; K = \frac{w}{\sqrt{3}},\; l = 1500$$

$$a = \frac{\pi^2 E}{S_y} = \frac{\pi^2 \times 20 \times 10^3}{246} = 8612$$

$$246 = \frac{720 \times 10^3}{2w^2}\left[1 + \frac{1}{8612}\left(\frac{1500 \times \sqrt{3}}{w}\right)^2\right]$$

$w^4 - 1463.4\,w^2 - 1202.3 \times 10^3 = 0.$; $\quad w^2 = 2049.9;\;\; w = 45.276$ mm.

Assume $w = 46,\; b = 92,\;\; K = \frac{46}{\sqrt{3}} = 26.56 \;\; \frac{l}{k} = \frac{1500}{26.56} = 56.5$

$$F_{cr} = \frac{246 \times 46 \times 92}{1 + \frac{56.5^2}{8612}} = 749.9 \times 10^3 \text{ which is greater than required } F_{cr}.$$

Prob. 4.4 : It is desired to determine the size of a cold drawn steel bar to withstand a tensile preload of 36 kN and a fluctuating tensile load varying from 0 to 72 kN. Owing to the design of the ends the bar will have a geometric stress concentration factor of 2.02. The properties of steel are $S_{yt} = 510$ MPa, $S_{ut} = 650$ MPa. Determine the suitable diameter for an infinite life and a factor of safety of at least 2.0.

Ref.	Calculations
	$F_{min.} = 36$ kN $\quad F_{max.} = 36 + 72 = 108$ kN
	Now $S_d = \frac{S_y}{N} = \frac{510}{2} = 255$ MPa

$\therefore$ For static load : $\frac{\pi}{4} d^2 \times S_d = F_{max}$

$$\therefore d = \sqrt{\frac{4 \times 108\,10^3}{\pi \times 255}} = 23.55 \text{ mm}$$

Now for fatigue design

$$F_m = \frac{108 + 36}{2} = 72 \text{ kN} \quad ; \quad F_v = \frac{108 - 36}{2} = 36 \text{ kN}$$

T-I-6A $S_{eb} = 0.5\, S_{ut} = \frac{650}{2} = 325$ MPa

T-I-12 $S_e = \frac{A . K_R . K_\theta . K_Z . K_S}{K_f} \times S_{eb}$ Taking $A = 0.7$ for axial load

$K_R = 1.0,\; K_\theta = 1$

For size factor,

T-I-13 Take $K_Z = K_Z' = .85$ for medium carbon steel.

F-1-7 K_S = For rough finished = .86 $\qquad K_f = 1 + q\,(K_t - 1),\; q = .35$

$= 1 + 35\,(2.02 - 1) = 1.357$

T-I-16 $S_e = \frac{0.7 \times .85 \times .86}{1.357} \times 325 = 122.5$ MPa

$$\therefore \sigma_m = \frac{72 \times 10^3}{A} \text{ and } \sigma_v = \frac{36 \times 10^3}{A} \quad ; \quad N = 2$$

$$\therefore \frac{1}{2} = \frac{\sigma_m}{S_y} + \frac{\sigma_v}{S_e} = \frac{72 \times 10^3}{510\,A} + \frac{36 \times 10^3}{122.5\,A}$$

$$\frac{A}{2} = [141.1 + 293.8] = 434.9$$

$$\therefore \frac{\pi}{4} d^2 = 2 \times 434.9 = 869.8 \qquad \therefore d = \sqrt{\frac{4 \times 869.8}{\pi}} = 33.3 \text{ mm}$$

So select d = 34 mm.

Prob. 4.5 : A cold drawn steel rod of circular cross section is subjected to a variable bending moment of 600 to 1200 N-m as the axial load varies from 4500 to 13,500 N. The maximum bending moment occurs at the same instant that the axial load is maximum. Determine the required diameter of the rod for a factor of safety FS = 2. Neglect any stress concentration and column effect. Design on the basis of the maximum normal stress. Assume $S_{ut} = 550$ MPa & $S_{yt} = 470$ MPa.

Ref.	Calculations
	1. Considering the bending load $M_m = \frac{600+1200}{2} = 900\ N-m$ $M_v = \frac{1200-600}{2} = 300\ N-m$ $\sigma_{bm} = \frac{900 \times 10^3 \times 32}{\pi d^3} = \frac{9172 \times 10^3}{d^3}$ MPa $\sigma_{bv} = \frac{300 \times 10^3 \times 32}{\pi d^3} = \frac{3057 \times 10^3}{d^3}$ MPa
T-I-6A	For steel $S_{eb} = 0.5 \times S_{ut} = 0.5 \times 550 = 275$ MPa
T-I-12	$S_e = \frac{A.K_R.K_\theta.K_Z.K_S}{K_f} S_{eb}$ Here A = 1.0 for bending load.
T-I-14	$= 275 \times .85 \times .88$ $K_R = K_\theta = K_f = 1$ $= 205.7$ MPa $K_Z = .85$ $K_S = .88$ for rough finished
F-I-7	$\sigma_{beq} = \sigma_{bm} + \frac{S_y}{S_e} . \sigma_{bv}$ $= \frac{9172 \times 10^3}{d^3} + \frac{470}{205.7} \times \frac{3057 \times 10^3}{d^3} = \frac{16157 \times 10^3}{d^3}$ MPa 2. Considering the axial load $F_m = \frac{13500+4500}{2} = 9000\ N$ $\sigma_{am} = \frac{4 \times 9000}{\pi d^2} = \frac{11465}{d^2}$ $F_v = \frac{13500-4500}{2} = 4500\ N$ $\sigma_{av} = \frac{4 \times 4500}{\pi d^2} = \frac{5732.5}{d^2}$ $\frac{S_e\ (bending)}{S_e\ (axial)} = \frac{A\ (bending)}{A\ (axial)} = \frac{1}{0.7}$. For axial loads A = 0.7 $S_e = 0.7 \times 205.7 = 144$ MPa

T-I-20	$\sigma_{aeq} = \sigma_{am} + \frac{S_y}{S_e} \sigma_{av}$ $\sigma_{aeq} = \frac{11465}{d^2} + \frac{470}{144} \times \frac{5732.5}{d^2} = \frac{30176}{d^2}$ $\therefore$ Total equivalent normal stress $= \sigma_{beq} + \sigma_{aeq}$ $\sigma_{eq} = \frac{16157 \times 10^3}{d^3} + \frac{30176}{d^2}$ $= \frac{S_y}{FS} = \frac{470}{2} = 235$ $\therefore \sigma_{eq} = 235$ $\therefore 235\ d^3 = 16157 \times 10^3 + 30176\ d$ or $d^3 - 128.4\ d = 68.75 \times 10^3$. By trial and error, d = 42 mm.

Prob. 4.6 : A hot rolled steel shaft is subjected to a torsional load that varies from 300 N-m clockwise to 100 N-m anticlockwise as an applied bending moment at a critical section varies from + 400 N-m to − 200 N-m. The shaft is of uniform cross section and no stress raisers are present at the critical section. Determine the required shaft diameter. Assume factor of safety FS = 1.5. The material properties are $S_{ut} = 550$ MPa, $S_{yt} = 410$ MPa.

Ref.	Calculations
	1. Considering the torsional load
T-I-4	$T_m = \frac{300-100}{2} = 100\ N-m$ $\tau_m = \frac{16 \times 100 \times 10^3}{\pi d^3} = \frac{509.6 \times 10^3}{d^3}$ $T_v = \frac{300+100}{2} = 200\ N-m$ $T_v = \frac{16 \times 200 \times 10^3}{\pi d^3} = \frac{1019.2 \times 10^3}{d^3}$
T-II-6A	$S_{eb} = 0.5\ S_{ut} = 0.5 \times 550 = 275$ MPa
T-I-12	$S_e = \frac{A.K_k.K_\theta.K_Z.K_S}{K_f} S_{eb}$ A = 0.577 for torsional load
T-I-7	$= .577 \times .85 \times .65 \times 275$ $K_k = K_\theta = K_f = 1$
F-1-7	$= 87.66$ MPa $K_Z = .85$; $K_S = .65$ $S_{ys} = 0.60\ S_{yt}$ $= .6 \times 410 = 246 = 246$ MPa
T-I-20	$\tau_{eq} = \tau_m + \frac{S_{ys}}{S_e} \tau_v$ $= \frac{509.6 \times 10^3}{d^3} + \frac{246}{87.66} \times \frac{1019.2 \times 10^3}{d^3} = \frac{3369.5 \times 10^3}{d^3}$

2. Considering the bending load

$$M_m = \frac{400-200}{2} = 100\ \text{N-m} \qquad \sigma_{bm} = \frac{32\times100\times10^3}{\pi d^3} = \frac{1019\times10^3}{d^3}$$

$$M_v = \frac{400+200}{2} = 300\ \text{N-m} \qquad \sigma_{bv} = \frac{32\times300\times10^3}{\pi d^3} = \frac{3057\times10^3}{d^3}$$

T-I-12 $$S_e = \frac{A.K_R.K_\theta.K_Z.K_S}{K_f} S_{eb} \qquad A = 1$$

T-I-14 $K_Z = .85$

F-1.7 $= 275 \times .85 \times .65 = 152$ MPa $\qquad K_S = .65$ (Hot rolled)

$K_\theta = K_f = 1$

T-I-20 $$\sigma_{beq} = \sigma_{bm} + \frac{S_y}{S_e} \qquad \sigma_{bv} = \frac{1019\times10^3}{d^3} + \frac{410}{152} \times \frac{3057\times10^3}{d^3}$$

$$= \frac{9264.8\times10^3}{d^3}$$

$$\therefore \tau_{max.} = \sqrt{\left(\frac{\sigma_{beq}}{2}\right)^2 + \tau_{eq}^2}$$

$$= \frac{10^3}{d^3}\sqrt{\left(\frac{9264}{2}\right)^2 + (3369.5)^2} = \frac{5.73\times10^6}{d^3}$$

For combined loading $S_{ys} = 0.5\ S_{yt} = 0.5 \times 410 = 205$ MPa

$$\therefore \frac{S_{ys}}{FS} = \tau_{max.} \text{ or } \frac{5.73\times10^6}{d^3} = \frac{205}{1.5}$$

$\therefore d = 34.7$ mm.

⁂

Chapter 5

SCREW FASTENERS & POWER TRANSMISSION SCREWS

5.1 Introduction :

Screw fastenings are used for holding two or more machine parts together or for adjusting one part in relation to another. In screw fastenings the threads are made in several forms but are always of triangular type, single thread. Screw threads are made right hand or left hand. Unless otherwise stated, it is always understood that the thread is right hand.

The fasteners are named according to how they are intended to be used rather that how they are actually employed in specific applications.

If a product is designed so that it's primary purpose is assembly into a tapped hole, it is a screw. The *screw* is tightened by exerting torque on the head. The special purpose screws are *cap screws* in which variety of shapes of head are available. *Machine screws* are cap screws slotted for a screw driver. *Setscrews* are used to prevent relative motion between parts.

If the product is designed so that it is intended to be used with a nut, it is a *bolt*. The bolt is tightened by exerting torque on the nut.

A *stud* resembles a threaded rod, one end assembles into a tapped hole, the other end receives a nut.

5.2 Thread Standards & Definitions :

The terminology of screw threads is explained as follows :

The *pitch* is the distance between a point on one thread to the corresponding point on the next adjacent thread measured parallel to the axis.

The *lead* is the distance the screw or the bolt will advance relative to the nut in one rotation. For a single thread screw, the lead is equal to the pitch.

The *major diameter or nominal diameter* is the largest diameter of a screw thread.

The *minor diameter or root diameter* is the smallest diameter of the screw thread.

Out of several thread forms, now a days, International metric threads are most commonly used. They are of two types -- *coarse and fine*. These are shown in (T-IV-1) and (T-VI-2) of data book along with their dimensions. These are specified with the help of their major diameter and the pitch. e.g. M18 × 2.5 means the major diameter is 18mm and the pitch is 2.5 mm. The form of thread used on pipes and called *pipe threads* are given in (T-VI-3) of data book.

5.3 Screw Torque and Axial Load :

The torque applied to the screw and the axial load built up in the screw are related by the equation (T-IX-1).

$$T = \frac{W}{2}\left[D_m \tan(\alpha + \phi) + D_c\, \mu_c\right]$$

Where α = Helix angle = $\tan^{-1}\frac{l}{\pi D_m}$, l = lead of the screw

D_m = mean diameter of the screw., D_c = mean collar diameter

ϕ = angle of friction of screw = $\tan^{-1}\left(\frac{\mu}{\sin\beta}\right)$

μ = coefficient of friction of screw and nut.

μ_c = coefficient of friction of collar.

β = Angle made by screw face with the axis.

Therefore the screw is subjected to two types of stresses.

(a) Tensile stress σ_t due to the resultant axial load which is the resultant of initial tightening load and the external load.

(b) Shear stress τ due to the torque applied during the initial tightening of the screw.

The initial tightening loads in bolts are highly indeterminate. The judgement of the mechanic in selecting a wrench and in applying the force cannot be predicted with accuracy. The initial tightening load in bolts, F_i, is approximately given by

$F_i = 3200\ d_b$ -- Metal to metal contact

$= 1600\ d_b$ -- For soft gasket between the members.

In ordinary fastenings, there is always metal to meal contact. The analysis of gasket joints for pressure vessels is dealt with in the chapter on cylinders. For normal screw fasteners, the allowable stress after considering the initial tightening load is given by the formula (T-X-2).

$S_d = .11\ (d_b^{\ 2} \div 25)$ MPa for rough joints

$= .18\ (1.2\ d_b^{\ 2} + 25)$ MPa for faced joints.

5.4 Bolts Subjected to Eccentric Loading :

Generally a group of bolts or rivets are subjected to eccentric loading. Two types of eccentric loading are generally encountered in practice which will be discussed below.

(A) In this type of joint the eccentricity of the load causes the turning of the bracket about the edge BB. This type of loading will cause two types of stresses in bolts. Fig. (5.1).

1. Direct shear stress
2. Tensile stress due to the turning action.

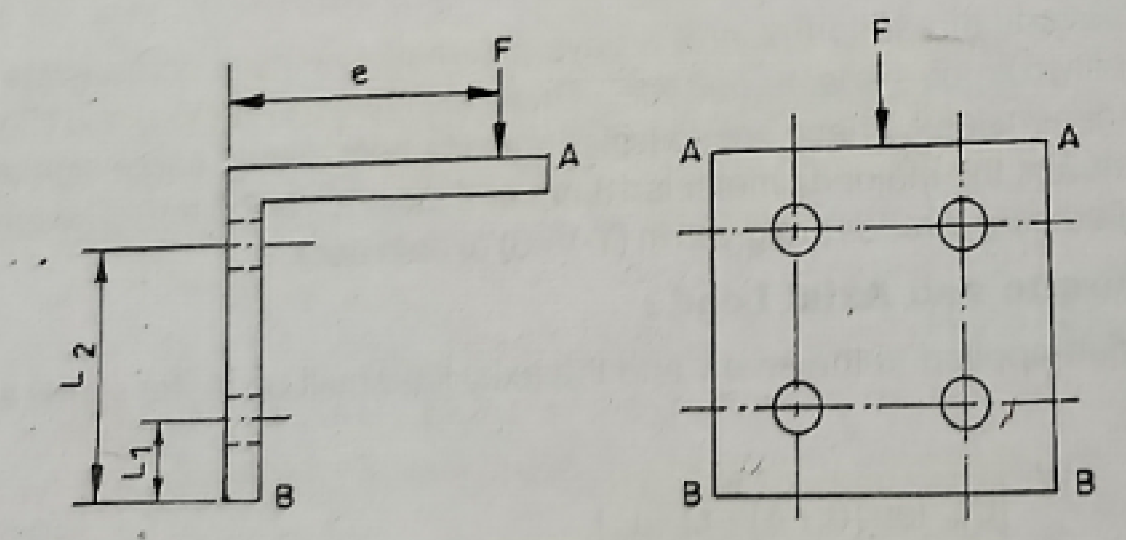


Fig. 5.1

The method of analysis is given below.

1. The shear load will be equal for all the bolts as the load is acting on the centre line of the bolts.

Let the area of the bolt be A mm^2.

$$\therefore \tau = \frac{F}{4A}$$

2. Let us assume that the tensile load on the bolts due to turning will be proportional to its distance from the edge BB. Let w be the load at unit distance. Therefore equating the turning moment by the external load to the resisting moment by the bolts about BB, we get,

$$w\left[2L_1^2 + 2L_2^2\right] = F \times e \quad \text{or} \quad w = \frac{F \times e}{2L_1^2 + 2L_2^2}$$

and maximum tensile load on the bolt $F_t(\text{max.}) = w\ l_2$.

$$\therefore \text{Tensile stress due to } F_t\ (\text{max.})\ \sigma_t = \frac{F_t\ (\text{max.})}{A}$$

The shear stress τ and the tensile stress σ_t are combined to find the principal stress, σ_1 and after selecting the design stress S_d we can find the diameter of the bolt.

(B) In this type of joint the eccentricity of the load will cause rotation of the part about the centroid of the bolts G. This loading will again cause two types of stress : Fig. (5.2).

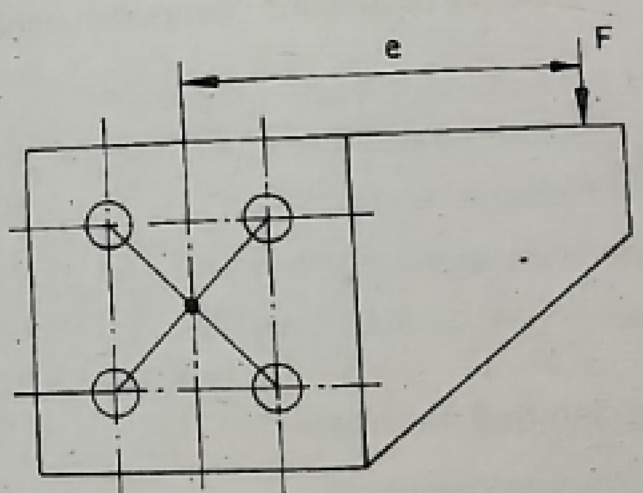


Fig. 5.2

1. Direct shear stress due to $F : \tau_1 = \frac{F}{4A}$ equal in all the four bolts.

2. Due to rotation about G : the shear force on the bolts will be proportional to their distance from G. Let the force at unit distance be w. Equating the turning moment by F to the resisting moment by the bolts we get,

$$w\left[l_1^2 + l_2^2 + l_3^2 + l_4^2\right] = F \times e$$

or $$W = \frac{F \times e}{\Sigma l^2}$$

The maximum shear force will be, F_{max} = wl (max.).

This will act perpendicular to the line joining the bolt centre to the centroid. The vector sum of the direct shear force and the force due to rotation will give the resultant shear force acting on the bolt. This will give the induced shear stress from which the diameter of the bolt can be found. The procedure for the design of screw fasteners will be illustrated with the help of solved problems.

5.5 Bolted Joint Subjected to Tensile Load :

These type of joints are frequently encountered in the design of fasteners specially in pressure vessels. The bolt is subjected to initial tension F_i due to tightening which is followed by the application of external load, F_e as shown in the Fig. (5.3).

Resultant bolt load $F_b = F_i + C F_e$

where $C = \frac{K_b}{K_b + K_c}$

K_b = spring stiffness of the bolt = $\frac{A_b E_b}{L}$

K_c = spring stiffness of the clamped members. The method for calculation of K_c is given below :

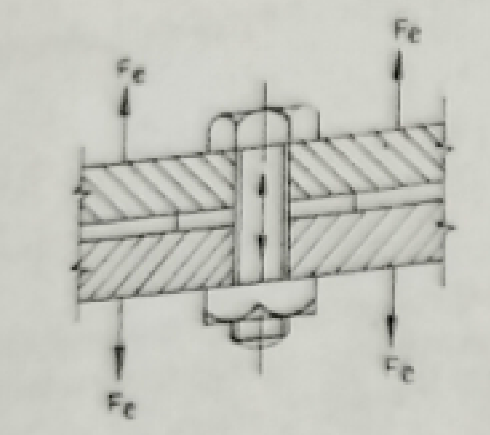


Fig. 5.3

Generally the clamped members will be the two flanges of metal and soft gasket. If we define the spring stiffness of flanges as K_1 and K_2 and that of gasket as Kg, then

$$\frac{1}{K_c} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_g}$$

If K_1 and K_2 are large $K_c = K_g$. The values of C are given in T-X-3 of data book. The joint will open when $F_e = F_b = F_i + C F_e$. When these joints are subjected to variable external load, F_e, the separation can be prevented if F_i is greater than $F_e (1 - C)$. With hard metal gasket $C \approx 0$ hence F_i should be greater than maximum external load. This is the principal used in tightening the cylinder studs in I.C. engines.

The value of K_c can be calculated if we can calculate the effective area of the clamped members as per formula given below.

$K_c = \frac{A_c E_c}{g}$ Where $A_c - d_b^2 + .68\, d_b \cdot g + .0.65\, g^2$

d_b – Major diameter of bolt

g – bolt grip

E_c – Modulus of elasticity of clamped members.

The design procedure will be illustrated with the help of solved problems.

5.6 Power Transmission Screws :

Power screws sometimes called *linear actuators* or *transmission screws*, are used to convert rotary motion of either the nut or the screw to relatively slow linear motion of the mating member along the screw axis. In many cases, the purpose is to obtain a great mechanical advantage to lift weights (as screw type jacks) or exert large forces (as presses or clamps). In other cases the purpose is to obtain precise positioning in axial direction (as lead screw of the lathe). The mechanism of all screws is based on the inclined plane theory. The equations for torque applied and the axial load are the same as given in art 5.3. For power screws generally two types of threads are used.

1. Square threads in which angle $\beta = 90°$. The standard proportions of metric square threads are given in (T-VI-4) and (T-VI-5) of data book. These threads are easier to cut and are used for screw jacks or other slow speed applications. They are difficult to manufacture, but are more efficient.

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2. Acme threads in which angle $\beta = 85.5°$. They are used for high speed applications. They can be used with split nut (split on an axial plane), the two halves can be moved together to compensate for thread wear. In addition another type of thread, Buttress thread is sometimes used for resisting large axial forces in one direction.

The design procedure of power screw consists of the following steps.

Step 1 : Calculate the maximum required core diameter D_c for the screw on the basis of axial load acting on the screw. If it is subjected to compressive load, it should be designed as a column as discussed in chapter-1.

Step 2 : Select the major diameter and the pitch as per (T-VI-5) so that the actual core diameter is more than that obtained in step 1.

Step 3 : Calculate the torque required to move the axial load acting on the screw as per T-IX-1 by selecting proper value of coefficient of friction as per (T-IX-2). Check that the screw is not reversible i.e. $\alpha < \phi$.

Step 4 : Calculate the shear stress τ due to the torque and the normal stress, σ due to the axial load W. Calculate the principal stress and the maximum shear stress and check whether they are less than allowable stresses.

Step 5 : Find the minimum number of threads required on the nut from the shearing strength of the threads on the nut as well as the screw and by the bearing consideration by selecting proper bearing pressure as per (T-IX-2). Select the larger number of threads and calculate the height of the nut.

Step 6 : Calculate the efficiency of screw.

The design procedure will be illustrated with the help of solved problems.

SOLVED PROBLEMS

Prob. 5.1 : The figure shows a 15 × 200 mm rectangular steel flat cantilevered to a 250 mm steel channel using four bolts. Based on external load of 16 kN find

(a) The maximum shear stress in the bolt.

(b) The maximum bearing stress in the bolt.

(c) The critical bending stress in the flat.

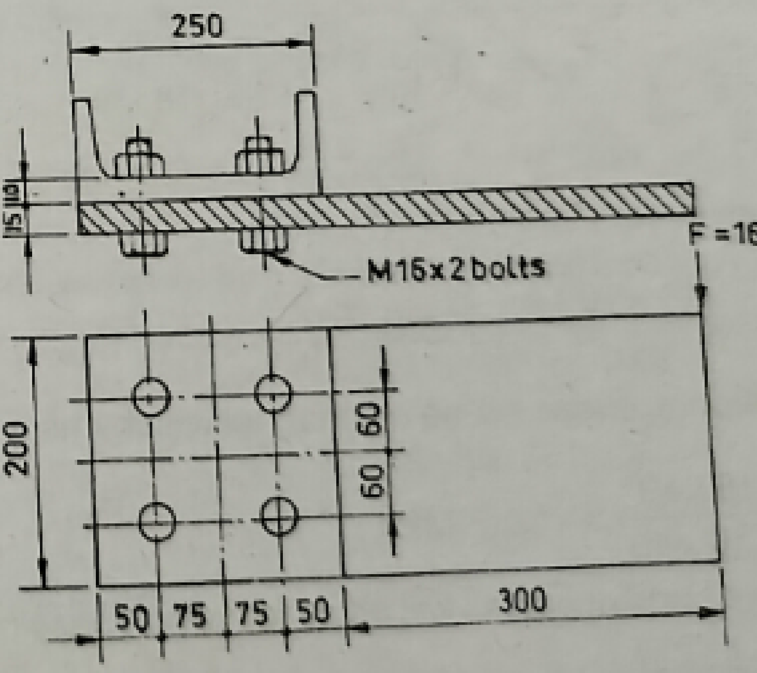


Fig.

Ref.	Calculations

T-VI-1

M16 × 2 bolts $\quad \therefore A = \frac{\pi}{4} 16^2 = 201 \text{mm}^2$

The bolt will be subjected to two forces

1. Direct shear force on the bolt $F_1 = \frac{16 \times 10^3}{4} = 4 \times 10^3$, N.

   It is equal on all the bolts.

2. Shear force due to rotation about the centroid of bolts G.

   The bolts are equidistant from G.

   $r = \sqrt{75^2 + 60^2} = 96$ mm.

$\therefore$ The force at unit distance, w, will be given by

$$w = \frac{16 \times 425 \times 10^3}{4 \times 96^2} = 184, \text{N}$$

The shear force on each bolt, $F_2 = w \times 96 = 184 \times 96 = 17.7 \times 10^3$, N

The direction of the shear forces F_1 and F_2 on each bolt is shown below :

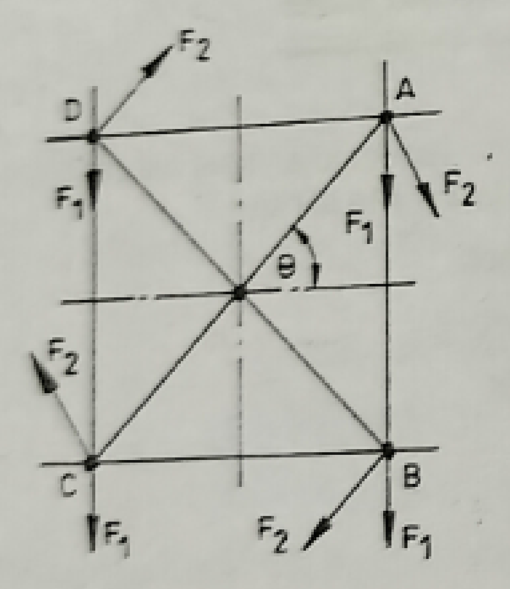


Fig.

$\tan\theta = \frac{60}{75} \qquad \therefore \theta = 38.66°$

The resultant shear force will be maximum on A and B.

Resultant shear force

$$F_R = \sqrt{(F_1 + F_2 \cos 38.66)^2 + (F_2 \sin 38.66)^2}$$

$$= \sqrt{(4 + 17.7 \times 7808)^2 + (17.7 \times .6247)^2}$$

$$= 18 \text{ kN.}$$

$\therefore$ Maximum shear stress,

$$\tau_{max.} = \frac{18 \times 10^3}{201} = 89.55 \text{MPa}$$

Maximum bearing stress will be on channel which has a thickness of 10 mm.

$$\sigma_b = \frac{18 \times 10^3}{16 \times 10} = 112.5, \text{ MPa}$$

The maximum bending stress will occur in section passing through bolts A and B.

BM = $16 \times 10^3 (300 + 50) = 5600$ N-m.

To find section modulus of the section

$$I = \frac{15}{12} . 200^2 - 2\left(\frac{15 \times 16^3}{12} + 15 \times 16 \times 60^2\right)$$

$$= 10^7 - 2 (5120 + 864000)$$

$$= 8.26 \times 10^6 \text{ mm}^4.$$

$$Z = \frac{I}{C} = \frac{8.26 \times 10^6}{100} = 8.26 \times 10^4 \text{ mm}^3$$

$$\therefore \text{Critical bending stress} = \frac{5600 \times 10^3}{8.26 \times 10^4} = 67.8 \text{MPa.}$$

Prob. 5.2 : The bracket shown in the figure is to be bolted to the flange of the beam by four identical bolts made from SAE 1020 steel. Assuming that the bolts are accurately fitted in reamed holes so that each bolt carries one fourth of the load in shear, what size of bolts should be used for this connection ?

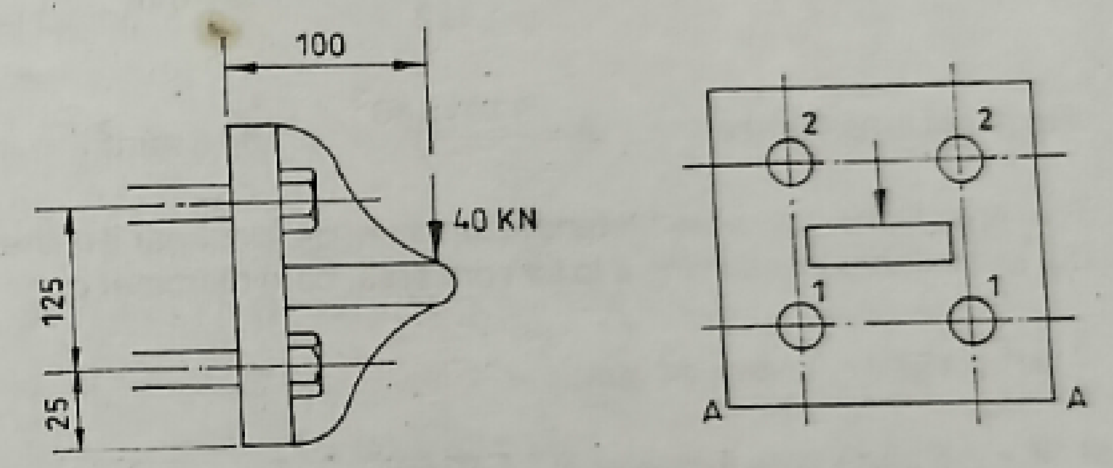


Fig.

Ref.	Calculations

1. The direct shear force on each bolt

   $$F_S = \frac{40}{4} = 10 \text{ kN} \qquad = \frac{10 \times 10^3}{A}$$

2. The force will try to turn the bracket about the edge AA.

This will be resisted by tensile forces in the bolts. Let the force at unit distance be w.

$$\therefore w = \frac{40 \times 100}{2\left(25^2 + 150^2\right)} = .0865 \text{ kN}$$

$\therefore$ Max. tensile force on bolts 2 will be

$$F_t = .0865 \times 150 \quad = 12.97 \text{ kN}$$

	$\therefore$ Principal stress $= \frac{12.97 \times 10^3}{2A} + \sqrt{\left(\frac{12.97 \times 10^3}{2A}\right)^2 + \left(\frac{10 \times 10^3}{A}\right)^2}$ $= \frac{10^3}{A}\left[6.485 + \sqrt{42 + 100}\right]$ $= \frac{10^3}{A}\left[6.485 + 11.91\right] = \frac{18.4 \times 10^3}{A}$ Max. shear stress $= \frac{11.91 \times 10^3}{A}$
T-II-7	SAE 1020 $S_{yt} = 246$, $S_{ys} = 154$, Fs = 2 $S_{dt} = \frac{246}{2} = 123$ and $S_{ds} = \frac{154}{2} = 77$ $\therefore$ Required area in tension $A = \frac{18.4 \times 10^3}{123} = 149.6\ \text{mm}^2$ Required area in shear $A = \frac{11.91 \times 10^3}{77} = 154.6\ \text{mm}^2$ This area is the core area for tensile stress while for shear it is the area of the shank. But for uniformity assuming it to be core area, core diameter dc. $\frac{\pi}{4} dc^2 = 154.6 \quad \therefore dc = 14\ \text{mm}$.
T-VI-I	M 18×2.5 has a core diameter of 14.48 mm. So select M18×2.5 bolts for the connection. The actual stress will be Major dia. area $A = \frac{\pi}{4} \times 18^2 = 254.34\ \text{mm}^2$ Core Area $A_c = \frac{\pi}{4} \times 14.48^2 = 164.59\ \text{mm}^2$ $\therefore$ Tensile stress $\sigma_1 = \frac{12.97 \times 10^3}{164.59} = 78.8\ \text{MPa}$ $\therefore$ Shear stress $\tau = \frac{10 \times 10^3}{254.34} = 39.3\ \text{MPa}$ $\therefore$ Max. shear stress $= \sqrt{\left(\frac{78.8}{2}\right)^2 + 39.3^2} = 55.65\ \text{MPa} < S_{ds}$ so o.k.

	Principal stress $= \frac{78.8}{2} + \sqrt{\left(\frac{78.8}{2}\right)^2 + 39.3^2}$ $= 39.4 + 55.65 = 95.05\ \text{MPa} < S_{dt}$ so o.k. So the bolts are safe.

Prob. 5.3 : The cover of a pressure vessel is held in place by ten M 20 × 1.5 bolts. The pressure is 1.4 MPa and the effective area of the cover exposed to the pressure is $0.2\ \text{m}^2$. The ratio of the stiffness of the bolts to connected parts is 1/4. Each bolt is tightened initially to 27 kN, before the pressure is applied. (a) Will the joint separate as a result of the load due to internal pressure ? (b) If the joint does not separate determine the tensile stress in each bolt due to the axial load.

Ref.	Calculations
	Given $P = 1.4\ \text{MPa}$, $A = 0.2\ \text{m}^2$, $K_b = \frac{1}{4} K_c$, $F_i = 27\ \text{kN}$ Ten M 20 × 1.5 bolts, Gasket factor, $C = \frac{K_b}{K_b + K_c} = 0.2$ External load per bolt, $F_e = \frac{0.2 \times 10^6 \times 1.4}{10} = 2.8 \times 10^4 = 28\ \text{kN}$ (a) For no separation $F_i > F_e (1 - C) = 28\,(1 - 0.2) = 22.4\ \text{kN}$. as $F_i = 27\ \text{kN}$ no separation will take place. (b) $\therefore F_b = F_i + C\,F_e$. $= 27 + 0.2 \times 28 = 32.6\ \text{kN}$
T-VI-2	$A_r = 251\ \text{mm}^2$, $\sigma_b = \frac{32.6 \times 10^3}{251} = 129.8\ \text{MPa}$

Prob. 5.4 : A 125 diameter hydraulic cylinder is being designed for use in an operation in which the pressure varies from 0 to 10 MPa every 10 sec. It is proposed to use 8 bolts to join the head to the cylinder. The bolts will be made from SAE 1030 steel and the threads will be die cut. What size of bolts would you recommend if (a) a compressible gasket is used and the joint is made leak proof and (b) an 'O' ring is used to provide sealing while the cylinder head makes metal to metal contact with the cylinder flange ? (c) what should be the minimum tightening loads on the bolts in (a) and (b) ?

Ref.	Calculations
	$D = 125\ \text{mm}$ $\quad \therefore A = \frac{\pi}{4} \times 125^2 = 12.265 \times 10^3\ \text{mm}^3$ $F_e\,(\text{max.}) = \frac{12.265 \times 10^3 \times 10}{8} = 15.3\ \text{kN}$ per bolt. $F_e(\text{min}) = 0$

T-II-7	For SAE 1030 steel : $S_{ut} = 527$, $S_{yt} = 296$, $S_{eb} = 225$
T-X-3	(a) Assuming soft copper corrugated gasket $C = .40$
	$\therefore F_b = F_i + C F_e$
	For no separation $F_i > F_e(1 - C) = 15.3(1 - 0.4) = 9.18$ kN.
	Let us assume $F_i = 10$ kN. The bolts will be subjected to fatigue loading. Hence the design will be for fatigue loading.
	$\therefore F_{b(max)} = 10 + .4 \times 15.3 = 16.12$ kN
	$F_{b(min.)} = 10 + .4 \times 0 = 10$ kN
	$\therefore F_{b(m)} = \frac{16.12 + 10}{2} = 13.06$ kN
	$F_{b(v)} = \frac{16.12 - 10}{2} = 3.06$ kN
T-I-12	$S_e = \frac{A \times K_R \times K_\theta \times K_Z \times K_S}{k_f} S_{eb}$ $A = 0.7$ for axial loads
T-I-17	$= \frac{0.7 \times 0.8 \times 225}{2.2} = 57.3$ MPa $K_R = K_\theta = K_Z = 1$
F-I-7	$\sigma_{b(m)} = \frac{F_b(m)}{A_r} = \frac{13.06 \times 10^3}{A_r}$ $K_S = .8$
	$\sigma_{b(v)} = \frac{F_b(v)}{A_r} = \frac{3.06 \times 10^3}{A_r}$ $K_f = 2.2$
T-I-20	$\frac{1}{N} = \frac{\sigma_m}{S_{yt}} + \frac{\sigma_v}{S_e}$ Assume $N = 2$.
	$\frac{1}{2} = \frac{13 \times 10^3}{A_r \times 296} + \frac{3 \times 10^3}{A_r \times 57.3}$; $A_r = 87.8 + 104.7 = 192.5$ mm^2.
T-VI-2	Select M 20 × 1.5 bolts for which $A_r = 251$ mm^2.
	(b) With metal to metal contact, $C = 0$.
T-X-3	$F_b = F_i + C F_e = F_i$.
	For no separation $F_i > F_e(1 - C) > 15.3$ kN
	$\therefore$ Assuming $F_i = 16$ kN
	This will remain constant and the bolt will not be subjected to fatigue loading.
	$\therefore \sigma_b = \frac{16 \times 10^3}{A_r} = \frac{S_{yt}}{N} = \frac{296}{2} = 148$ MPa ; $A_r = \frac{F_i}{\sigma_b} = \frac{16 \times 10^3}{148} = 108$ mm^2

T-VI-2	For M14 × 1.5 bolts $A_r = 111$ mm^2 so o.k.
	The final results are :
	(a) Use M 20 × 1.5 bolts with $F_i = 10$ kN
	(b) Use M 14 × 1.5 bolts with $F_i = 16$ kN.

Prob. 5.5 : Bell bottom screw jacks as shown in Fig. are used in a number of industries such as construction industry, automobiles, erection etc. Design a screw with square threads for use in 200 kN capacity jack with a maximum lift of 150 mm. Design also the steel bar and the nut portion of the cast iron base. Select material for screw and the bar.

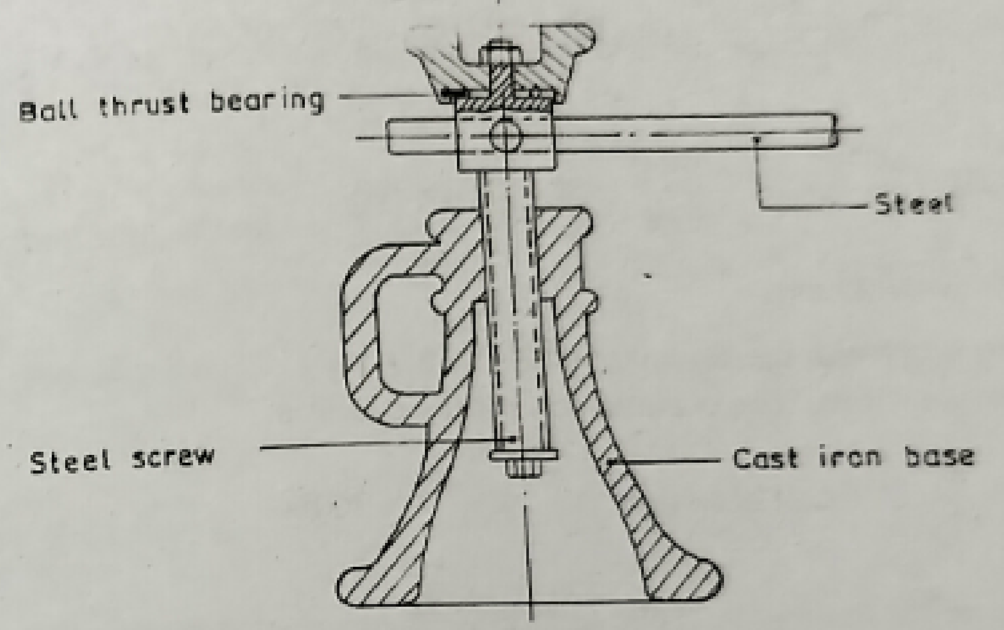


Fig.

Ref.	Calculations
	Given $W = 200$ kN Maximum lift = 150 mm.
	Material for screw and bar -- SAE 1020 steel.
	Material for base -- High grade C.I.
T-II-7	For SAE 1020 steel – $S_{ut} = 435$, $S_{yt} = S_{yc} = 246$, $S_{ys} = 154$, $E = 206 \times 10^3$.
T-II-3	For I.S. grade 35 C.I. – $S_{ut} = 350$, $S_{uc} = 1000$.
	S_{dt} for screw and bar $= \frac{246}{2} = 123$
	S_{ds} for screw and bar $= \frac{154}{2} = 77$
T-I-24	Using Rankine's formula for columns :
	$S_y = \frac{F_{cr}}{A}\left[1 + \frac{1}{an}\left(\frac{l}{k}\right)^2\right]$

Let D_c be the core diameter of the screw.

$A = \frac{\pi}{4} D_c^2$, $K = \frac{D_c}{4}$, $l = 150$ mm, $n = 0.25$ (one end fixed other end free)

$a = \frac{\pi^2 E}{S_{yt}} = \frac{\pi^2 \times 206 \times 10^3}{246} = 8256$, $F_{cr} = 200 \times 2 = 400$ kN

$$246 = \frac{400 \times 10^3 \times 4}{\pi D_c^2}\left[1 + \frac{1}{8256 \times .25}\left(\frac{150 \times 4}{D_c}\right)^2\right]$$

$$D_c^2 = 2071 + \frac{36 \times 10^4}{D_c^2} \text{ or } D_c^4 - 2071 D_c^2 - 36 \times 10^4 = 0$$

$$\therefore D_c^2 = \frac{2071}{2} + \sqrt{\left(\frac{2071}{2}\right)^2 + 36 \times 10^4} = 2231$$

$\therefore D_c = 47.23$ mm.

T-VI-5 Selecting major dia. $D_o = 60$ mm & $P = 8$ mm $D_c = 60 - 8 = 52$ mm, large value is selected to take care of shear stress due to torque.

$A_c = \frac{\pi}{4} \times 52^2 = 2122.64$ mm^2 $\quad K = \frac{52}{4} = 13$ mm

$\therefore \frac{l}{k} = \frac{150}{13} = 11.53$ as this is greater than 10, so Rankine formula is O.K.

$$\therefore F_{cr} = \frac{S_{yt} \times A}{1 + \frac{1}{an}\left(\frac{l}{k}\right)^2} = \frac{246 \times 2122.64}{1 + \frac{11.54^2}{8256 \times .25}}$$

$= 490.5 \times 10^3$ N or 490.5 kN

We want $F_{cr} = 400$ kN so o.k.

T-IX-2 For a screw jack with steel screw and C.I. nut : safe bearing pressure = 12.5 to 17.5 MPa $\mu = .10 - .16$ assume $\mu = .13$ $\therefore$ $\phi = \tan^{-1} .13 = 7.4°$

$$D_m = \frac{D_o + D_c}{2} = \frac{60 + 52}{2} = 56 \text{ mm} \quad P = 8 \text{ mm}$$

$$\alpha = \tan^{-1}\frac{P}{\pi D_m} = \tan^{-1}\frac{8}{\pi \times 56} = 2.6^0$$

$$\therefore \tan(\alpha + \phi) = \tan 10^0 = .176$$

$\therefore$ Screw torque $T_S = \frac{W D_m}{2} \tan(\alpha + \phi) = \frac{200 \times 56}{2} \times .176 = 985.5$ N-m.

T-IX-1 For collar torque, assume the diameter of screw head as $1.5 \times D_o = 90$ mm.

$\therefore$ Collar torque, $T_c = \frac{\mu_c W D_c}{2}$ $\qquad D_c = \frac{2}{3} \times 90 = 60$ mm.

$= \frac{.13 \times 200 \times 60}{2} = 780$ N – m

$\therefore$ Total torque, $\quad T = 985.6 + 780 = 1765.6$ N-m.

$$\therefore \text{Shear stress } \tau = \frac{16 \times T}{\pi D_c^3} = \frac{16 \times 1765.6 \times 10^3}{\pi \times 52^3} = 64 \text{ MPa}$$

If we use a ball thrust bearing, the collar friction is negligible.

$\therefore \qquad T_c = 0$ & $T = 985.6$ N-m.

$$\text{Direct Comp. stress} = \frac{200 \times 10^3}{2122.64} = 94.2 \text{ MPa}$$

$$\therefore \text{Maximum shear stress} = \sqrt{\left(\frac{94.2}{2}\right)^2 + 64} = 79.5 \text{ so O.K.}$$

$$\text{Principal stress } \sigma = \left(\frac{94.2}{2}\right) + 79.5 = 126.6 \text{ so O.K.}$$

Assuming one person can apply a force 300 N, the length of the bar, L :

$300 \times l = 1765.6$

$$l = \frac{1765.6}{300} = 5.9 \text{ m (very large)}$$

So we must use a ball thrust bearing at the collar which will reduce the torque required to a value of 985.6 N-m.

$\therefore l = \frac{985.6}{300} = 3.28$ or with two persons $l = 1.64$ m.

This torque will be acting as B.M. on the rod. Let d be the diameter of the rod.

$$\sigma_b = \frac{M}{Z}, \quad Z = \frac{\pi d^3}{32}$$

Putting $\sigma_b = 123$, $\quad 123 = \frac{985.6 \times 10^3 \times 32}{\pi d^3}$

$$\therefore d = \sqrt[3]{\frac{985.6 \times 10^3 \times 32}{\pi \times 123}} = 43.3 \text{ mm.}$$

To find the height of the nut.

T-IX-2	Assume bearing pressure = 15 MPa $\frac{\pi}{4}\left(D_o^2-D_c^2\right)\times n\times P_b$; $\frac{\pi}{4}\left(60^2-52^2\right)\times n\times 15=200\times 10^3$ $\therefore$ n = 18.9 say 19 threads. From shearing considerations $\pi D_o\times\frac{P}{2}\times n\times\tau_d=200\times 10^3$ for nut $\tau_d=\frac{S_{ut}}{4}=\frac{350}{4}=87.5$ $\pi\times 60\times\frac{8}{2}\times n\times 87.5=200\times 10^3$ $n=3$ For screw $\pi D_c\times\frac{P}{2}\times n\times\tau_d=200\times 10^3, \tau_d=77$ $\therefore n=3.97$ $\therefore$ select $n=19$ $\therefore$ Height of nut = 19 × 8 = 152 mm. Final results : D_o = 60 mm P = 8mm dia of bar = 52 mm Height of nut = 152 mm. Efficiency $\eta=\frac{D_m\tan\alpha}{D_m\tan(\alpha+\phi)+\mu_c D_c}=\frac{56\tan 2.6}{56\tan 10^0+.13\times 60}=\frac{2.54}{16.95}$ $=.1437$ or 14.37% with collar torque. If $T_c=0$; $\eta=\frac{2.54}{9.87}=25.7\%$

Prob. 5.6 : In a large gate valve used in a high pressure water line, the gate weights 6 kN and friction force due to water pressure resisting opening is 2 kN. The valve steam is non rotating and is raised by a rotating wheel with internal threads acting as a rotating nut on the valve steam. The outside diameter of the valve stem screw is 38 mm. The wheel presses against a supporting collar of 40 mm inside diameter and 75 mm outside dia. The stem has square threads having an 8.5 mm pitch. Assuming μ_s = .10 and μ_c = .25 determine.

(a) the torque that must be applied to the wheel to raise the gate.
(b) the efficiency of the screw and collar mechanism.
(c) the maximum shear stress in the body of the screw.
(d) the length of the nut for an allowable P_b = 16 MPa and τ = 28 MPa.

Ref.	Calculations
T-IX-1	$W=6+2=8$ kN $D_o=38, P=8.5, D_c=38-8.5=29.5$ mm. $D_m=\frac{38+29.5}{2}=33.75$ mm

	$\alpha=\tan^{-1}\frac{8.5}{\pi\times 33.75}=4.59^0$ $\phi=\tan^{-1}0.10=5.71^0$ $\therefore\alpha+\phi=10.3^0$ $\therefore\tan 103^0=.182$ Screw torque $T_s=\frac{W D_m}{2}\tan(\alpha+\phi)$ $=\frac{8\times 10^3\times 33.75}{2}\times .182=24.5$ N − m Collar torque $T_c=\mu_c\frac{W D_c}{2}$ $D_c=\frac{2}{3}\left[\frac{D_0^3-D_1^3}{D_0^2-D_1^2}\right]$ $=\frac{.25\times 8\times 10^3\times 59.27}{2}$ $=\frac{2}{3}\left[\frac{75^3-40^3}{75^2-40^2}\right]=59.27$ mm. $=59.27$ N-m. (a) $\therefore$ Total torque $T=T_s+T_c=24.5+59.27=83.77$ N-m. (b) Efficiency $\eta=\frac{D_m\tan\alpha}{D_m\tan(\alpha+\phi)+D_c\mu_c}=\frac{33.75\tan 4.59}{33.75\tan 10.3+.25\times 59.27}$ $=.129$ or 12.9%. (c) Maximum shear stress in the body of the screw : $A_c=\frac{\pi}{4}\times 29.5^2=683.15$ mm^2 $\sigma_c=\frac{8\times 10^3}{683.15}=11.7$ MPa $\tau=\frac{16\times 83.77\times 10^3}{\pi\times 29.5^3}=16.63$ MPa $\therefore$ Maximum shear stress $=\sqrt{\left(\frac{11.7}{2}\right)^2+16.63^2}=17.62$ MPa (d) Length of nut depends on the number of threads, n. For bearing : $\therefore\frac{\pi}{4}(38-29.5)^2\times n\times 16=8\times 10^3$ $n=1.11$ For shear : $\pi\times 29.5\times\frac{8.5}{2}\times n\times 29=8\times 10^3$ $n=.725$ But minimum height of nut = D_o = 38 mm.

⁂

Chapter 6

RIVETED & WELDED JOINTS

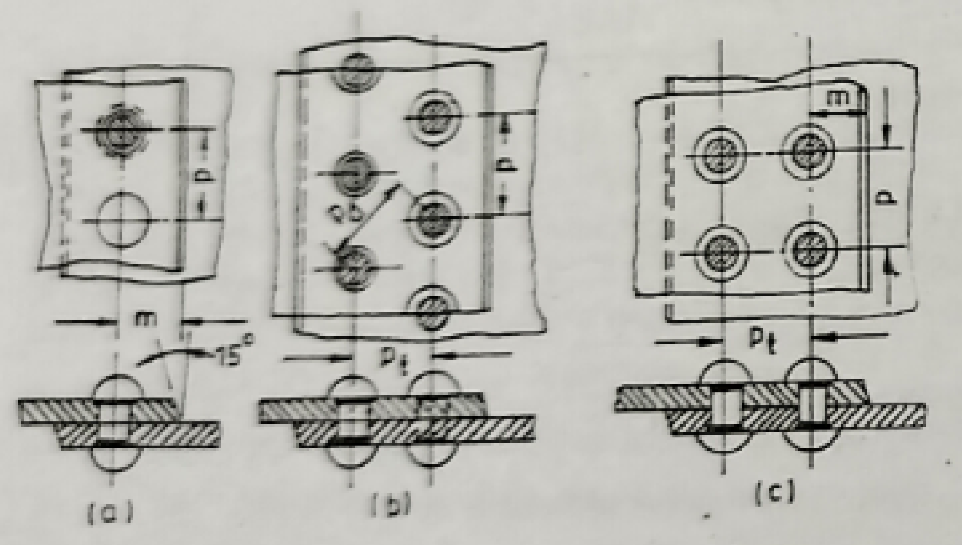
6.1 Introduction :

Riveting was the standard method of joining plates and structural parts before welding began to replace it with increasing rapidity. Both types of joint are used to permanently joins structural and machine parts that are made as separate units for manufacturing, assembling or transportation reasons.

The function of rivets in a joint is to make a connection that has *strength* and *tightness*. Strength is necessary to prevent failure of the joint while tightness is necessary in order to contribute to strength and to prevent leakage. Rivets may be driven by hand or by riveting machines. Structures in which riveted joints are used may be grouped as follows :

(a) Tanks or pressure vessels, (b) Bridges, buildings, cranes and machinery in general

(c) Hulls of ships.



In structure of the first group, the joints must be leakproof. In structures in the second group, the joints must resist given outside loads and have sufficient rigidity. In structures in the third group it is important to consider strength, rigidity and durability, as well as security against leakage. Two arrangements used in joining plates by means of rivets are equally well adapted to all three groups of structures namely *lap joints* and *butt joints.*

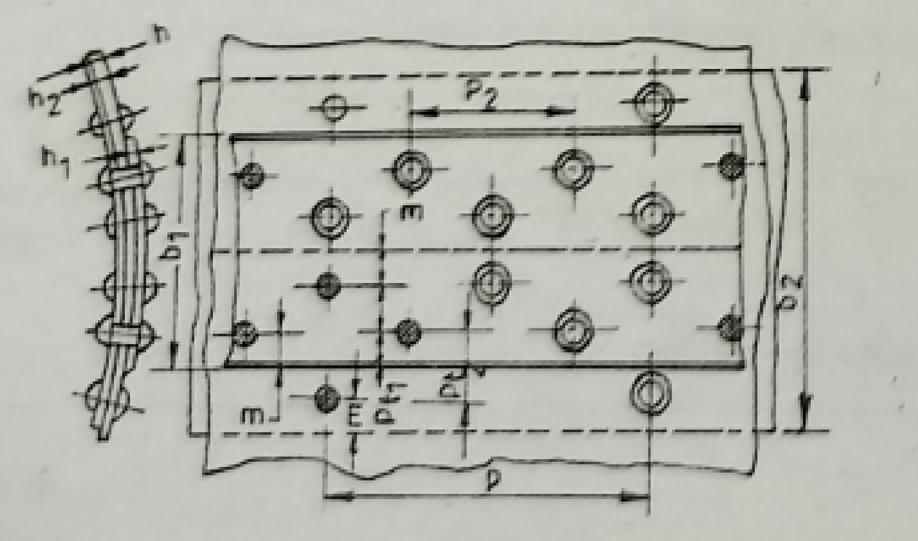


Fig. 6.2 Triple riveted double-strap butt joint

Lap joints shown in Fig. 6.1 consist of overlapping plates held together by one or more rows of rivets. *Butt joints* shown in Fig. 6.2 are formed by butting the plates against each other and joining them by overlapping straps or cover plates with single row of rivets or multiple row of rivets on each plate.

6.2 Terminology :

The following terms are used for riveted joints.

1. **Rivet :** A rivet is a round bar consisting of an upset end called the *head* and a long part called *shank*. The projecting end of the shank is formed into a second head called the *point*. The various shapes of head and their proportions in terms of shank diameter are given in F - 5.2 of data book.

2. **Gage line :** The line through the centres of a row of rivets, parallel to the edge of the plate is called the *gage line*.

3. **Pitch :** It is the distance between the centres of adjacent rivets measured on the gage line and is denoted by P. If the spacing on one gage line is greater than on another, the pitch is the maximum rivet spacing.

4. **Transverse or Back Pitch, Pt :** It is the distance between two adjacent gage lines on the same plate.

5. **Diagonal Pitch, Pd :** It is the distance between the centres of adjacent rivets on adjacent gage line, in staggered rivetting.

6. **Margin, m :** It is the distance from the gage line to the edge of the plate.

7. **Unit Strip :** It is a section of the joint with width equal to the pitch.

8. **Calking :** It is the process of making the joint leak proof. It is good practice to calk not only the plates but also around rivet heads.

6.3 Stresses in Riveted Joints :

Rivets should always be placed at right angles to the acting forces, and the maximum stress induced in them should be either shear or crushing. According to the conventional theory, the failure of joints may occur in one of the following ways :

(a) Shearing of rivets (b) Rupturing of the plate by tension

(c) Crushing of the plate or rivets

(d) Shearing of the margin

(e) Tearing of the margin

(f) Rupturing of the plate by tension in a zigzag line passing diagonally between the rivet holes in staggered riveting.

In a multiple row joint, ultimate failure may be due to one or more of the causes just cited. In a theoretical analysis of strength the problem is to find the general relation between the thickness of the plates, the rivet diameter, and the pitch when a riveted joint is equally strong against failure in any of the six possible ways. Since the margin and transverse pitch do not affect the longitudinal pitch P and can be assigned values based on the diameter of the rivet, it follows that, the last three ways of failure, d, e, and f may be omitted from the general analysis. Hence the problem narrows down to the analysis of the remaining three types of failure or a combination of these.

One more term used for the design of the rivetted joints is *efficiency* of the joint. It is ratio of the strength of the weakest section of a unit strip of the joint to the tensile strength of an equally wide unperforated plate.

6.4 Design of Pressure Vessels Joints :

There are two types of joints in a cylindrical pressure vessel (a) Longitudinal joint and (b) Circumferential joint or girth joint. The stress set up by the pressure on the ends of a circumferential section or joint of a pressure vessel is only half as great as the stress set up by the radial pressure in a longitudinal joint. Therefore a circumferential or girth joint does not need to be more than half as strong as a longitudinal joint. The steps for design are as follows :

Step 1 : Select the type of longitudinal joint to be used, the choice depending on the diameter D of the vessel and the internal fluid pressure P as per T-V-3 of data book. For circumferential joint generally a lap joint is used.

Step 2 : Determine the number of rivet rows. A greater number of rows gives a higher efficiency and permits the use of thinner plates, but it is costly. Assume suitable efficiency of the joints as per T-V-1.

Step 3 : Select proper material of plates and rivets. Also select allowable stresses for these as per T-V-4.

Step 4 : Determine the thickness of the plate by the formula.

$t = \frac{PD_i}{2S_t \times \eta}$ Where η = efficiency of the joint.

S_t = allowable stress in tension.

Step 5 : Select the size of the rivets as per T-V-2.

Step 6 : Determine the main pitch P as per T-V-2.

Step 7 : Calculate minimum strength and efficiency of the joint by considering all possible cases of failure. This should be equal to or greater than the efficiency assumed in step 3.

Step 8 : Calculate the margin and transverse pitch as per T-V-2.

A solved problem will illustrate the design procedure.

6.5 Design of Structural Joints :

The design of riveted joints for structural work is the subject of a separate branch of engineering. However, the main points will be discussed below.

1. **Rivets :** The diameter of river for these joints is calculated by the relation (T-V-2)

$$d = 7\sqrt{t} - 2$$

Where t = thickness of connected members.

2. **Pitch :** Here the requirement of leak proof joint is not there. However the rivets are placed at a pitch determined by the relation.

$$16t \geq P \geq 3d$$

where t = thickness of the thinnest plate used in the joint.

An excessively large pitch prevents intimate contact between the members. Water may then collect in the crack and deteriorate the joint by rusting.

3. **Margin :** If the force is acting parallel to the edge, the margin m = 1.5 d, while for the force acting normal to the edge, m = 2d.

For connecting two structural members, overlapping plate or plates are used as shown in the Fig. (6.3). These plates are :

(a) **Splice plates** are used to connect two members placed end to end in a straight line as 'a' in the Fig.

(b) **Gusset plates** are used to connect members that intersect or meet at an angle, as 'g' in the Fig.

(c) **Filler plates** are used to line up members of different thickness as 'c' in the figure.

(d) **Clipangles** are intended to secure a central loading of angle shaped members as 'i' in the Fig.

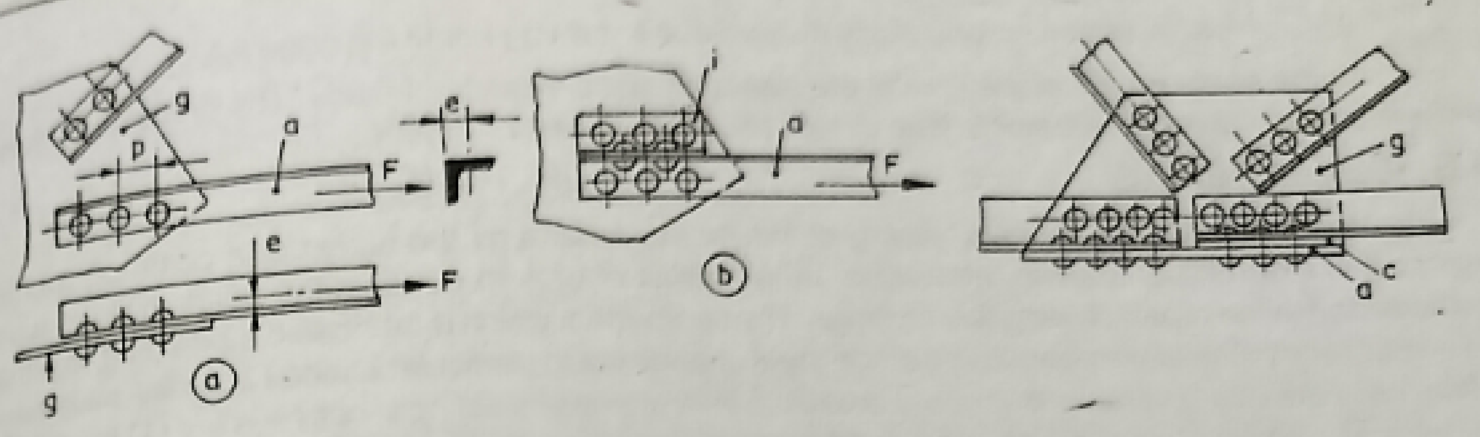


Fig. 6.3

6.6 Uniform Strength or Lozenge Joint :

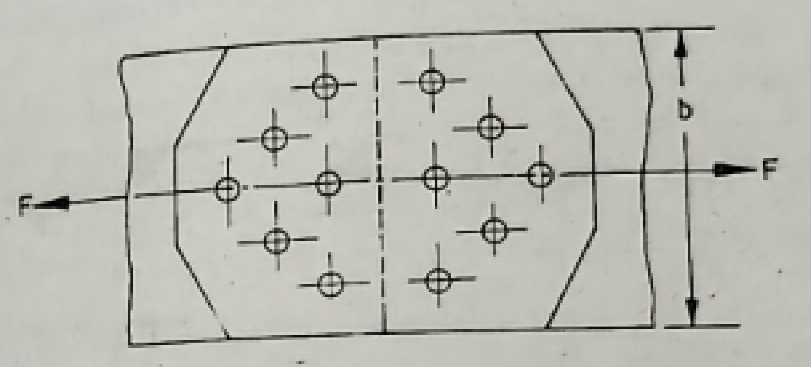


Fig. 6.4

A riveted joint known as Lozenge joint is used for roof, bridge work or girders as shown in Fig. (6.4). In such a joint diamond riveting is employed so that the joint is made of uniform strength.

The Fig. shows a triple rivetted double strap butt joint.

The design procedure will be illustrated with the help of solved problems.

6.7 Joints Under Eccentric Loading :

In an eccentrically loaded joint, the line of action of the load does not pass through the centroid of the rivet areas and thus all rivets are not equally loaded. The two types of eccentricities are shown in Fig. (6.5).

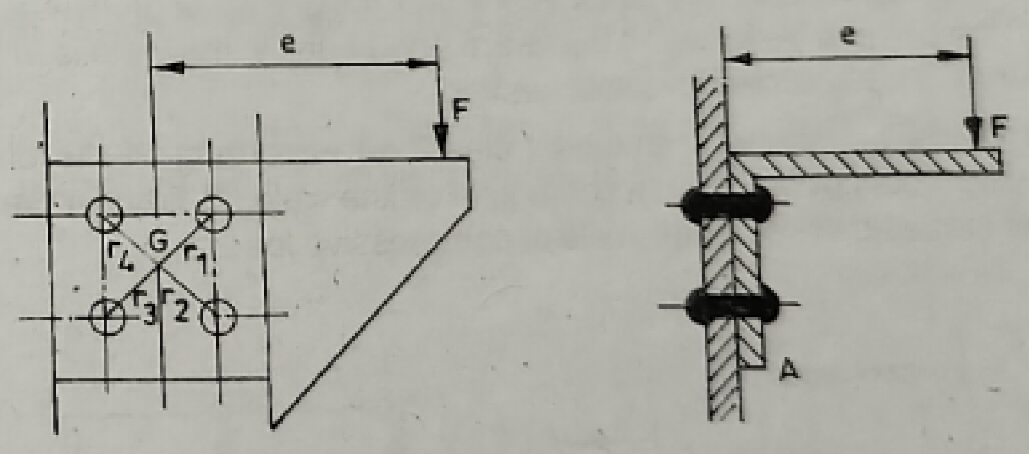


Fig. 6.5

In the first type of joint, the eccentricity produces the rotation about G causing two types of shear stresses.

(a) Primary shear stress τ_1 equally distributed in all the rivets.

(b) Secondary shear stress τ_2 due to rotation of the member about G.

In the second type of joint the member turns about the lower edge causing two types of stresses.

(a) Primary or direct shear stress, τ equally distributed in all the rivets.

(b) Normal stress, σ, unequally divided due to turning about the edge AA.

In the second type of joint, bolts are generally used in place of rivets. The analysis of these joints is already given in chapter 5. The design procedure is exactly same.

6.8 Welded Joints :

Welding is the permanent joining of two or more parts by the fusion of a common molten region. It is extensively used in fabrication, in the repair of broken parts, in the building up of worn surfaces and in the repair of defective castings. The component which is fabricated by welding together a number of hot rolled low or medium carbon steel shapes cut to particular shapes is called *weldment*. While there are many variations, the two broad classifications of welding processes are (1) those that use heat alone and (2) those that use a combination of heat and pressure. Out of these, the first is of more interest to the machine designer and it is called *fusion welding*. The fusion welding process using the electric arc is the most widely used process for the manufacture of weldments. In the metal arc welding process the filler metal is supplied by a welding electrode. There are five basic forms of welded joints, 'edge', 'butt', 'lap', 'corner' and 'tee'. They are shown in Fig. (6.6).

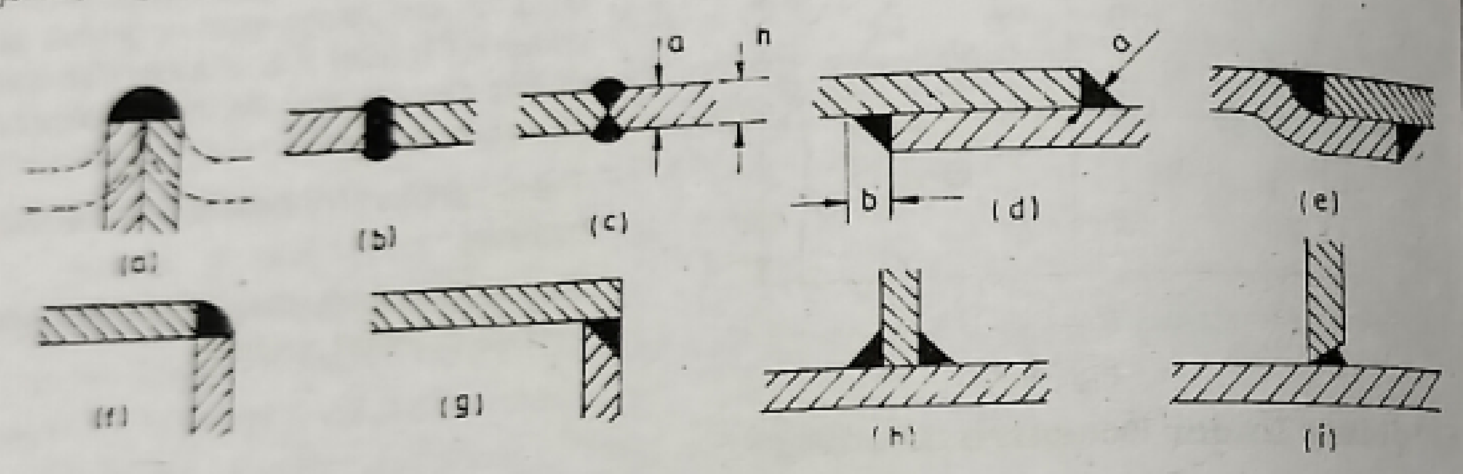


Fig. 6.6

These joints can be classified in two groups :

(a) Butt or Groove Welds : The shape and size of the groove or grooves is decided according to the thickness of the parts to be joined. If these are good quality, the weld is fully as strong as the plate and its efficiency is 100 percent for static loading.

(b) Fillet welds : They are commonly classified according to the direction of loading (i) Parallel load in which both plates exert a shear load on the weld (ii) Transverse load in which one plate exerts a shear load and the other a tensile or compressive load.

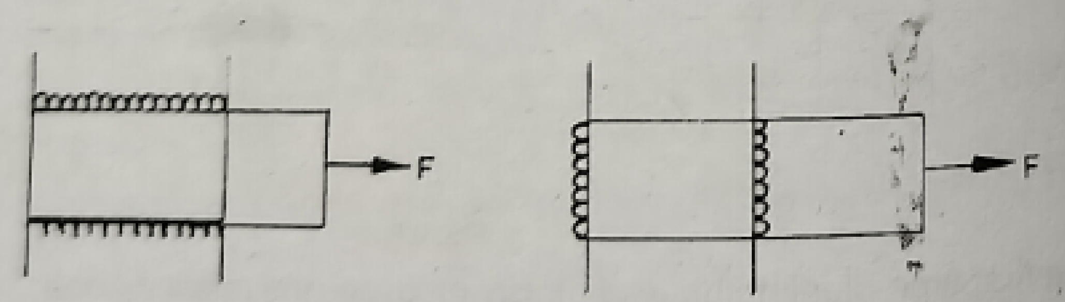


Fig. 6.7

6.9 Design of Steel Welds :

Since welds usually fail at the minimum section, it is logical to base design calculations upon the throat area of the weld, even though the exact stress distribution is not known. The Fig. (6.8) shows the location of the throat area for butt and fillet welds.

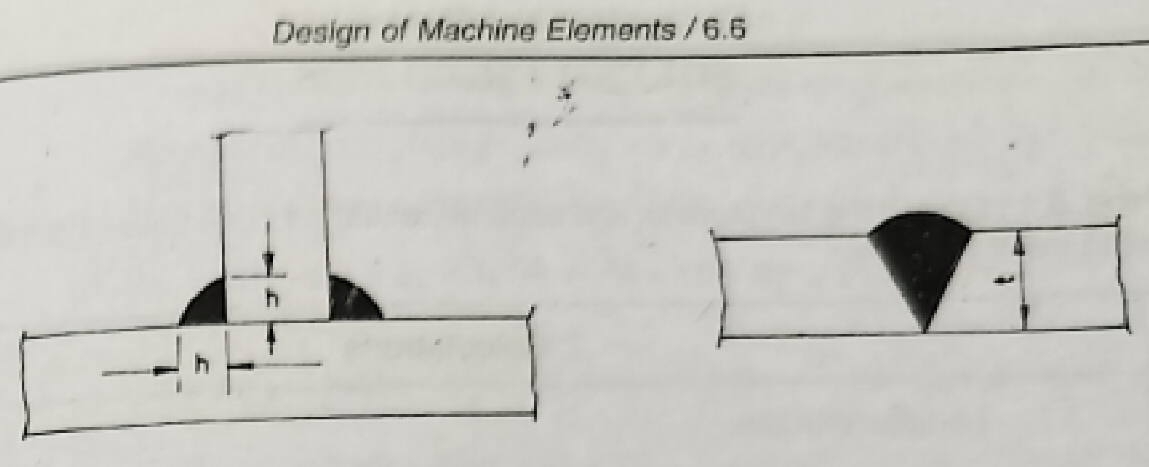
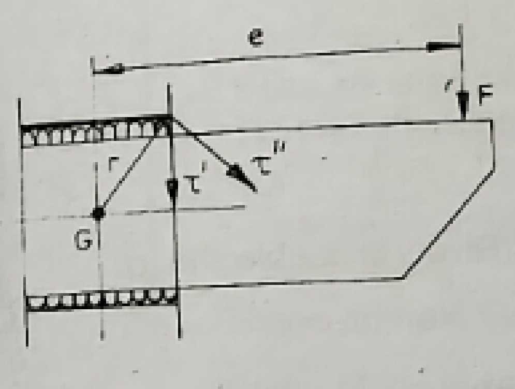


Fig. 6.8

Conventional engineering practice considers the significant weld stress to be the shear stress in the throat section for either transverse or parallel load. The recommended design stresses are given in T-V-5 of data book.

For the cases when welded joints are subjected to torsion and bending, the stress analysis of welded joints of these types is given below.

(a) Torsion in Welded Joints : The Fig. (6.9) illustrates a cantilever welded to a column by two fillet welds. The welds will be subjected to a shear force equal to F and a torsional moment.



$\tau' = \frac{F}{A}$ where A = throat area of all welds.

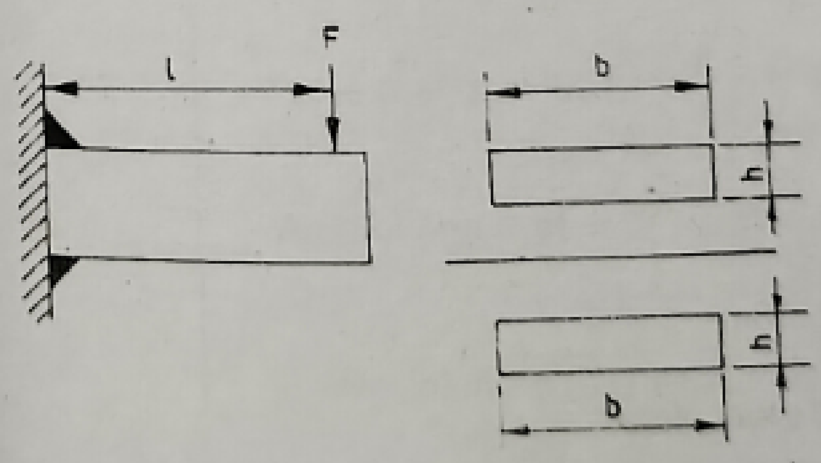
$\tau'' = \frac{T \cdot r}{I_p}$ where I_p = Polar moment of inertia of the weld group about the centroid of the group.

r = distance from the centroid of the group to the point of interest in the weld. T = Torque = F × e.

Fig. 6.9

After finding τ' and τ'' they are combined vectorically. The polar moment of inertia for some common weld patterns are given in T-V-10.

(b) Bending in Welded Joints : The Fig. (6.10) shows a cantilever welded to a support by fillet welds at top and bottom. There will be two types of stresses in the weld :



i) Shear stress $\tau = F/A$

where A = throat area of all welds

ii) Normal stress $\sigma = MC/I$

where I = moment of inertia of all the welds about their centroidal axis.

C = distance of the farthest point from the centroidal axis.

M = moment = F × L

After finding τ and σ, principal stress and maximum shear stress is calculated and joints checked for safety.

Fig. 6.10

The procedure will be illustrated with the help of solved problems.

SOLVED PROBLEMS

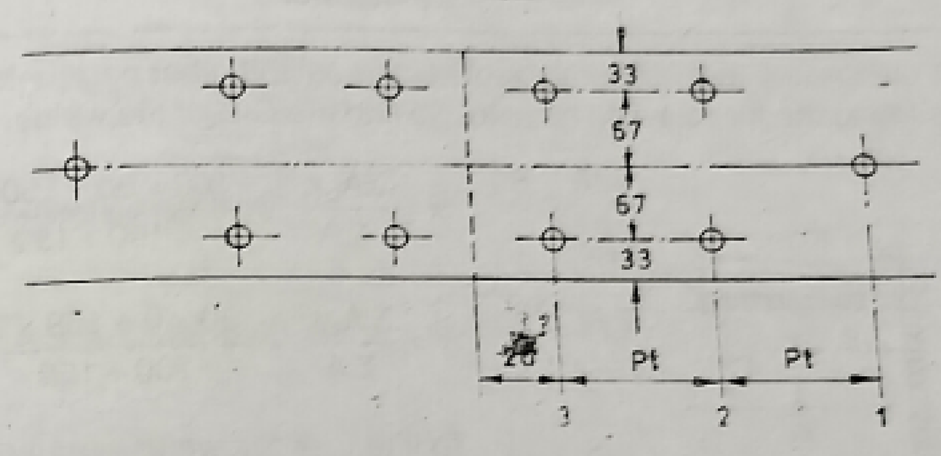
Prob. 6.1 : Design the longitudinal and circumferential joint of a boiler 1.5 m in diameter for a working pressure of 1.4 MPa.

Ref.	Calculations
	1. Longitudinal joint
T-V-3	For no joint, plate thickness $t=\frac{PD}{2S_t}$, $S_1 = 82$ for SAE 1020 steel
	$=\frac{1.4\times1500}{2\times82} = 12.8$ mm.
T-V-4	
T-V-1	So the actual thickness will be about 15mm for D = 1500 and t = 15mm. Select triple rivetted but joint. For which η = 80 to 90 percent. Assume η = 85 percent.
	Hence $t=\frac{PD}{2S_t\eta}=\frac{1.4\times1500}{2\times82\times.85} = 15.06$ mm.
	The triple rivetted but joint is shown in Fig. (6.2)
T-V-2	$d=6\sqrt{t}$ to $7\sqrt{t} = 6.5\sqrt{15}=25.17$ select $d=25$ mm.
T-V-2	$P=\frac{\pi}{4}d^2\frac{(2n_2+n_1)S_s}{tS_t}+d$ $n_2=4$ (Rivets in double shear)
	$=\frac{\pi}{4}25^2\frac{(2\times4+1)}{15\times82}+25$ $n_1=1$ (Rivets in single shear)
	$=197.3$ mm $S_t = 82$, $S_s = 48$ for SAE 1020 steel
	But for leak proof joints $P_1 < 8t = 120$ mm so select $P = 200$, $P_1 = 100$
	$P_{t_1} = 2d = 50$ mm For staggered riveting $P_t = (.33P_1 + .67d)$
	$P_{t_2} = 2.75\times25 = 68.75$ mm $= 49.75 = 50$ mm.
	For inner rows where pitch P_1 :
	$P_1=\frac{P}{2}=\frac{200}{2}=100$ mm.
	$P_d=\frac{P_1+d}{2}=\frac{100+25}{2} = 62.5$ mm.
	Thickness of cover plates -- Unequal cover plates
	$t_o = .625\times t = .625\times15 = 9.375 \approx 10$ mm.
	$t_i = .75\times t = .75\times15 = 11.25 \approx 12$ mm.
	Now we will calculate the strengths for various modes of failure.
	1. Strength of solid plate $F_p = 200\times15\times82 = 246$ kN.
	2. Tearing of plate on the outer gage line.
	$F_{t_1} = (P-d)\,t\times S_t = (200-25)-15\times82 = 215.25$ kN.
	3. Shearing of rivets $=\frac{\pi}{4}d^2(2+n_2+n_1)\,S_s=\frac{\pi}{4}28^2(2+4+1)48 = 211.95$ kN.
	4. Resistance to crushing $= (4\times15+1\times10)\,25\times135 = 236.25$ kN.
	5. Tearing of plate through second row and shearing of rivets in first row.
	$F_{St}=(P-2d)\,t_{St}+\frac{\pi}{4}d^2\times1\times S_s=(200-2\times25)\times15\times82+490\times1\times48$
	$=184.5+23.52=208.02$ kN.
	6. Tearing of plate through second row and crushing of rivets in first row.
	$F_{ct}=(P-2d)\,t_{St}+t_o\,.\,d\times S_c=184.5+10\times25\times135=218.25$ kN.
	7. Shearing of rivets in the first row and crushing of rivets in the other two rows.
	$F_{sc} = 490\times48+4\times25\times15\times135 = 226.0$ kN.
	$\therefore$ Lowest strength is = 208.02 kN for tearing of plate through second row and shearing of rivets in first row.
	$\therefore$ Minimum efficiency $=\frac{208.02}{246.00} = .846$ or 84.6%
	This is approximately equal to assumed value of 85% so O.K.
	Width of the outer cover plate $W_o = 4\times m + 2P_t = 4\times37.5+2\times50 = 250$ mm.
	Width of the inner cover plate $W_i = W_o + 2P_{t_2} = 250+2\times68.75 = 386$ mm
	2. Circumferential Joint : Use a lap joint. Stress in the longitudinal direction is ;
	$\sigma_t=\frac{PD}{4t}$ or $\eta=\frac{PD}{4tS_t}=\frac{1.4\times1500}{4\times82\times15}=.427$
T-V-1	Keeping diameter of rivet same and thickness of plate same d = 25 mm, t = 15 mm.
T-V-2	Force acting on the joint $\frac{\pi}{4}\times1500^2\times1.4=2472.7$ kN
	Area of one rivet $=\frac{\pi}{4}\times25^2=490.6\text{ mm}^2$
	$\therefore$ No. of rivets to support the load $n=\frac{2472.7\times10^3}{490.6\times48}=105.13$
	$\therefore$ For single row, pitch $=\frac{\pi(1500+2\times15)}{105}=45.75$ mm.

	Geometrically minimum pitch = 2.25 × 25 = 56.25 mm.
	So we should use double rivetted lap joint with 53 in each row
	$\therefore$ Pitch $= \frac{\pi \times 1530}{53} = 90.64$ mm.
	Minimum pitch for leak proof joint $P < 8\,t = 8 \times 15 = 120$ so O.K.
	Crushing stress on each rivet.
	$\sigma_c = \frac{2493 \times 10^3}{25 \times 15 \times 106} = 62.2$ MPa so O.K.
	Margin $= 1.5 \times 25 = 37.5$ mm.
	Efficiency $= \frac{P - d}{P} = \frac{90 - 25}{90} = 72.2\%$ so o.k.

Prob. 6.2 : Two lengths of a steel tie rod having width of 200 mm and thickness of 12 mm are to be connected by means of a butt joint with double cover straps. Design a uniform strength joint for this connection.

Ref.	Calculations
T-V-4	Select wrought iron for rivets $S_t = 80$, $S_c = 135$ and $S_s = 63$
	SAE 1020 for plate $S_t = 82$
T-V-2	For double cover strap, thickness of straps = .75t = .75 × 12 = 9 mm.
T-V-2	Diameter of rivet, $d = 7\sqrt{t} - 2 = 7\sqrt{12} - 2 = 22.5$ mm select $d = 22$ mm.
	$\therefore$ Maximum pull which the plate can take,
	$F = (b - d)\,tS_t = (200 - 22)\ 12 \times 82 = 175$ kN
	Area of one rivet $= \frac{\pi}{4} \times 22^2 = 380\ \text{mm}^2$
	As the rivets are in double shear, we can assume their shearing strength as ;
	$F_s = 1.75 \times 380 \times 63 = 41.9$ kN per rivet.
	Crushing resistance of one rivet $F_c = 22 \times 12 \times 135 = 35.64$ kN per rivet.
	$\therefore$ No. of rivets $n = \frac{175}{35.64} = 4.91$ say 5.
	$\therefore$ It will be triple row with 2,2 and 1 rivet in the three rows.
	Taking margin = 1.5 × 22 = 33 mm.
	P should be between 3d = 3 × 22 = 66 and 16 × t = 16 × 12 = 192
	Here P = 200 − 66 = 134 mm so. o.k.
	Resistance to tearing along 33 and shearing of rivets.

	
	Fig.
	$F_{t_{3s}} = (D - 2d)\,tS_t + 3 \times 41.90 = (200 - 2 \times 22) \times 12 \times 82 + 125.7$
	$= 279.2$ kN
	Resistance to tearing along 33 and crushing of rivets
	$F_{t_{3c}} = (b - 2d)\,tS_t + 3 \times 35.64 = 260.42$ kN
	Resistance to tearing along 22 and crushing of rivets
	$F_{t_{2c}} = (b - 2c)\,tS_t + 1 \times 35.64 = 189.14$ kN
	Resistance to tearing along 22 and shearing of rivets
	$F_{t_{2s}} = (b - 2d)\,tS_t + 1 \times 41.90 = 195.4$ kN
	$\therefore$ Minimum resistance = 175 kN
	Strength of solid plate, $F_p = 200 \times 12 \times 82 = 196.8$ kN
	$\therefore$ Efficiency of the joint $= \frac{175}{196.8} = 88.9\%$

Prob. 6.3 : The Fig. shows a bracket welded to a column along with the load acting. Determine the required weld size.

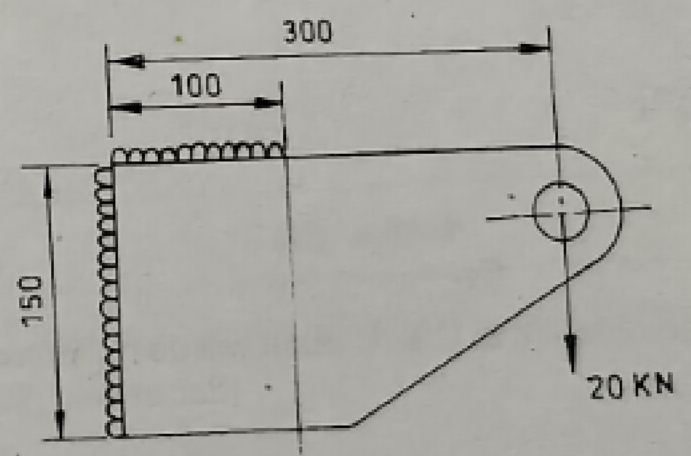


Fig.

Ref.	Calculations

It is customary to find the moment of inertias and other parameters of the welds by assuming the throat width as unity. To find the C-G of the welds.

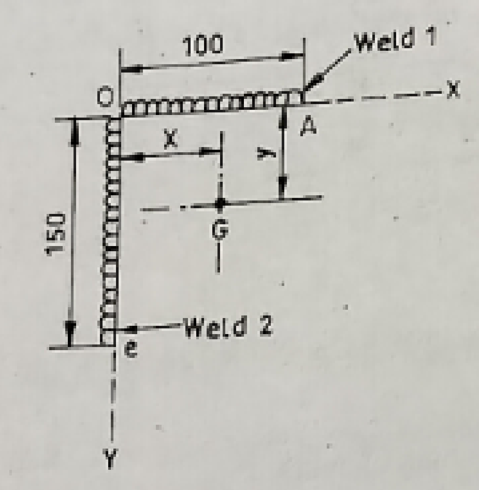


Fig.

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A} = \frac{100 \times 50 + 150 \times 0}{100 + 150} = 20\,\text{mm}$$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A} = \frac{100 \times 0 + 150 \times 75}{100 + 150} = 45\,\text{mm.}$$

To find I_p of the welds about G : $I_p = (I_x + I_y)$

$$I_x = \frac{150^3}{12} + 150 \times 30^2 + \frac{100}{12} + 100 \times 45^2$$

$$= 618758.3$$

$$I_y = \frac{150}{12} + 150 \times 20^2 + \frac{100^3}{12} + 100 \times 30^2$$

$$= 233345.8$$

$$I_p = I_x + I_y = 618758.3 + 233345.8 = 852104.1$$

It is customary to neglect the quantities $\frac{100}{12}$ & $\frac{150}{12}$ as they are small.

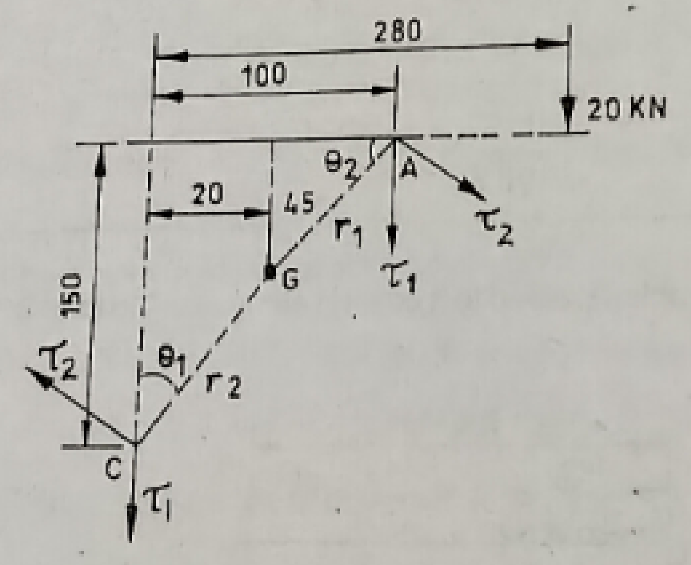


Fig.

Assuming throat width as 't'

ΣA of welds = 100 × t + 150 × t

= 250 t

∴ Primary shear

$$\tau_1 = \frac{20 \times 10^3}{250\,t} = \frac{80}{t}\,\text{MPa}$$

Torque

T = 20 × 280 = 5600 N-m.

$$r_1 = \sqrt{45^2 + 80^2} = 91.8\,\text{mm.}$$

$$r_2 = \sqrt{20^2 + 105^2} = 106.88\,\text{mm}$$

∴ τ_2 will be different at C & A. But it will be maximum at C.

1 AT C :

$$\tau_2 = \frac{T \times r_2}{I_p} = \frac{5600 \times 10^3 \times 106.9}{852104\,t} = \frac{702.5}{t}\,;\ \tan\theta_1 = \frac{20}{105},\ \theta_1 = 10.78^\circ$$

∴ Horizontal component of $\tau_2 = \tau_2 \cos\theta_1 = \frac{702.5}{t} \cos 10.78^\circ = \frac{690.1}{t}$

Vertical component of $\tau_2 = \tau_2 \sin\theta_1 = \frac{702.5}{t} \sin 10.78^\circ = \frac{131.4}{t}$

∴ Net vertical component $= \frac{131.4}{t} - \frac{80}{t} = \frac{51.4}{t}$

∴ Net resultant shear stress $= \frac{1}{t}\sqrt{690.1^2 + 51.4^2} = \frac{692}{t}$

2 AT A : $\tau_2 = \frac{702.5}{t} \times \frac{91.8}{106.9} = \frac{603.4}{t}$; $\theta_2 = \tan^{-1} 45/80 = 29.36^\circ$ Result. Shear stress $= \frac{674.25}{t}$ Therefore using the greater value of resultant shear stress.

T-V-5 Allowable shear stress for steady load = 80 MPa

∴ $\frac{692}{t} = 80$ or $t = \frac{692}{80} = 8.65\,\text{mm.}$

∴ size of weld $h = \frac{t}{.707} = \frac{8.65}{.707} = 12.23\,\text{mm.}$

We can select the size of weld as 13 mm.

Prob. 6.4 : A bracket is welded to the column as shown in Fig. with the load acting on it. Determine the size of the weld.

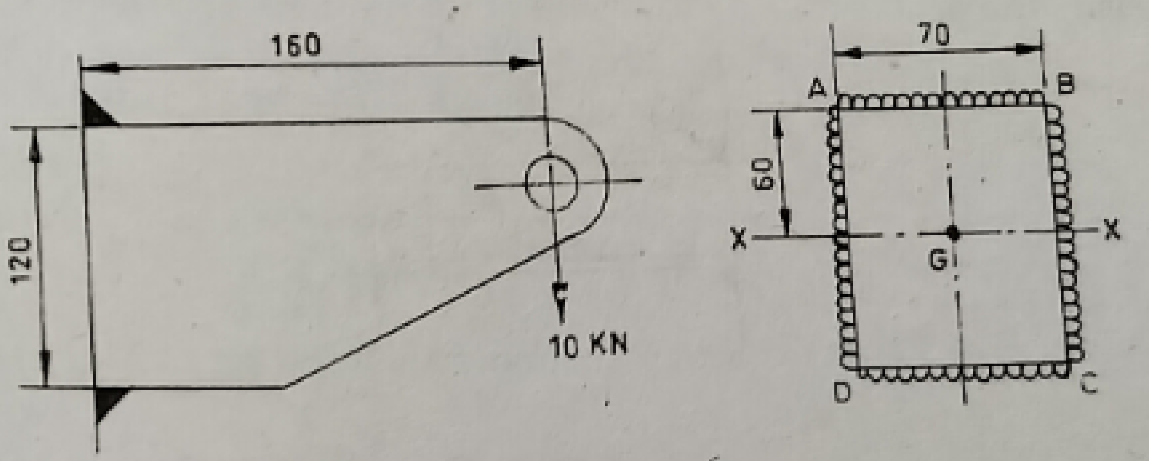


Fig.

Ref.	Calculations

The loading involves direct shear and bending. For bending we find the I_{xx} passing through G. Assuming unit throat.

$$I_{xx} = \left[\frac{70}{12} + 70 \times 60^2\right] \times 2 + \frac{2 \times 120^3}{12} = 792011,\ \Sigma A = [2 \times 70 + 2 \times 120]\,t = 340\,t$$

	$\therefore$ Bending stress in weld AB $\sigma_t = \frac{10 \times 10^3 \times 160 \times 60}{792011t} = \frac{121.2}{t}$
	Shear stress on all the welds $\tau = \frac{10 \times 10^3}{340} = \frac{29.4}{t}$
T-I-6	$\therefore$ Principal stress, $\sigma_1 = \frac{121.2}{2t} \pm \frac{1}{t}\sqrt{\left(\frac{121.2}{2}\right)^2 + 29.4^2} = \frac{128}{t}$
	Max. shear stress $\tau_{max} = \frac{1}{t}\sqrt{\left(\frac{121.2}{2}\right)^2 + 29.4^2} = \frac{67.35}{t}$
T-V-5	Allowable stress : tension = 115 MPa. shear = 80 MPa.
	$\therefore \frac{128}{t} = 115 \qquad t = \frac{128}{115} = 1.1\text{ mm}$
	$\frac{67.35}{t} = 80 \qquad t = \frac{67.35}{80} = .84\text{ mm}$
	So selecting $t = 1.1$, $h = \frac{1.1}{.707} = 1.55$ mm.
T-V-9	But minimum recommended size = 3 mm.

Prob. 6.5 : Find the torque T that can be applied to the weldment shown in Fig. If the permissible weld shear is 140 MPa.

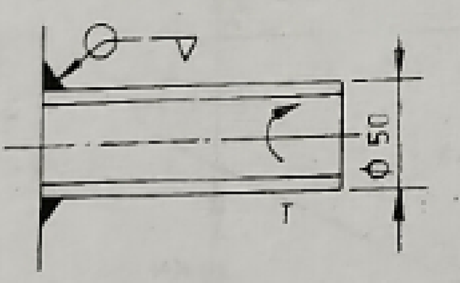


Fig.

Ref.	Calculations
	To find the polar moment of inertia of the weld $I_p = \frac{\pi}{32}(d+2t)^4 - \frac{\pi}{32}d^4 = \frac{\pi}{32}d^4\left[\left(1+\frac{2t}{d}\right)-1\right]$ $= \frac{\pi}{4}d^3t$ (Neglecting higher order terms of t/d)

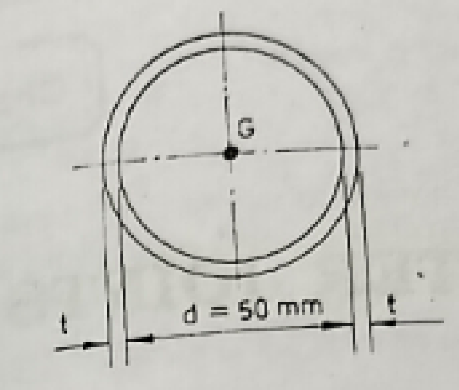


Fig.

	If $t = 1$, $I_p = \frac{\pi}{4}d^3$; $t = .707 \times h$ $= .707 \times 6 = 4.242$ mm $\therefore$ Shear stress $\tau = \frac{T \times r}{I_p} = \frac{T \times 25 \times 4}{\pi \times 50^3 \times 4.242}$ If $\tau = 140$, $T = \frac{140 \times \pi \times 50^3 \times 4.242}{100}$ $= 2331 \times 10^3$ N-mm = 2331 N-m.

Prob. 6.6 : Determine the length of welds required to transmit a reversed load of 45 kN between 12 mm thick plates when the plates are to be joined by the use of (a) parallel fillet welds and (b) transverse fillet welds.

Ref.	Calculations
T-V-9	For 12 mm plate thickness, weld size = 6 mm.
T-V-5	For reversed load, allowable shear stress = 35 MPa
T-V-6	Stress concentration factor K_f
	Parallel fillet weld = 2.7
	Transverse fillet weld – 1.5
	(a) Let L be the length of the weld Throat size $t = .707 \times 6 = 4.24$ mm $\therefore 2 \times L \times 4.24 \times \frac{35}{2.7} = 45 \times 10^3$ $\therefore L = \frac{45 \times 10^3 \times 2.7}{2 \times 4.24 \times 35} = 409$ mm.
	(b) Let L be the length of the welds $t = 4.24$, $K_f = 1.5$ $\therefore 2 \times L \times 4.24 \times \frac{35}{1.5} = 45 \times 10^3$ $L = \frac{45 \times 10^3 \times 1.5}{2 \times 4.24 \times 35} = 227.4$ mm.

* * *

Chapter 7

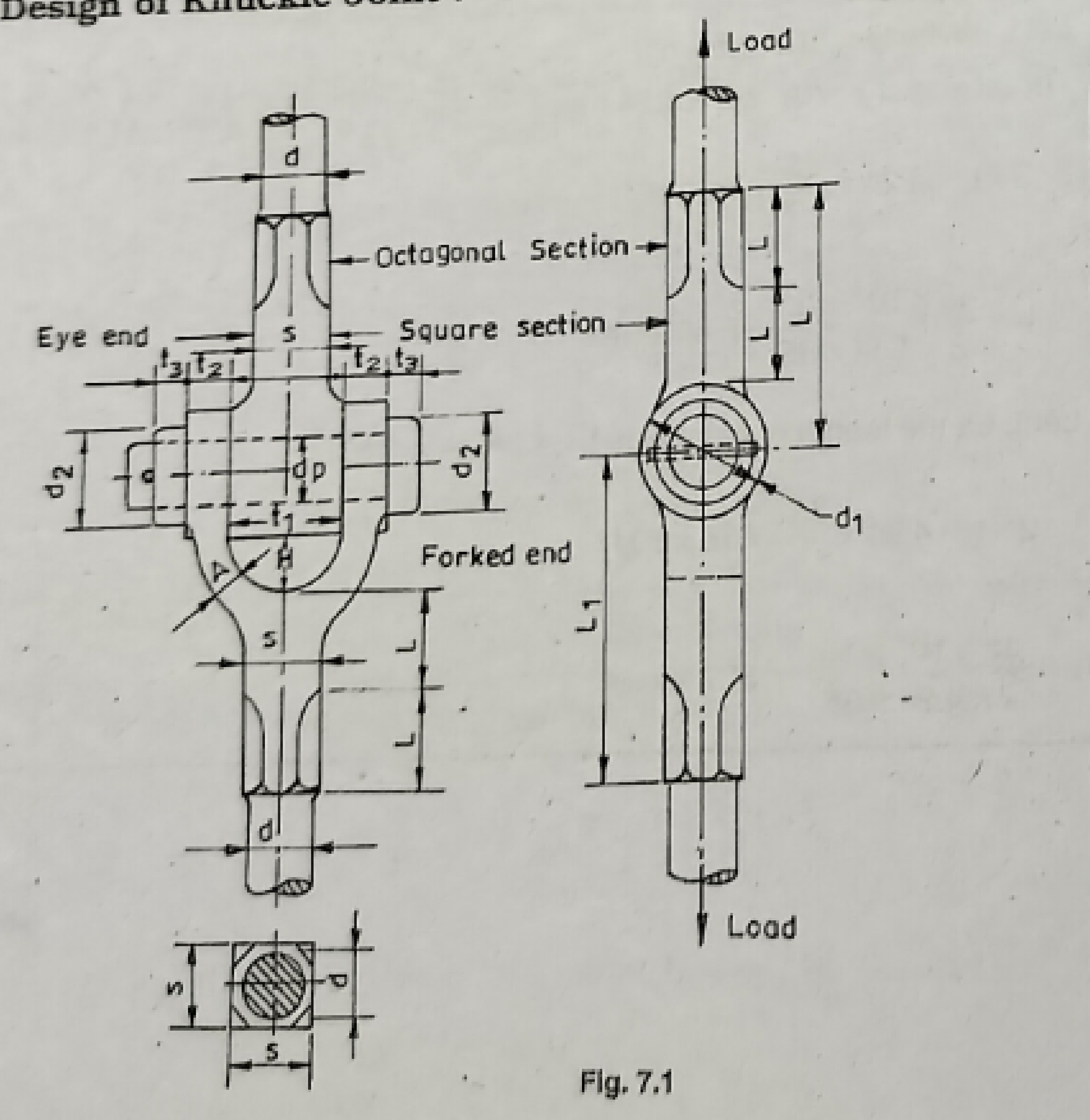
PIN JOINTS & COTTER JOINTS

7.1 Introduction :

Geometrically pins may be divided into cylindrical pins, called *straight pins* & *conical pins* or *taper pins*. The pins may be classified as those used only to locate the relative position of two parts, called *dowel pins* & those that fasten two or more parts together & are subjected to considerable stresses, which are mostly shear stresses but sometimes bending stresses. All connecting pins must be fitted well, using standard pins & reamers, & they must be driven home to prevent them from working loose. *Grooved pins* are now finding wide application because they do not require reaming of the drilled holes.

In other type of joints the pins are used as fulcrums in knuckle joints. A *Knuckle joint* is used to connect two rods in tension and the joint is not expected to transmit any rotary motion. However small oscillatory motion is permitted and it can connect two rods which are not in a straight line.

7.2 Design of Knuckle Joint :



s	1.2 d
d_p	d
d_1	2d
t_1	1.2 d
t_2	0.75 d
d_2	1.5 d
t_3	0.5 d
A	0.6 d
L	4d
L_1	4.8 d

Fig. 7.1

The standard proportions of a knuckle joint in terms of the diameters of the rods to be connected are shown in Fig. (7.1). Essentially the ends of the two rods at the joint are made into a *eye end* and *fork end* through which a connecting pin is inserted. The pin has a head at one end and a collar is fitted at the other end with a solid or split pin. The pin may rotate in both the eye and forked end or it may be fixed in the forked end.

After the diameter of the rod is determined, the other dimensions are worked out by the empirical proportions given in Fig. Then the dimensions are checked for failure of the joint in the following modes.

1. Failure of the knuckle pin in shear and bending.
2. Failure of the eye end in shear.
3. Failure of the eye end in tension.
4. Failure of the eye end in crushing.
5. Failure of the forked end in shear.
6. Failure of the forked end in crushing.
7. Failure of the forked end in tension.

The procedure will be illustrated with the help of an example.

7.3 Cotter Joints :

A *cotter* is a cross key used for fastening a rod to some part having a socket, the rod being subjected to tension or compression. The rod end is called the *spigot end*. It is used in reciprocating engines, pump sets to connect the piston rod to the cross head or strap end of the connecting rod. If only a cotter is used there is likely hood of the ends of the socket opening up. To prevent this a *Gib* is used which also gives parallel sides for the slots. A cotter is shown in Fig. (7.2).

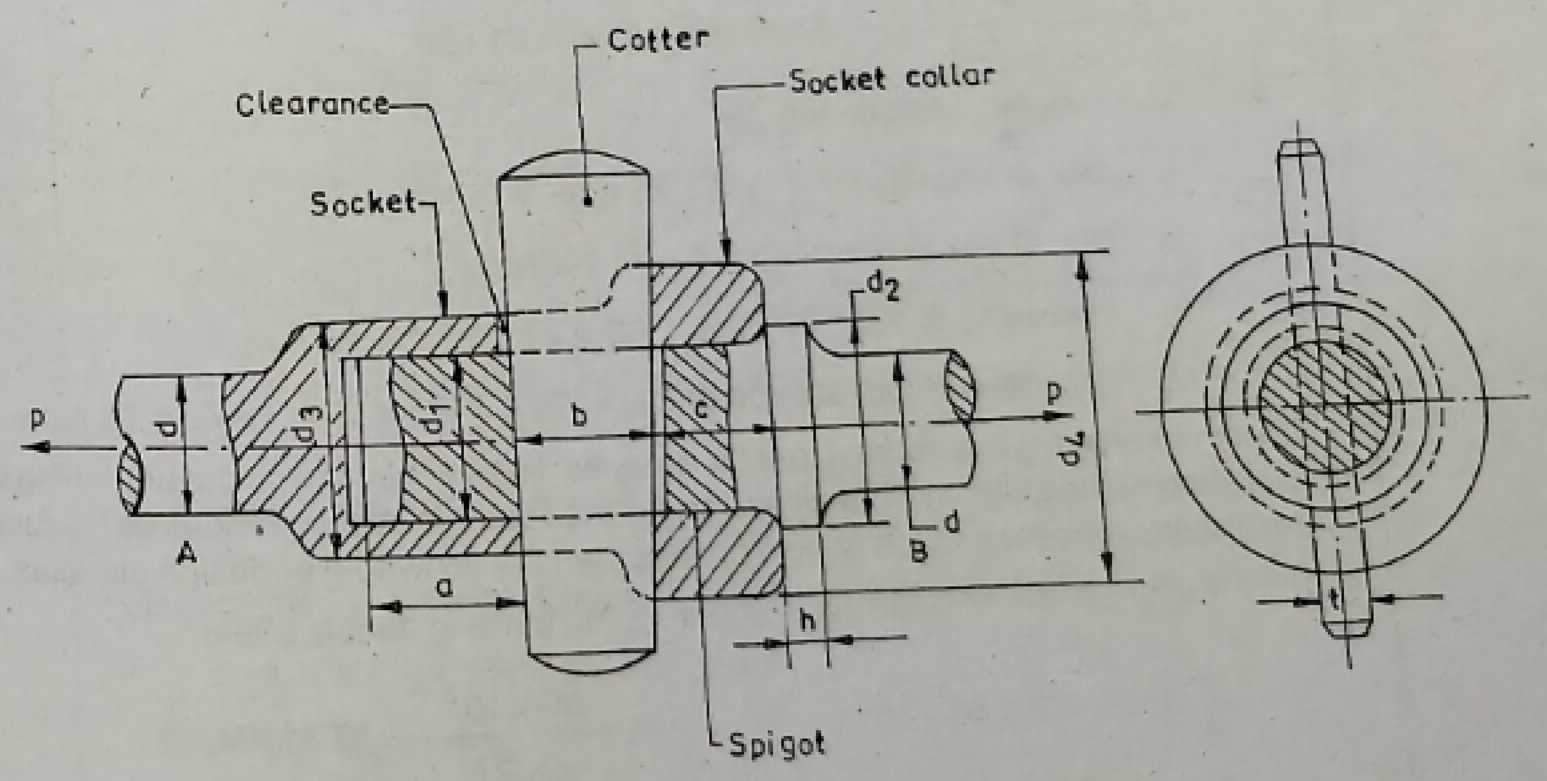


Fig. 7.2

7.4 Design of Cotter Joints :

First the rod diameter 'd' is calculated. Next the dimensions of the cotter are assumed as ;

Thickness of cotter, $t = 0.4\ d$

Width of cotter, $b = 4t = 1.6\ d$

The taper is given on the width and generally the taper is 1:24 if the joint is not subjected to vibration. After this it is necessary to check the failure of the three important parts in tension, shear and crushing at various places. The parts are :

(a) Cotter (b) Spigot (c) Socket

The procedure will be illustrated with the help of an example.

SOLVED PROBLEMS

Prob. 7.1 : Design a knuckle joint to carry a tensile load of 40 kN. The rods and pins are made of SAE 1020 steel. Assume a factor of safety of 2. Draw a neat sketch for the same.

Ref.	Calculations
T-II-7	For SAE 1020 steel $S_{ut} = 435,\ S_{yt} = 246,\ S_{ys} = 154$

$\therefore$ Allowable stresses $S_{dt} = \frac{246}{2} = 123\ \text{MPa}$ $\quad S_{ds} = \frac{154}{2} = 77\ \text{MPa}$

$S_{dc} = 2\ S\ ds = 2 \times 77 = 154$ MPa.

1. Dia of rods ; d :

$$\frac{\pi}{4} d^2 \times S_{dt} = 40 \times 10^3$$

$$d = \frac{40 \times 4 \times 10^3}{\pi \times 123} = 20.35 \text{ mm assume 21 mm.}$$

2. The empirical proportions are ;

i) Dia. of knuckle pin $d_p = d = 21$ mm.

ii) Dia. of eye end $d_1 = 2d = 2 \times 21 = 42$ mm.

iii) Thickness of eye $t_1 = 1.2\ d = 1.2 \times 21 = 25.2 = 26$ mm.

iv) Thickness of fork at each end $t_2 = .75\ d = .75 \times 21 = 15.75 = .16$ mm.

Other dimensions are not required for analysis. Hence they are calculated at the end and shown in the sketch. Now we will check the failure in various modes by calculating the induced stresses which should be less than the allowable or design stresses.

1. Failure of the knuckle pin in shear As the pin is in double shear.

$$\frac{\pi}{4} d_p^2 \times \tau \times 2 = 40 \times 10^3 \quad ; \quad \therefore \tau = \frac{80 \times 10^3}{\pi \times 21^2} = 57.8\ \text{MPa.}$$

$S_{ds} = 77$ so o.k.

If there is no slack in the fork and pin there will be no bending. But if the bending is present the loading on the pin is assumed as follows. The resultant loading is also shown.

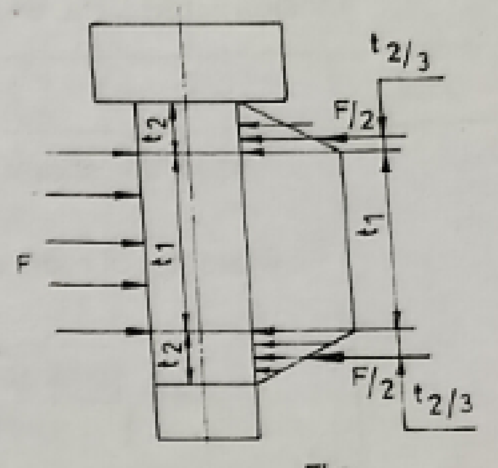


Fig.

This gives the maximum B.M. as

$$M_{max.} = \frac{F}{4}\left(t_1 + \frac{2t_2}{3}\right) = \frac{40 \times 10^3}{4}\left(26 + \frac{2 \times 16}{3}\right)$$

$$= 366.6 \times 10^3 \text{ N-mm.}$$

$$\therefore \text{ Bending stress } \sigma = \frac{M}{Z} = \frac{32 \times 366.6 \times 10^3}{\pi \times 21^3}$$

$$= 403.5\ \text{MPa}$$

This is too large. Hence the pin should not be slack in the fork.

2. Failure of the eye end in shear

$$t_1\,(d_1 - d_p)\,\tau = F \quad ;. \quad \tau = \frac{40 \times 10^3}{26\,(42 - 21)} = 73.2\ \text{MPa so o.k.}$$

3. Failure of the eye end in tension

$$t_1\left(d_1 - d_p\right) = \sigma_t = F \quad ; \quad \sigma_t = \frac{40 \times 10^3}{26(42 - 21)} = 73.2\ \text{MPa so o.k.}$$

4. Failure of the eye end in crushing.

$$d_p \times t_1 \times \sigma_{cr} = F \quad ; \quad \sigma_{cr} = \frac{40 \times 10^3}{21 \times 26} = 73.2\ \text{MPa so o.k.}$$

5. Failure of the fork end in tension.

$$\left(d_1 - d_p\right) 2t_2 \times \sigma_t = F \quad ; \quad \sigma_t = \frac{40 \times 10^3}{21 \times 2 \times 16} = 59.52\ \text{MPa so o.k.}$$

6. Failure of the forked end in shear.

$$\left(d_p - d_p\right) 2t_2 \times \tau = F \quad ; \quad \tau = \frac{40 \times 10^3}{21 \times 2 \times 16} = 59.52\ \text{MPa so o.k.}$$

7. Failure of the forked end in crushing

$$d_p \times 2t_2 \times \sigma_{cr} = F \quad ; \quad \sigma_{cr} = \frac{40 \times 10^3}{21 \times 2 \times 16} = 59.52\ \text{MPa so o.k.}$$

Hence the joint is safe. Other dimensions are dia. of knuckle pin head and collar,

$d_2 = 1.5\ d = 1.5 \times 21 = 31.5$ mm.

Thickness of pin head

$= 0.5\ d = 0.5 \times 21 = 10.5$ mm.

Prob. 7.2 : Determine all the dimensions of a cotter joint for an external load of 90 kN which changes continuously from tension to compression. The allowable stresses in all the parts are : Tension – 160 MPa, Shear – 100 MPa and crushing – 200 MPa

Calculations

Ref.

The joint is shown in Fig. (7.2)

Diameter of rods, d : $\frac{\pi}{4} \times d^2 \times S_{dt} = F$

$$d = \sqrt{\frac{90 \times 10^3 \times 4}{\pi \times 160}} = 26.76 = 28 \text{ mm.}$$

$t = 0.4\, d = 0.4 \times 28 = 11.2$ mm

$b = 4\, t = 4 \times 11 = 44$ mm.

Now we will calculate the other dimensions as shown in Fig.

1. Diameter of spigot. Failure in tension,

$$\left(\frac{\pi}{4} d_1^2 - d_1 t\right) S_{dt} = F$$

$$\text{OR } .785\, d_1^{\,2} - 11\, d_1 = \frac{90 \times 10^3}{160} = 0 \quad \text{OR} \qquad d_1^2 - 14 d_1 - 720 = 0$$

$$\therefore d_1 = \frac{14}{2} \pm \sqrt{\left(\frac{14}{2}\right)^2 + 720} = 34.7 \text{ say } 35\text{mm.}$$

1. (a) Failure in crushing

$$d_1 t\, \sigma_{cr} = F$$

$$\sigma_{cr} = \frac{90 \times 10^3}{35 \times 11} = 233.76 \text{ MPa.}$$ This is more than allowable stress of 200 MPa

$$\therefore d_1 = \frac{90 \times 10^3}{11 \times 200} = 40.9 \text{ say } 41\text{mm.}$$

$\therefore$ Assume $d_1 = 41$ mm.

2. Outside diameter of socket d_3 : Failure in tension

$$\left[\frac{\pi}{4}\left(d_3^2 - d_1^2\right) - \left(d_3 - d_1\right) t\right] S_{dt} = F$$

$$\left[.785\, d_3^2 - .785 \times 41^2 - 11 d_3 + 11 \times 451\right] = \frac{90 \times 10^3}{160}$$

$$d_3^2 - 14\, d_3 - 1588.6 = 0 \quad ; \quad d_3 = 7 + \sqrt{7^2 + 1588.6} = 46 \text{ mm.}$$

2. Outside dia. of socket collar, d_4 :

2. (a) Crushing failure

$$\left(d_4 - d_3\right) t\, S_{cr} = F \qquad \therefore d_4 = \frac{F}{t\, S_{cr}} + d_3 = \frac{90 \times 10^3}{11 \times 200} + 46 = 82 \text{ mm.}$$

Assume $d_3 = 82$ mm.

3. Distance of the spigot end from slot end

(a) Shear failure

$$2 \times a \times d_1 \times S_{ds} = F \quad ; \quad a = \frac{90 \times 10^3}{2 \times 41 \times 100} = 10.97 = 11 \text{ mm.}$$

4. Diameter of the collar : Failure in crushing for comp. loading

$$\frac{\pi}{4}\left(d_2^2 - d_1^2\right) S_{dcr} = F \quad ; \quad d_2^2 = \frac{90 \times 10^3}{.785 \times 200} + 41^2$$

$d_2 = 47.5$ mm say 48 mm.

5. Thickness of collar h : Failure in shear

$$\pi d_1 \times h \times S_{ds} = F \quad ; \quad h = \frac{90 \times 10^3}{\pi \times 41 \times 100} = 7.0 \text{ mm.}$$

6. Distance of socket end from the slot end c : Failure in shear

$$\left(d_3 - d_1\right) \times c \times 2 \times S_{ds} = F \quad ; \quad c = \frac{90 \times 10^3}{\left(82 - 41\right) \times 2 \times 100} = 10.97 = 11\text{mm.}$$

Check

7. Failure of cotter in shear

$$2 \times b \times t \times \tau = F \quad ; \quad \tau = \frac{90 \times 10^3}{2 \times 11 \times 44} = 92.97 \text{ so o.k.}$$

8. Failure of socket in crushing

$$\left(d_3 - d_1\right) t \times \sigma_{cr} = F \quad ; \quad \sigma_{cr} = \frac{90 \times 10^3}{\left(82 - 41\right) \times 11} = 199.5 \text{ so o.k.}$$

⁂

Chapter 8

FITS, TOLERANCES, PRESS & SHRINK JOINTS

8.1 Introduction :

Metal fits must be specified to ensure the proper assembly and functioning of mating machine members. Since it is impossible to manufacture large quantities of machine parts with exactly the same dimensions, systems have been devised to tolerate small dimensional variations of the mating parts without sacrificing their proper functioning. The terms used in this connection and their meanings are discussed below.

The principal terms used in describing limits and fits are :

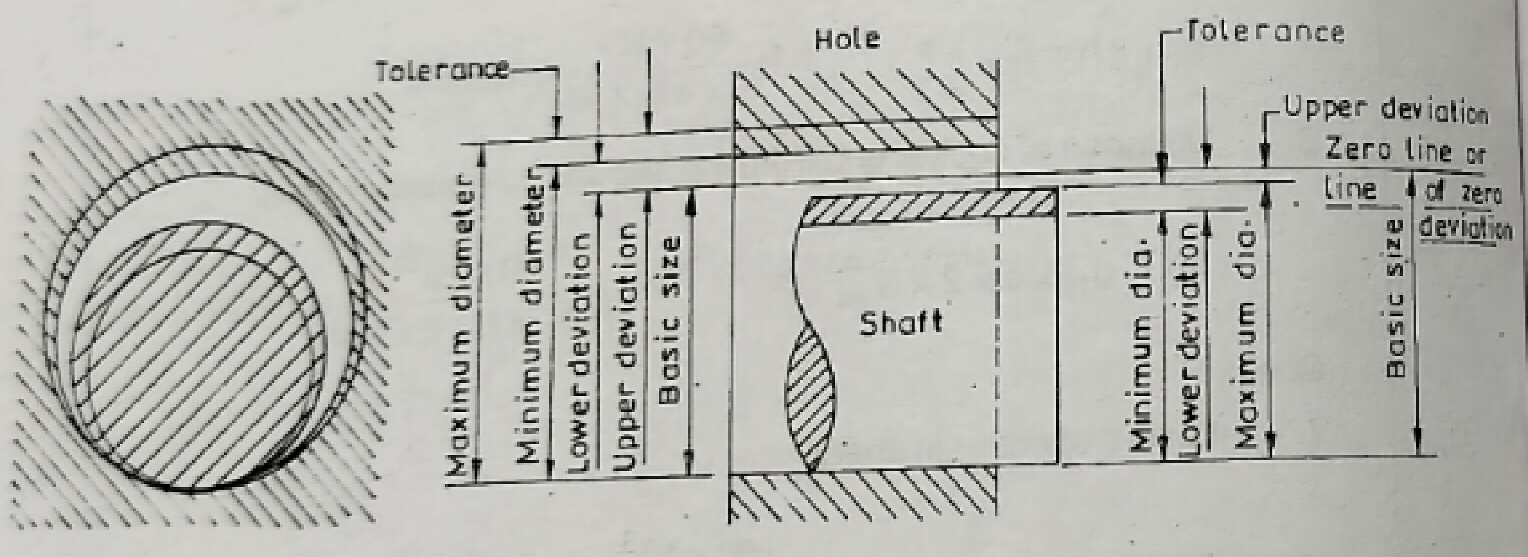


Fig. 8.1 Principal terms used in describing limits and fits

1. **Basic or Nominal Size :** This is the theoretical size decided upon by the designer and to which tolerances are applied to arrive at final dimensioning of mating parts.
2. **Tolerance :** This is the maximum permissible variation in the size of the part. The tolerance may be *bilateral,* in which case the size of the part is permitted to vary above and below the basic size such as 30.000 ± .0200 or it may be *unilateral*, in which case the part may be either larger or smaller than the basic size, but not both, such as

   $30.000 \pm {}^{0.000}_{0.040}$
3. **Allowance :** This is the difference in the basic sizes of mating parts. The difference in the size of hole and the shaft can be positive when it is called *Clearance* or it could be negative when it is called *Interference.*
4. **Limits :** These are the maximum & minimum sizes permitted for the parts such as 30.020/29.980.

5. **Upper Deviation :** The algebraic difference between the maximum limit of the basic size and the corresponding basic size.
6. **Lower Deviation :** The algebraic difference between the minimum limit of the basic size and the corresponding basic size.

8.2 Fits & Their Uses :

There are three classes of fits which are used in practice.

They are :

(a) **Clearance Fit :** In this the hole diameter is always larger than the shaft diameter so that there is always a clearance between the mating parts. In clearance fit we have three types.

i) Loose fit ii) Free fit iii) Medium fit.

(b) **Interference Fit :** In this the hole diameter is always less than the shaft diameter so that there is always an interference between the mating parts. In this also we have three types.

i) Tight fit ii) Medium force fit iii) Heavy force fit.

(c) **Transition Fit :** In this, depending on the actual dimensions of the finished products, either there is clearance or interference between the mating parts.

The commonly used fits, allowances, tolerances, tolernces and their uses are given in T-IV-1 & T-IV-2.

8.3 Fundamental Aspects of Tolerance System & Fits :

A tolerance is designated by a letter and a number. Capital letters are used for holes and small letters for shafts. The letter symbol indicates the position of the zone of tolerance in relation to the zero line representing the basic size while the numerical symbol represents the value of this zone of tolerance and is called the grade of tolerance. Both the position and the grade of tolerance are functions of the basic size. Thus a toleranced size of hole and shaft are written as :

$$30^{H-7} \text{ and } 30_g - 6$$

Similarly a fit is indicated by the basic size common to both the mating parts, followed by the appropriate symbols corresponding to each component.

$$30\,{}^{H-7}_{g-6} \text{ or } 30 - H - 7/g - 6$$

The letter symbols range from A to ZC for holes and a to zc for shafts. The actual deviations for each letter symbol are given in T-IV-7 while their position with respect to the basic size is shown in F-4-1 of data book. Similarly there are 18 grades of tolerances designated as IT-01 to IT-18. Basic sizes have been grouped in steps shown below.

From 1 mm to 500 mm -- 13 steps

From 500 mm to 3150 mm -- 8 steps

The use of different grades of tolerance is as follows :

(a) Grades 1 to 7 are mainly used for gauges.

(b) Grades 5 to 11 are primarily for the fit of the work pieces achieved through metal-cutting machining processes.

(c) Grades 12 to 16 are found suitable for rougher production tolerance in processes not involving metal cutting. The actual tolerances for basic sizes of 1mm to 500 mm and for grades 5 to 11 are given in T-IV-6.

The use of these tables will be illustrated with the help of solved problems.

8.4 Press Fit or Press Joints :

This is obtained by forcing the shaft into a hole that is slightly smaller than the shaft. This is possible because of the elasticity of the material. The tendency of the material to return to their original dimensions produces the grip that holds the shaft and the hub together. The stresses due to interference fits may be calculated by considering the fitted parts as thick-walled cylinders. First we calculate the contact pressure P_c developed between the parts and then determine the stresses with the help of equations given in T-I-22. The maximum axial force required to assemble a force fit is approximated by

$$F_a = \mu \pi d L P_c$$

where F_a = axial force, N

μ = coefficient of friction

d = nominal shaft diameter, mm

L = length of outer member, mm

P_c = contact pressure, MPa

The torque that can be transmitted by a interference fit without slippage can be estimated by

$$T = \frac{\mu \pi d^2 L P_c}{2}$$

8.5 Shrink Joint :

This joint differs from the press joint in the method of assembling the mating parts. The hub is heated to expand its bore in order to slip it on the shaft. When the hub cools down, the grip is produced in the same way as a press joint. It must be pointed out that the press joints are easier to make especially if a hydraulic press of sufficient capacity is available. Shrink joints are used mainly in places where it is difficult or impossible to use a press joint. However tests have shown that shrink joins with the same interference as press joints give more than three times the holding power against both torsion and axial pull. This superior effectiveness is due to the absence of abrasion between the surfaces of the shaft and the hub during assembly. The temperature change ΔT required to effect an increase δ in the inside diameter of the hub may be determined by ;

$$\Delta T = \frac{\delta}{\alpha d_i}$$

where δ = diameteral interference

α = coeff. of linear expansion

d_i = initial diameter of hole.

An alternative to heating the hub is to cool the shaft by means of a coolant such as dry ice. Now we will illustrate the use of these equations with the help of solved problems.

SOLVED PROBLEMS

Prob. 8.1 : Give the dimensions for the hole and shaft for the following (a) 12 mm electric motor sleeve bearing (b) 50 mm sleeve bearing on the elevating mechanism of a road grader (c) Medium force fit on a 200 mm shaft.

Ref.	Calculations
T-IV-1	For a sleeve bearing of electric motor, select free fit for which Allowance $= -.0040\,(D)^{2/3} = -.0040\,(12)^{2/3} = -.021$ mm Hole tolerance $= +.01\,(D)^{1/3} = +.01\,(12)^{1/3} = +.0229$ mm Shaft tolerance $= -.01\,(D)^{1/3} = -.01\,(12)^{1/3} = -.0229$ mm
T-IV-6	For tolerance of 23 microns use grade 7 which gives tolerance of 18 microns for dia. of 12 mm.
I-IV-7	For deviations use H for hole which is 0 microns. For shaft use f which gives – 16 microns So we select 12 H – 7/f 7
F-4.1	Hole dimensions : 12.000 and 12.018 mm Shaft dimensions : $12.000 - 0.16 = 11.984$ $11.984 - .018 = 11.966$ This will give the clearances of : Max. $= 12.018 - 11.966 = .052$ mm. Min. $= 12.000 - 11.984 = .016$ mm. Average clearance $= \frac{52+16}{2} = 34$ microns.
T-IV-2	Commonly used normal running fit - H-7/ f-7
T-IV-1	(b) For sleeve bearing of a road grader use loose running fit. Allowance $= -.0075\,(D)^{2/3} = -.0075\,(50)^{2/3} = -.102$ mm Hole tolerance $= +.02\,(D)^{1/3} = +.02\,(50)^{1/3} = +.0736$ mm Shaft tolerance $= -.02\,(D)^{1/3} = -0.2\,(50)^{1/3} = -.736$ mm.
T-IV-6	For 50 to 80 dia. and 74 microns tolerance, use grade 9 where tolerance = 74 microns.
T-IV-7	For deviations : Hole – H – deviation = 0 shaft – d – deviation = – 100 so select 50 H-9/d-9.
F-4-1	∴ Hole dimensions : 50.000 and 50.074 Shaft dimensions : $50{:}000 - 0.100 = 49.900$ $49.900 - 0.074 = 49.826$ Max. clearance $= 50.074 - 49.826 = .248$ mm Min. clearance $= 50.000 - 49.900 = .100$ mm Average clearance $= \frac{248+100}{2} = 174$ microns

T-IV-2	Commonly used loose running fit - H-8/f-8
	(c) Medium force fit :
	Allowance = + .0005 D = .0005 × 200 = 0.100 mm
	Hole tolerance = + .005 $(D)^{1/3}$ = .005 $(200)^{1/3}$ = + .029 mm
	Shaft tolerance = + .005 $(D)^{1/3}$ = .005 $(200)^{1/3}$ = + .029 mm
	For tolerance of 29 microns on 200 mm dia. use grade 6 where tolerance is 28 microns
T-IV-6	For deviation : Hole – H – deviation 0
T-IV-7	Shaft – s – deviation + 122
	200 H-6 / s-6
	Hole dimensions = 20.000 & 20.028
	Shaft dimensions = 20.000 + .122 = 20.122 mm
	= 20.122 + .028 = 20.150 mm
	Max. interference = 20.000 – 20.150 = – .150
	Min. interference = 20.028 – 20.122 = –.094
T-IV-2	Commonly used medium force fit - H-7/r-6

Prob. 8.2 : A 150 mm diameter steel shaft is to have a press fit with a 300 mm O.D. by 250 mm long hub of cast iron. The max. tangential stress is to be 35 MPa. E = 200 GPa for steel and 100 GPa for cast iron. Poisson's ratio = 0.3 for both and coefficient of friction = 0.12 (1) Determine the maximum diametral interference (2) What axial force Fa will be required to press the hub on the shaft and (3) What torque may be transmitted with this fit ? Design the fit for the joint.

Ref.	Calculations
T-I-22	The assembly can be seen as :
	$R_1 = \frac{150}{2} = 75$ mm. $R_2 = \frac{300}{2} = 150$ mm
	Let the contact pressure be P_c
	∴ Tangential stress will be max. at the inner surface of the C.I. hub.
	$\sigma_1 = \frac{R_1^2 P_c}{R_2^2 - R_1^2}\left[1 + \frac{R_2^2}{R_1^2}\right] = \frac{75^2 P_c}{150^2 - 75^2}\left[1 + \frac{150^2}{75^2}\right]$
	$35 = 1.66\, P_c \quad \therefore P_c = 21$ MPa
	Total deformation $\delta = \frac{R_1 P_c}{E_o}\left[\frac{R_2^2 + R_1^2}{R_2^2 - R_1^2} + \mu_o\right] + \frac{R_1 P_c}{E_i}(1 - \mu_i)$
	$= \frac{75 \times 21}{100 \times 10^3}\left[\frac{150^2 + 75^2}{150 - 75^2} + 0.3\right] + \frac{75 \times 21}{300 \times 10^3}[1 - 0.3] = 0.3517$ mm

	∴ Total diametral interference = 2 × .03517 = .07034 mm
	Axial force to assemble $F_a = \mu \pi d L P_c$
	$= .12 \times \pi \times 150 \times 250 \times 21 = 296.73$ kN
	Torque transmission capacity $= F_a \times \frac{d}{2} = 296.73 \times \frac{150}{2} = 22.25 \times 10^3$ kN – m.
	Maximum interference = 70 microns
T-IV-2	Select $H_7\, s_6$ fit.
	For 120 – 180 dia. – Grade 7 tolerance = 40 microns
	Grade 6 tolerance = 25 microns
	Deviation Hole – H = 0
	Shaft – s = + 65
	∴ Hole dimensions = 150.000 & 150.040
	Shaft dimensions = 150.00 + 0.065 = 150.065 & 150.065 + 0.025 = 150.090
	∴ Max. interference = 150.000 – 150.090 = – .090 mm
	Min. interference = 150.040 – 150.065 = – .025 mm
	So selective assembly will be requital.

Prob. 8.3 : Two circular rings are to be shrunk on the ends of the hub of a split flywheel as shown in Fig. The outside diameter of the hub is 405 mm and the centrifugal force that must be resisted by the rings is 800 kN. Determine the inside diameter of the rings and their cross-section, provided the maximum tensile stress does not exceed 215 MPa. Take E = 200 GPa. Also determine the temperature to which the rings must be heated for assembling.

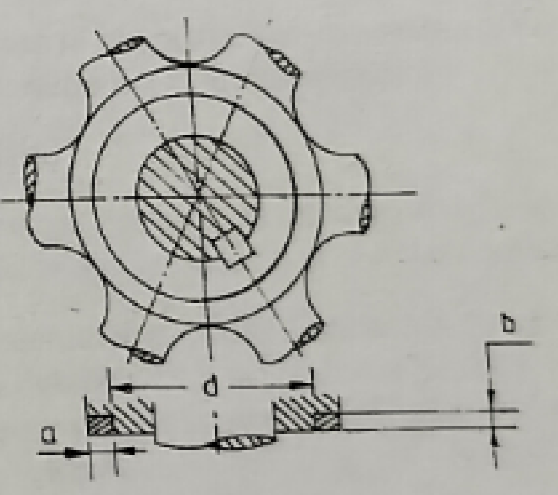


Fig.

Ref.	Calculations
	d = 405 mm, cross section = a × b
	Treating the hub as incompressible. The radial interference will be ;
	$\delta = \frac{\sigma_t \times d}{2E} = \frac{215 \times 405}{2 \times 200 \times 10^3} = 0.217$ mm
	∴ Inside diameter of ring = 405 – 2 × 0.217 = 404.566 mm.

The initial tension set up by the shrinkage must be greater than the centrifugal force to hold two halves of the hub together. Assuming a margin of safety of 30 percent, the force that each ring must resist is :

$$F = \frac{1}{2} \times 800 \times 1.3 = 520 \text{ kN.}$$

∴ For an allowable tensile stress of 215 MPa the cross section of the ring should be

$a \times b \times 215 = 520 \times 10^3$ ∴ $a \times b = 2418.6 \text{ mm}^2$

Taking $a = b$, $a = 49.17$ mm say $a = b = 50$ mm

T-II-2

Coefficient of linear expansion for steel $\alpha = 10.7 \times 10^{-6} \dfrac{\text{mm}}{\text{mm}^\circ\text{C}}$

Total expansion $= .217 \times 2 = .434$ ∴ $d \times \alpha \times \Delta T = .434$

$$\therefore \Delta T = \frac{.434 \times 10^6}{405 \times 10.7} = 100^\circ\text{C.}$$

⁂

Chapter 9

MECHANICAL SPRINGS

9.1 Introduction :

Spring are elastic members that exert forces (or torques) & *absorb energy* (which is usually stored & later released). Springs are usually, but not necessarily, made of metal. Plastics can be used when loads are light. Blocks of rubber often constitute springs, as in bumpers & in vibration - isolation mountings of various machines such as electric motors, refrigerators, compressors. There are pneumatic & Hydraulic springs which make use of the compressibility of fluids. However springs made of solids, metals or plastics only will be discussed in this book. Generally springs may be classified in three types :

(a) *Wire springs* which include coil springs of round or square wire and are made to resist tensile, compressive or torsional loads.

(b) *Beam springs* (commonly fabricated as multileaf springs) are usually arranged as cantilevers & simply supported beams in the form of a quarter, half or full ellipse. These springs are also called *flat springs,* although they usually have some curvature when unloaded.

(c) *Disc or Belleville springs* are made in the form of a series of flat, tapered or dish shaped disks. These are used for large loads and deflections.

The major functions of springs are :

(a) To cushion, absorb or control energy due either to shock, as in car springs or railways buffers, or to vibration, as in spring support and vibration dampers.

(b) To control motion by maintaining contact between two elements such as cam and follower ; by restoring a machine part to its normal position when the disturbing force is removed, as in a governor or valve; by producing the necessary pressure in a friction device, as in a brake or clutch.

(c) To store energy as in clock works or starters.

(d) To measure forces, as in spring balances, gages or engine indicators.

9.2 Helical Springs :

They are usually made of circular cross section wire and are subjected to either compressive or tensile loads. The design procedure for both the loads is same except for the effect of ends on the selection of allowable stress for tension springs. Hence general procedure for stress and deflection analysis will be discussed first and then the design of ends will be discussed later.

A helical coil spring under a compressive load is shown in Fig. (9.1).

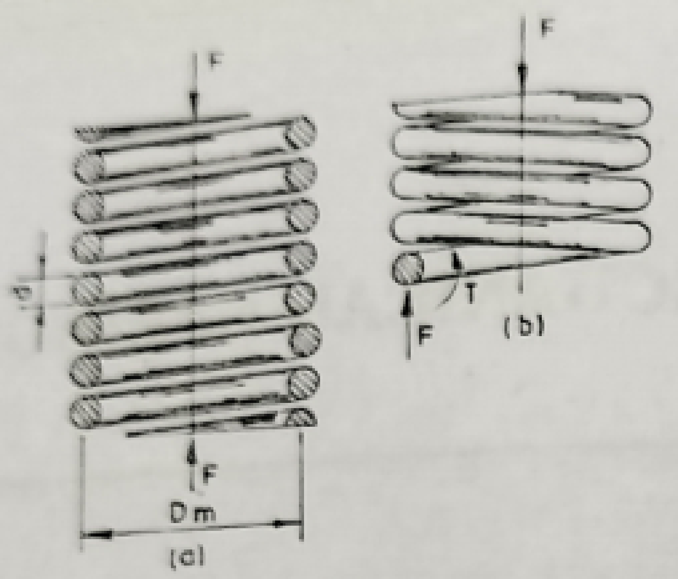


Fig. 9.1

Let $\frac{D_m}{d} = C$ called *spring index.*

The cut section as shown in Fig. will be subjected to a direct shear load F and a torque T. The resultant shear stress on the section will be :

$$\tau_{max} = \frac{T}{Z_p} + \frac{F}{A} = \frac{8FD_m}{\pi d^3} + \frac{4F}{\pi d^2} \qquad \text{For round wire } Z_p = \frac{\pi}{16} d^3$$

$$= \frac{8FD_m}{\pi d^3}\left[1 + \frac{0.5\,d}{D_m}\right] \qquad A = \frac{\pi}{4} d^2$$

$$= \frac{8FD_m}{\pi d^3}\left[1 + \frac{0.5}{C}\right] = K_1 \frac{8FD_m}{\pi d^3} \qquad T = \frac{FD_m}{2}$$

The quantity $\left(1 + \frac{0.5}{C}\right)$ is called shear stress multiplication factor and is denoted by K_1. There is another effect due to the curvature of the wire which further increases the stress induced at the inner fibre as shown in Fig. (9.2).

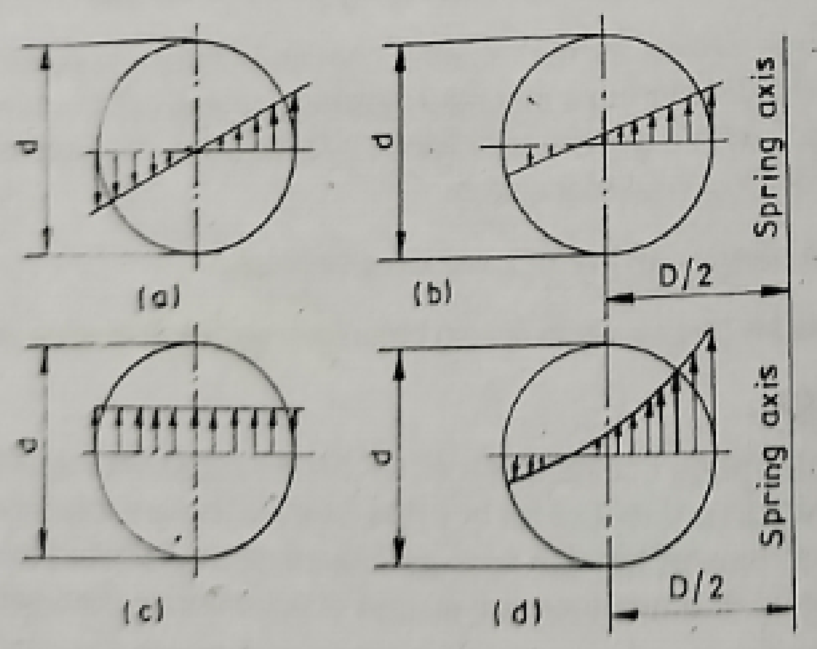


Fig. 9.2

Therefore the maximum shear stress induced at the inside fibre of wire is given by (T-VIII-1).

$$\tau_{max.} = K\frac{8FD_m}{\pi d^3} = K\frac{8FC}{\pi d^2}$$

where K = Wahl's correction factor $= \frac{4C-1}{4C-4} + \frac{0.615}{C}$

By defining $K = K_1 . K_2$ where K_2 is the effect of curvature alone, K_2 can be found out by the relation.

$$K_2 = \frac{K}{K_1}$$

Investigation reveal that the effect of curvature is highly localized on the inside of the spring. Hence, for static loads only K_1 should be considered.

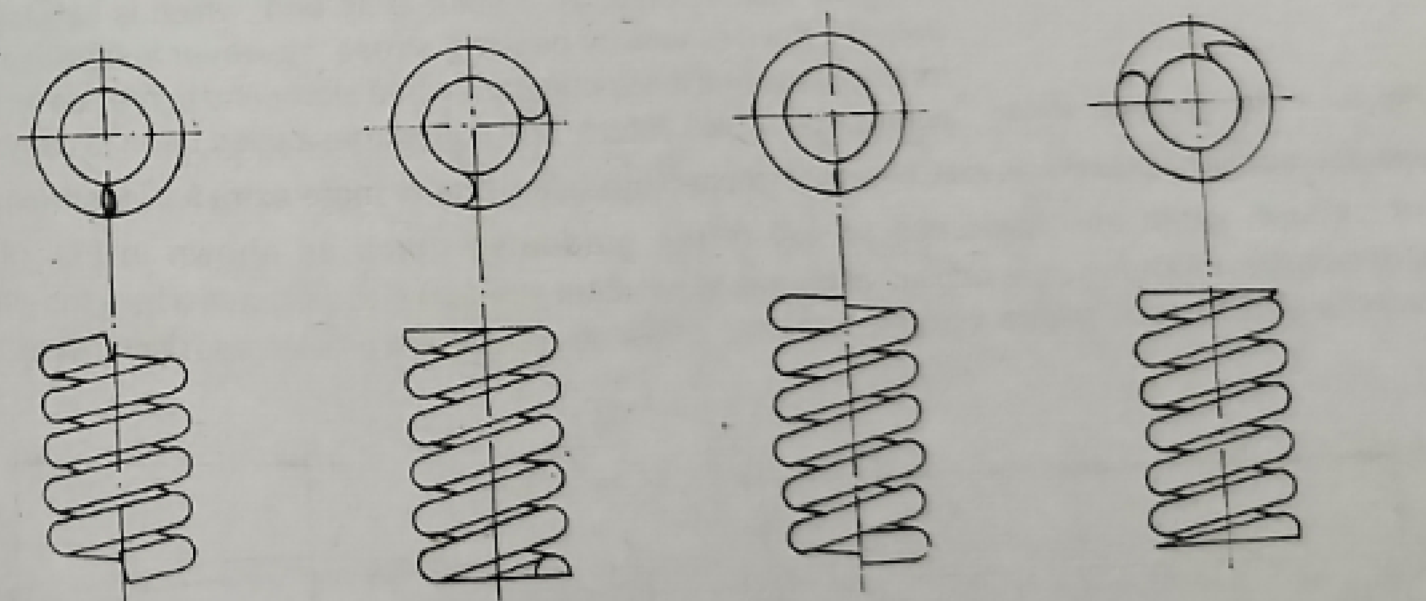
The deflection of a helical spring having n, effective turns, can be shown to be (T-VIII-1).

$$\delta = \frac{8FD_m^3\,n}{Gd^4} = \frac{8FC^3\,n}{G\,d}$$

The equation for the spring rate or stiffness denoted by K_s is given by,

$$K_s = \frac{F}{\delta} = \frac{Gd}{8\,C^3\,n}$$

Spring ends for compressive loads may be either plain and ground, squared or squared and ground as shown in Fig. (9.3) The relation between the number of active coils and the total number of coils is given in (T-VIII-3) of data book.

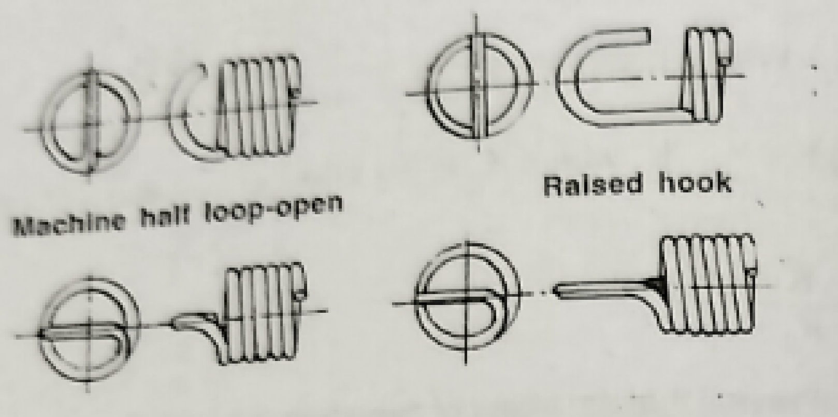


Plain End **Plain & Ground Ends** **Squared Ends** **Squared & Ground Ends**

Fig. 9.3

Spring ends for extension springs are of various types and are shown in Fig. (9.4). The ends add to the stress concentration which depends on the type of end. Another important consideration in the design of compression springs is to check the design from the buckling effect which may occur if the free length is more than four times the mean diameter, unless the spring is properly guided.

(F-8.2 of data book). The springs are generally made from steels whose properties are given in T-VIII-9. For special purposes, they are made of non-ferrous metals such as brass, bronze whose properties are also given in the table. The design procedure is illustrated with the help of solved problems.


Machine half loop-open

Raised hook

Short twisted loop

Full twisted loop

Fig. 9.4

9.3 Beam or Leaf Springs :

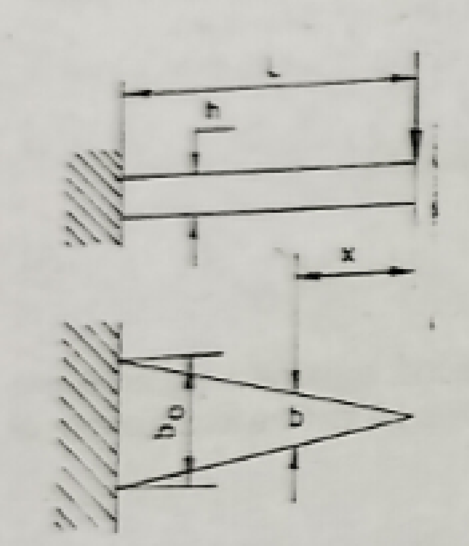


Fig. 9.5 A flat triangular spring

These springs are made out of flat strips which are given some initial curvature. Any flat strip loaded as cantilever has maximum bending moment at the support. Hence the bending stresses also vary linearly from the support to the free end. If the strip has uniform width, the stresses also vary in the same manner. Hence, to utilize the material properly, width of the strip should also be changed. Hence, a constant strength beam should have a triangular shape as shown in Fig. (9.5). This triangular shape is obtained with the help of multileaf graduated or laminated springs. The multileaf spring may be considered as a triangular plate cut into strips of width b and stacked in a graduated manner as shown in Fig. (9.6). A graduated spring made from a triangular beam comes to a point at its end, which is satisfactory from the point of view of bending stress. However sufficient metal must be provided to support transverse shear and to provide for load connection which inturn are sometimes called upon to carry end thrust and twisting action. This may be accomplished by adding one or more extra full length leaves, of uniform width and thickness on top of the graduated stack as shown in Fig. (9.6). The extra full length leaves are not part of beams of constant strength and will have a bending stress approximately 50 percent greater than the graduated leaves unless they are prestressed during assembly.

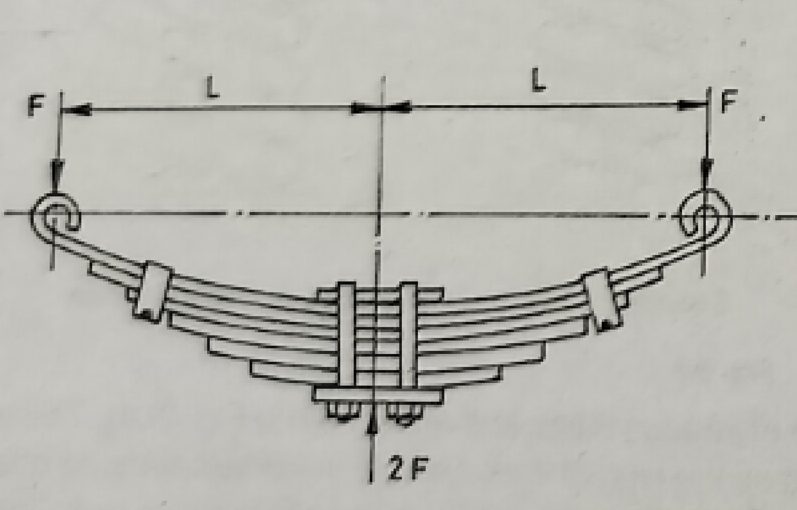


Fig. 9.6

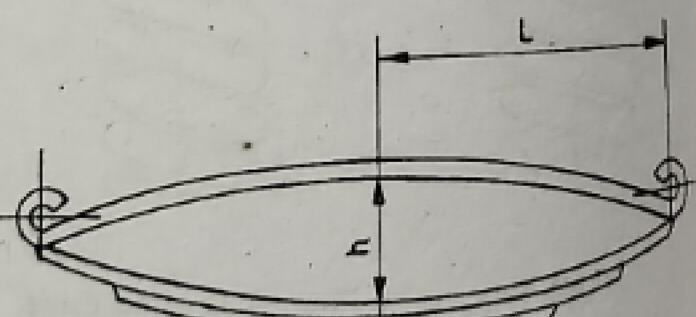


Fig. 9.7

Prestressing can be obtained by having extra full length leaves formed with a different radius of curvature than the graduated leaves. This will leave a gap 'h' betwеen the extra full length leaves and the graduated leaves before assembly as shown in Fig. (9.7). Then, upon assembly, the extra full length leaves will have an initial stress of oppositive sign to that which occurs when the load is applied. The gap may be determined so that all the leaves will be equally stressed after the full load is applied.

9.4 Design :

The basic element for stress and deflection analysis is taken a cantilever of length L and loaded by a force. The semielliptic spring can be considered as two cantilevers sharing the load in parallel. The stress and deflection equations for two cases are given below :

1. Cantilever

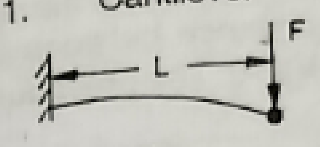


Fig.

$$\sigma_b = \frac{6\,FL}{bt^2}$$

$$\delta = \frac{6\,FL^3}{Ebt^3}$$

2. Semielliptic

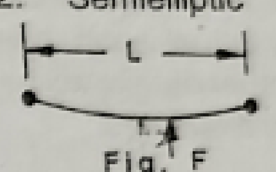


Fig.

$$\sigma_b = \frac{3FL}{2bt^2}$$

$$\delta = \frac{3FL^3}{8Ebt^3}$$

For graduated leaf spring consisting of n leaves, the total b is to be taken as 'nb' in these equations. Then the stress and deflection equations for semielliptic and cantilever springs are (T-VIII-4). Their derivation is shown in prob. 9.7.

1. Semielliptic spring without extra full length leaves

$$\sigma_b = \frac{3}{2}\,\frac{FL}{nbt^2}\,; \quad \delta = \frac{3}{8}\,\frac{FL^3}{Enbt^3}$$

2. Semielliptic spring with n_g graduated and n_f extra full length leaves without prestressing.

$$\sigma_b \text{ in } n_f = \frac{3\,FL}{2\,n_g\,bt^2}\left[\frac{1.5}{1+1.5\,\frac{n_f}{n_g}}\right]\,; \quad \sigma_b \text{ in } n_g = \frac{3\,FL}{2\,n_g\,bt^2}\cdot\left[\frac{1}{1+1.5\,\frac{n_f}{n_g}}\right]$$

$$\delta = \frac{3\,FL^3}{8E\,n_g\,bt^3}\left[\frac{1}{1+1.5\,\frac{n_f}{n_g}}\right]$$

3. If they are prestressed in assembly by forming the leaves with a gap 'h' as mentioned earlier, then the stress in all the leaves will be equal and the deflection remains unchanged.

Then,

$$\sigma_b = \frac{3\,FL}{2\left(n_f+n_g\right)bt^2}\,; \quad \delta = \frac{3\,FL}{8F\,n_g\,bt^3}\left[\frac{1}{1+1.5\,\frac{n_f}{n_g}}\right]$$

Initial gap $h = \frac{FL^3}{8 E n_g b t^3}$

4. For cantilever spring

$$\sigma_b = \frac{6FL}{n b t^2} \qquad \delta = \frac{6FL^3}{E n b t^3}$$

In semielliptic springs, the graduated leaves are assembled together by means of central clamp of certain width or U bolts put at a distance. To account for this strengthening at the centre the effective span is taken as the actual span, minus the clamp width (distance between U bolts). The selection of materials and other details will be as per T-VIII-9 and T-VIII-5. The procedure will be illustrated with the help of solved examples.

SOLVED PROBLEMS

Prob. 9.1 : In some machine component spring operates between two compressed lengths of 65 mm and 45 mm providing loads of 300 N and 700 N respectively. Following springs are available in stock. Select suitable spring.

Item	1	2	3
Wire diam., d, mm	2.0	3.5	5.0
Mean coil dia. D_m, mm	15	15	30
No. of effective turns, n	9.5	8.25	11.6
Free length, l_0, mm	80	80	80

The material of all springs is music wire and ends are squared and ground for all the springs.

Ref.	Calculations			
	Required spring stiffness, $K_s = \frac{700-300}{65-45} = 20$ N/mm			
	∴ Maximum deflection, $\delta_{max} = \frac{700}{20} = 35$ mm			
	Now the total number of turns for the three springs :			
		1	2	3
T-VIII-3	$n_1 = n + 1.75$	11.25	10	13.35
	∴ Minimum free length required : $n_1 \times d + \delta_{max}$			
		1	2	3
	$L_{o\,(min)}$ mm	62.5	74.5	107.75
	∴ spring no. 3 is not suitable.			
	Now checking spring 1 and 2 from stress considerations.			

Ref.	Item	1	2	3
T-VIII-1	Spring index, $C = \frac{D_m}{d}$	7.5	4.286	
	Wahl's factor, K	1.197	1.37	
	Max. shear stress max MPa	4002.7	854.86	
T-VIII-9	For music wire S_{ys} = 1060 MPa. Therefore only spring number 2 is suitable.			

Prob. 9.2 : Design a helical spring for the front-wheel suspension of an automobile using Cr-Vanadium steel. The spring must have a scale of 40 N per mm, an inside diameter of 100 mm and a free length between 300 to 350 mm with squared and ground ends. The spring must be able to carry a static load of 3.6 kN and when compressed solid, it should not receive a permanent deformation.

Ref.	Calculations
	Given spring stiffness, $K_s = 40$ N/mm, $F_{max.} = 3600$ N
	$D_i = 100$ mm, Free length = 350 mm
	Material Cr – V steel.
T-VIII-9	$S_{ys} = 770$ MPa G = 85 GPa
	$D_i = D_m - d = d\,(c-1) = 100 \qquad \therefore d = \frac{100}{c-1}$
	$K_s = \frac{Gd}{8c^3 n}$ or $n = \frac{Gd}{8c^3 K_s} = \frac{85 \times 10^3 \times 100}{8c^3 \times 40 \times (c-1)} = \frac{26 \times 10^3}{c^3 (c-1)}$
T-VIII-3	Total no. of turns $n_1 = n + 1.75$
	As there is restriction free length and then add clash allow an ce.
	∴ Minimum free length $= n_1 \times d + \delta_{max}$
	$= \frac{100}{c-1}\left[\frac{26 \times 10^3}{c^3 (c-1)} + 1.75\right] + \frac{3600}{40} = 300$
	or $c^5 - 2.83\,c^4 + 1.83\,c^3 - 12.38 \times 10^3 = 0$
	By trial and error, c = 7.25
	$\therefore d = \frac{100}{7.25-1} = 16$ mm $\qquad n = \frac{26 \times 10^3}{7.25^3\,(7.25-1)} = 10.91$
	$n_1 = 10.91 + 1.75 = 12.66 \approx 13$
	∴ Free length, $L_o = 13 \times 16 + 90 + 12 \times 1 = 310$ mm so o.k.
	Wahls factor, $K = \frac{4 \times 7.25 - 1}{4 \times 7.25 - 4} + \frac{0.615}{7.25} = 1.2$

T-VIII-1 $\tau = \frac{K.8Fc}{\pi d^2} = \frac{1.2 \times 8 \times 3600 \times 7.25}{\pi \times 16^2} = 324.69$ MPa so. o.k.

$\therefore$ Final results : $d = 16$ mm, $D_m = 16 \times 7.25 = 116$, $D_i = 100$

$n_1 = 13$, Free length $= 310$ mm.

Prob. 9.3 : A force of 4.45 kN is required to engage an automotive type disc clutch. The force is to provided by nine identical springs equally spaced around the pressure plate of the clutch. Because of space limitations the O.D. of the coils can not exceed 40 mm and the length of the springs when the clutch is engaged cannot exceed 52 mm. The pressure plate must move 3 mm to disengage the friction surfaces and the lowest spring rate reasonably possible is desired. Deisgn the springs. Determine a satisfactory combination of D_m, d, n and material.

Ref | **Calculations**

Total force = 4.45 kN, No. of springs = 9.

$$\text{Force/spring} = \frac{4.45 \times 10^3}{9} = 494 \text{ N}$$

$D_o < 40$ mm

T-VIII-9 For automotive clutch springs subjected to fatigue loading select

Cr – V steel SAE 6150.

$S_{ys} = 770$, $G = 85 \times 10^3$

Select Fs = 1.25, $\therefore \tau_{max} = \frac{770}{1.25} = 616$ MPa

T-VIII-I Select spring index, C = 8, $\therefore K = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{6} = 1.21$

$\therefore \tau_{max} = K.\frac{8Fc}{\pi d^2}$, $616 = \frac{1.21 \times 8 \times 494 \times 8}{\pi d^2}$

$d = 4.445 \approx 4.5$ mm $\quad D_m = 4.5 \times 8 = 34$, $\quad D_o = 34 + 4.5 = 39.5$ so o.k.

Generally the max deflection should be equal to D_m i.e. 40 mm.

$\therefore$ Spring rate, $K_s - \frac{494}{40} = 12.35$ N/mm ≈ 12 N/mm

$$K_s = \frac{Gd}{8c^3n} \text{ or } n = \frac{G \times d}{8c^3K_s} = \frac{85 \times 10^3 \times 4.5}{8 \times 8^3 \times 1200}$$

$\therefore n_1 = 7.78 + 1.75 = 7.78$

$= 9.53$ say 10 $\quad \therefore = 10 - 1.75 = 8.25$

$$\therefore \text{Actual } K_s = \frac{85 \times 10^3 \times 4.5}{8 \times 8^3 \times 8.25} = 11.32 \text{ N/mm}$$

$\therefore F_{max} = 494 + 3 \times 11.32 = 527.96$ N

$\therefore \tau_{max} = \frac{1.21 \times 8 \times 527.96 \times 8}{\pi \times 4.5^2} = 643$ N $\quad \therefore FS = \frac{770}{643} = 1.2$ so o.k.

$$\delta_{max} = \frac{527.96}{11.32} = 46.64 \text{ mm}$$

Min free length $= 10 \times 4.5 + 46.64$

without closed allowance $= 45 + 46.64 = 91.64$ mm

Deflection for 494 N, $\delta = \frac{494}{11.32} = 43.64$ mm

Max. free length $= 52 + 43.64 = 95.64$ mm.

Assume free length = 95 mm

$\therefore$ Clash allowance $= 95 - 91.64 = 3.36$ mm so o.k.

Prob. 9.4 : A rigid bar hinged at one end is supported by two identical spring as shwon in Fig. Initially the rod is horizontal. A vertical downward load of 400 N at free end results in downward deflection of free end by 60 mm. Selecting spring index, C = 6.5 and permissible shear stress as 400 MPa, determine wire dia., outer diameter and number of turns for the springs.

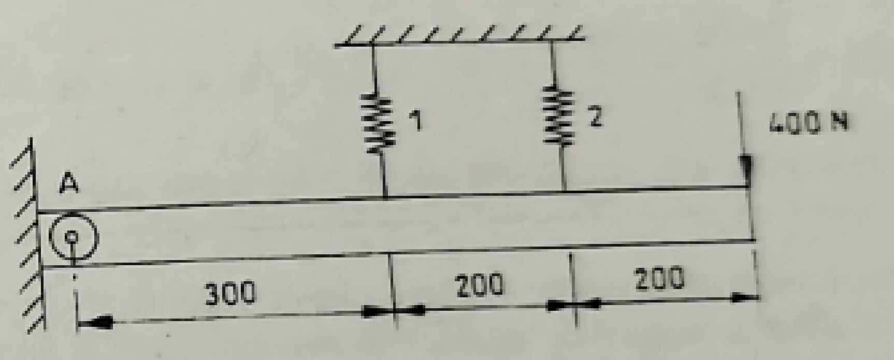


Fig.

Ref | **Calculations**

The F.B.D. of the bar is as shown in below. The deflections are also shown.

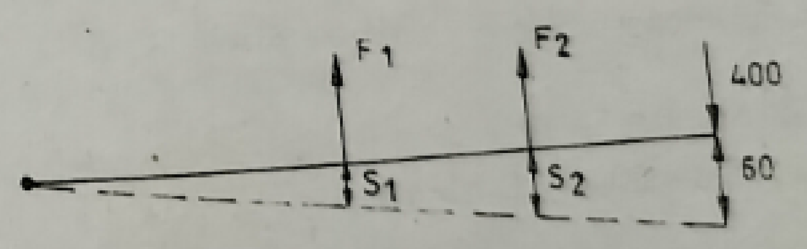


Fig.

Now by similar triangles : $\frac{\delta_1}{300} = \frac{\delta_2}{500} = \frac{60}{700}$

$\therefore \delta_1 = 25.7$ mm $\quad \delta_2 = 42.86$ mm

As the springs are identical, their stiffness will be same.

	Let it be K_s N/mm. $\therefore K_s\delta_1 = F_1$ and $K_s\ \delta_2 = F_2$ $\therefore \frac{F_1}{F_2} = \frac{\delta_1}{\delta_2} = \frac{25.7}{42.86}$ or $F_2 = 1.666\,F_1$ Now taking moments about A : $+ \circlearrowright \Sigma M_A$: $400 \times 700 - F_2 \times 500 - F_1 \times 300 = 0$ or $5 \times 1.666\,F_1 + 3F_1 = 2800$ $\therefore F_1 = 247.13$ N $F_2 = 411.72$ N. $\therefore$ Spring stiffness, $K_s = \frac{247.13}{25.7} = \frac{411.72}{42.86} = 9.6$ N/mm Therefore for spring design : $C = 6.5, F_{max} = 411.72, K_s = 9.6$
T-VIII-1	$K = \frac{4 \times 6.5 - 1}{4 \times 6.5 - 4} + \frac{0.615}{6.5} = 1.23$ $\tau_{max} = 400 = \frac{1.23 \times 8 \times 411.72 \times 6.5}{\pi d^2}$ $\therefore$ d = 4.58 select d = 5 mm $\therefore D_m = 6.5 \times 5 = 32.5$; $d_o = 32.5 + 5 = 37.5$ mm $K_s = 9.6 = \frac{G \times d}{8\,C^3 n} = \frac{80 \times 10^3 \times 5}{8 \times 6.5^3 \times n}$ $\therefore n = 18.96 \approx 19$ As this is an extension spring, the effective turns are same as total number of turns.

Prob 9.5 : At the bottom of a elevator shaft a group of 8 identical springs are set in parallel to absorb the shock of the elevator in case of failure. The elevator weighs 28 kN. Assuming that the elevator has a free fall of 1.2 m from rest, determine the maximum stress in each spring if each spring is made from 30 mm diameter rod. The spring index is 6 and the number of active turns is 15. Neglect any effect of counterweights in the system and take $G = 83 \times 10^3$ MPa.

Ref.	Calculations
	For the general impact problem $W(h+\delta) = \frac{1}{2} K_s \delta^2$ Given $W = 28$ kN, $h = 1.2$ m For each spring, $d = 30, C = 6, n = 15$
T-VIII-1	$\therefore K_s = \frac{Gd}{8\,C^3 n} = \frac{83 \times 10^3 \times 30}{8 \times 6^3 \times 15} = 96$ N/mm $= 96$ kN/m $K_{se} = 8 \times 96 = 768$ kN/m. $\therefore 28(1.2 + \delta) = \frac{1}{2} \times 768 \times \delta^2$ $\delta^2 - .073\delta - .0875 = 0$ $\therefore \delta = .3345$ m.

	$\therefore F_{max.}$ on each spring $= .3345 \times 96 = 32.11$ kN. $\tau_{max.} = K\frac{8\,F_{max.}\,C}{\pi d^2}$, $K\ \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{C} = 1.25$ $= \frac{1.25 \times 8 \times 32.11 \times 10^3 \times 6}{\pi\,30^2} = 682$ MPa

Prob. 9.6 : A coil spring with squared and ground ends is to operate with a load that fluctuates between 450 N to 900 N during which the deflection is to vary by 25 mm. We wish to use the available supply of Cr-V steel wire of 5 mm diameter. Design the spring with a clash allowance of 6 mm and determine the safety factors based on static loading and fatigue loading.

Ref.	Calculations
	$F_1 = 450$ N $F_2 = 900$ N $\delta = 25$ mm $\therefore K_s = \frac{900 - 450}{25} = 18$ N/mm
T-VIII-9	For Cr – V steel $S_{ys} = 770, S_{es} = 390, G = 85$ GPa $S_z = 0.86 + \frac{1.75}{5} = 1.21$ so $S_{ys} = 931.7$ $S_{es} = 472$
T-VIII-1	$K_s = \frac{Gd}{8\,C^3 n} = 18$ Assume C = 6 $\therefore n = \frac{85{,}\times 10^3 \times 5}{8 \times 6^3 \times 18}$ $K = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{C} = 1.25$ $= 13.66$ $K_1 = 1 + \frac{0.5}{6} = 1.08$ (Shear stress multiplication factor)
T-VIII-3	$n = 13.66 + 1.75 = 15.41$ say 16 $\therefore$ Free length $= 16 \times 5 + \frac{900}{18} + 6 = 136$ mm. $D_m = 6 \times 5 = 30$ mm. As free length is greater than 4 D_m it is likely to buckle. So we increase C = 7. $n = \frac{Gd}{8\,C^3 K_s} = \frac{85 \times 10^3 \times 5}{8 \times 7^3 \times 18} = 8.6$ $n = 8.6 + 1.75 = 10.3.5$ say 11. $\therefore$ Free length $= 11 \times 5 + \frac{900}{18} + 6 = 111$ mm. $D_m = 7 \times 5 = 35$ mm $\therefore \frac{111}{35} = 3.174$ so o.k.

T-VIII-1

For calculating safety factors :

For static loading :

$$\tau_{max.} = K_1 \frac{8\,FC}{\pi d^2} \qquad K = \frac{4 \times 7 - 1}{4 \times 7 - 4} + \frac{.615}{C} = 1.21$$

$$= \frac{1.07 \times 900 \times 7 \times 8}{\pi \times 5^2}$$

K_1 = shear multiplication factor

$$= 688.5 \text{ MPa} \qquad = 1 + \frac{0.5}{7} = 1.07$$

$$\text{Factor of safety} = \frac{931.7}{688.5} = 1.35$$

$$\text{For fatigue loading, } F_m = \frac{450 + 900}{2} = 675 \text{ N}$$

$$F_v = \frac{900 - 450}{2} = 225 \text{ N}$$

$$\therefore \tau_m = \frac{K_1\, 8\, F_m C}{\pi d^2} = \frac{1.07 \times 8 \times 675 \times 7}{\pi \times 5^2} = 515.23 \text{ MPa}$$

$$\tau_v = \frac{K\, 8\, F_v C}{\pi d^2} = \frac{1.21 \times 8 \times 225 \times 7}{\pi \times 5^2} = 194.21 \text{ MPa}$$

$$\therefore \frac{1}{N} = \frac{\tau_m}{S_{ys}} + \frac{\tau_v}{S_{es}} = \frac{515.23}{931.7} + \frac{194.21}{472} = .553 + .411$$

$\therefore N = 1.03$ (Low)

Suppose we design with N = 2 and find out C. First neglecting K_1 and K.

$$\tau_m = \frac{8 \times 675 \times C}{\pi \times 5^2} = 68.8\,C \quad ; \quad \tau_v = \frac{8 \times 225 \times C}{\pi \times 5^2} = 22.9\,C$$

$$\frac{1}{N} = \frac{\tau_m}{S_{ys}} + \frac{\tau_v}{S_{es}} = \frac{68.8\,C}{931.7} + \frac{22.9\,C}{472} = .1224\,C$$

$$\frac{1}{2} = .1224\,C \qquad \therefore C = 4$$

$$\text{Now } K = \frac{4 \times 4 - 1}{4 \times 4 - 4} + \frac{0.615}{4} = 1.4 \quad ; \quad K_1 = 1 + \frac{0.5}{4} = 1.125$$

$$\therefore \tau_m = 68.8 \times 4 \times 1.125 = 309.6 \qquad \tau_v = 22.9 \times 4 \times 1.4 = 128.24$$

$$\therefore \frac{1}{N} = \frac{309.6}{931.7} + \frac{128.4}{472} = .604 \qquad \therefore N = 1.655 \text{ so o.k.}$$

$$\text{For static loading, } \tau_{max.} = \frac{1.125 \times 900 \times 4 \times 8}{\pi \times 5^2} = 412.73 \text{ MPa}$$

$$\therefore FS = \frac{931.7}{412.73} = 2.25 \text{ so o.k.}$$

Prob. 9.7 : Show that the stress and deflection in a laminated semielliptic spring are given by :

$$\sigma_b = \frac{3}{2} \frac{F\,l}{nbt^2} \quad \text{and} \quad \delta = \frac{3}{8} \frac{FL^3}{Enbt^3}$$

Also show that the stresses and deflection in the same spring with extra full length leaves are given by,

$$\sigma_b \text{ in } n_f = \frac{3FL}{2n_g bt^2} \left[\frac{1.5}{1 + 1.5 \frac{n_f}{n_g}} \right]$$

$$\sigma_n \text{ in } n_g = \frac{3\,FL}{2n_g\, bt^2} \left[\frac{1}{1 + 1.5 \frac{n_f}{n_g}} \right]$$

$$\delta = \frac{3\,FL^3}{8\,En_g\, bt^3} \left[\frac{1}{1 + 1.5 \frac{n_f}{n_g}} \right]$$

where,

F = load acting the centre, N.

L = span of the spring, mm.

n = Total number of leaves.

n_f = Extra full length leaves.

n_g = Graduated leaves including master leaf.

b = width of leaves mm.

t = thickness of leaves, mm.

E = modulus of elasticity, MPa.

Calculations

Ref.

The laminated spring is a beam of const. strength as shown in Fig.

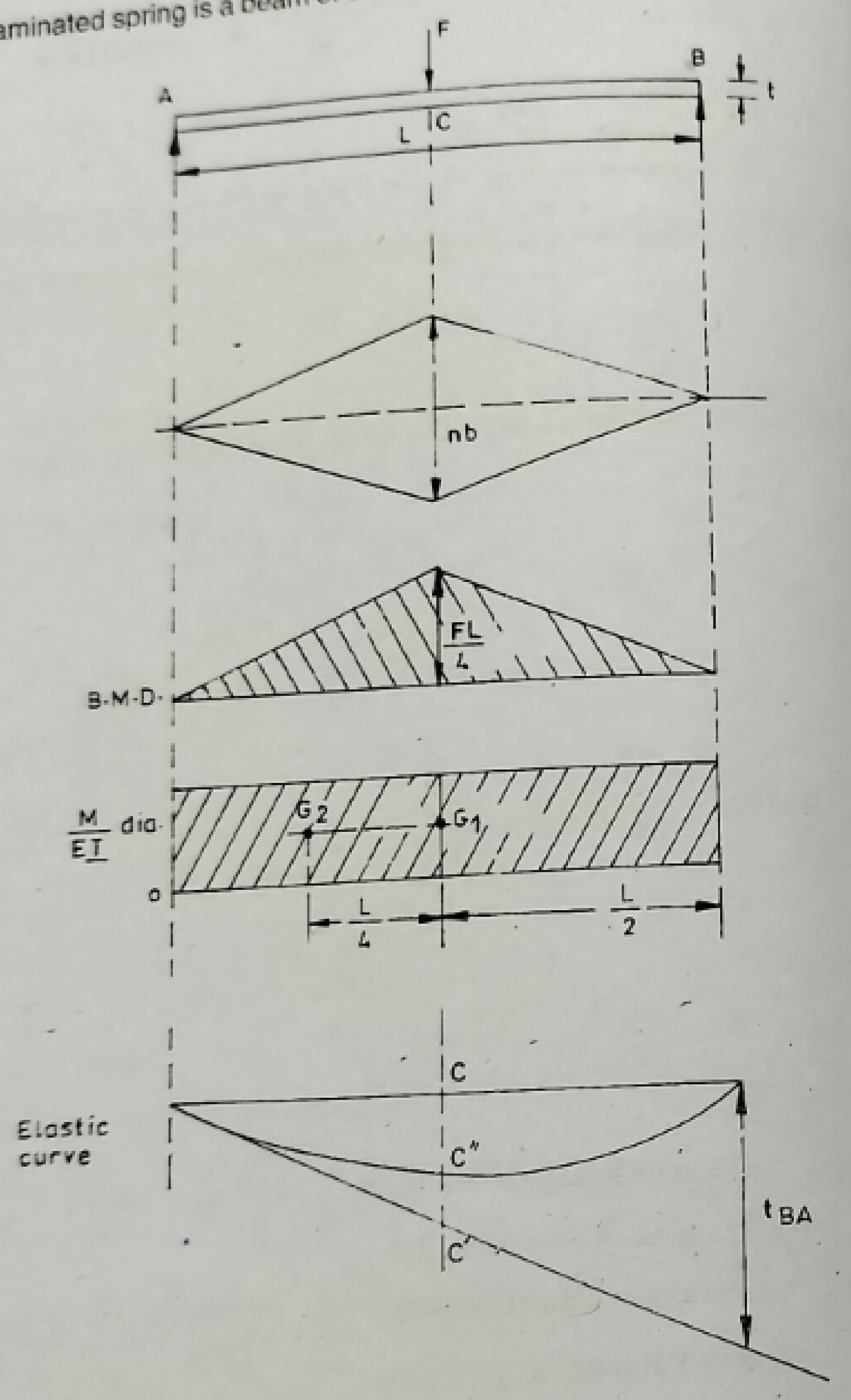


Fig.

The B.M.D. for the beam is shown below :

As the I of the beam is changing uniformly as per the B.M. the $\frac{M}{EI}$ diagram will be rectangle with the width = $\frac{Fl}{4\,EI_{(max)}}$

$$I_{max} = \frac{nbt^3}{12} \qquad \therefore \sigma_b = \frac{Fl \times 12 \times t}{4\,E_n\,bt^3 \times 2} = \frac{3}{\cancel{2}}\,\frac{Fl}{Enbt^2}$$

For deflection applying area moment theorem II :

$$\delta_c = CC' - C''C' = \frac{t_{BA}}{2} - t_{CA}$$

$$t_{BA} = \frac{FL}{4\,EI} . L \times \frac{1}{2} = \frac{FL^3}{8\,EI} \qquad t_{CA} = \frac{FL}{4EI} \times \frac{1}{2} \times \frac{L}{4} = \frac{FL^3}{32\,EI}$$

$$\therefore \delta_c = \frac{FL^3}{16\,EI} - \frac{FL^3}{32\,EI} = \frac{FL^3}{32\,EI} \; ; \; I = \frac{nbt^3}{12}$$

$$= \frac{12\,FL^3}{32\,Enbt^3} = \frac{3\,FL^3}{8\,Enbt^3}$$

Now when their are extra full length leaves, the beam can be considered as two beams acting in parallel.

1. Beam of constant strength as shown below :

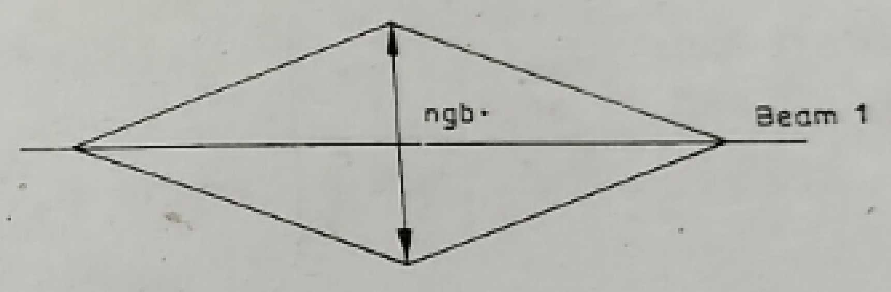


Fig.

2. Beam of constant width as shown in below :

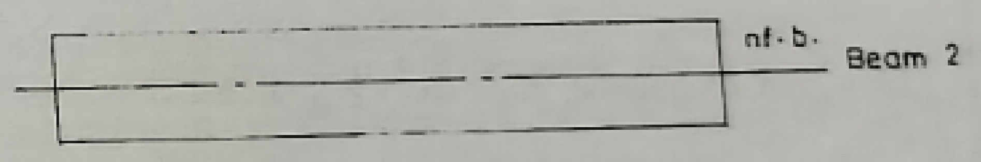


Fig.

Therfore we can treat the beam as two springs in parallel.

$\therefore$ Deflection in beam 1, $\delta_1 = \frac{3}{8} \cdot \frac{FL^3}{Enbt^3}$;

Spring rate, $K_1 = \frac{F}{\delta_1} = \frac{8\,Enbt^3}{3L^3}$

T-I-2

Deflection in beam 2, $\delta_2 = \frac{FL^3}{48\,EI}$; $I = \frac{n_f\,b.t^3}{12} = \frac{FL^3}{4\,En_f\,bt^3}$

$\therefore$ spring rate, $K_2 = \dfrac{F}{\delta_2} = \dfrac{4 E n_f bt^3}{L^3}$

$\therefore$ Combined spring rate, $K_e = K_1 + K_2 \; \dfrac{8 E n_g bt^3}{L^3}\left[1 + \dfrac{3 n_f}{2 n_g}\right]$

$\therefore$ Deflection $\delta = \dfrac{F}{K_e} = \dfrac{3FL^3}{8 E n_g bt^3}\left[\dfrac{1}{1 + 1.5\dfrac{n_f}{n_g}}\right]$

$\therefore$ Force acting on beam 1, $F_1 = K_1 \,.\, \delta = \dfrac{F}{1 + 1.5\dfrac{n_f}{n_g}}$

Max. B.M. on beam 1, $M_1 = \dfrac{F_1 l}{4}$ $\quad Z_{max} = \dfrac{n_g bt^2}{6}$

$$\therefore \sigma_b = \frac{3 F_1 l}{2 n_g bt^2} = \frac{3Fl}{2 n_g bt^2}\left[\frac{1}{1 + 1.5\dfrac{n_f}{n_g}}\right]$$

Force acting on beam 2 : $F_2 = K_2 \,.\, \delta = \dfrac{3}{2}\dfrac{n_f}{n_g} \,.\, F\left[\dfrac{1}{1 + 1.5\dfrac{n_f}{n_g}}\right]$

Max. B.M. on beam 2, $M_2 = \dfrac{F_2 l}{4}$, $Z_{max} = \dfrac{n_f bt^2}{6}$

$$\therefore \sigma_b = \frac{3}{2} \cdot \frac{F_2 l}{n_f bt^2} = \frac{3Fl}{2 n_g bt^2}\left[\frac{1.5}{1 + 1.5\dfrac{n_f}{n_g}}\right]$$

Prob. 9.8 : A laminated semielliptical leaf spring under a central load of 12 kN is to have an effective length of 1 m & is to deflect not more than 75 mm. The spring has 10 leaves, two of which are full length and have been pre-stressed so that all leaves have the same stress after full load has been applied. All leaves have the same width and thickness. The maximum stress in the leaves is not to exceed 350 MPa. Calculate the width and thickness of leaves. Also calculate the initial gap between the extra full length leaves and the master leaf before assembly.

Ref. | **Calculations**

T-VIII-4

Given F = 12000 N $\delta < 75$ mm L = 1000 mm

$n = n_f + n_g = 10$ $\; n_f = 1$, $n_g = 9$ $\; \sigma_b = 350$ MPa

To find b and t.

When extra full length leaves are prestressed.

$$\sigma_b = \frac{3FL}{2nbt^2}, \quad 350 = \frac{3 \times 12000 \times 1000}{2 \times 10 \times bt^2}$$

$bt^2 = 5.142 \times 10^3$

$$\delta = \frac{3FL^2}{8 n_g bt^3}\left[\frac{1}{1 + 1.5\dfrac{n_f}{n_g}}\right] \; ; \quad 75 = \frac{3 \times 12 \times 10^{12}}{8 \times 200 \times 10^3 \times 9 \times bt^3}\left[\frac{1}{1 + \dfrac{1.5 \times 1}{9}}\right]$$

$\therefore bt^3 = 28.6 \times 10^3$ $\qquad \therefore t = \dfrac{bt^3}{bt^2} = \dfrac{28.6 \times 10^3}{5.412 \times 10^3} = 5.56 \text{ mm} = 5.5 \text{ mm}$

$b = \dfrac{5142}{5.5^2} = 170$ mm $\quad$ or $\quad b = \dfrac{28.6 \times 10^3}{5.5^3} = 172$ mm

Assume $b = 172$ mm

$$h = \frac{FL^3}{8 E n_g bt^3} = \frac{12 \times 10^{12}}{8 \times 200 \times 10^3 \times 9 \times 170 \times 5.5^3} = 29.12 \text{ mm} = 30 \text{ mm.}$$

Prob. 9.9 : A semielliptic street car spring has a length of 1m and carries a load of 40 kN. It is made up of 18 leaves, 75 mm wide, three of which are full length. Use SAE 6150 steel with a safety factor of 2.25. Suggest suitable thickness for the leaves and calculate the resulting deflection when

(a) The extra full length leaves are prestressed.

(b) There is no prestress in the extra full length leaves.

Ref. | **Calculations**

T-VIII-4

Given $F = 40 \times 10^3$ N, $n = n_g + n_f = 18$ $\; n_f = 2$, $n_g = 16$

$b = 75$ mm, FS = 2.25, L = 1m.

T-VIII-9

Mat. SAE 6150 steel $S_{yt} = 1340$, $E = 220 \times 10^3$ MPa

Now with FS = 2.25

Allowable bending stress $= \dfrac{1340}{2.25} = 600$ MPa

$$\sigma_b \text{ in } n_f = \frac{3\,FL}{2 n_g bt^2}\left[\frac{1.5}{1+1.5\dfrac{n_f}{n_g}}\right] \text{ without prestressing}$$

$$t^2 = \frac{3\times 40\times 10^3\times 10^3}{2\times 16\times 75\times 600}\left[\frac{1.5}{1+\dfrac{1.5\times 2}{16}}\right] \qquad \therefore\ t = 10.25 \text{ mm}$$

$$\sigma_b = \frac{3\,FL}{2\left(n_f+n_g\right)bt^2} \text{ with prestressing}$$

$$t^2 = \frac{3\times 40\times 10^3\times 10^3}{2\times 18\times 75\times 600} = 74 \qquad \therefore\ t = 8.6 \text{ mm}$$

$$\delta = \frac{3\,FL^3}{8\,E n_g bt^3}\left[\frac{1}{1+1.5\dfrac{n_f}{n_g}}\right]$$

With $t = 10.25$, $\delta = \dfrac{3\times 40\times 10^3\times 10^3}{8\times 220\times 10^3\times 16\times 75\times 0.25^3}\times\left[\dfrac{1}{1+\dfrac{1.5\times 2}{16}}\right]$

$$= 44.43 \text{ mm}$$

For deflection with $t = 8.6$ mm

$$\delta = 44.43\times\left(\frac{10.25}{8.6}\right)^3 = 75.22 \text{ mm.}$$

Prob. 9.10 : A semi-elliptic leaf spring for use in a light trailer is to be made of steel having $S_{ut} = 1200$ MPa, $S_{yt} = 1080$ MPa and a fully corrected endurance limit of 550 MPa. The spring is 1 m long and has 5 leaves of 5 mm thickness and 100 mm width. Assume $K_f = 1.4$. When the trailer is fully loaded, the static load applied to the cêntre of the srping is 3500 N.

(a) Estimate the alternating load (due to driving over a rough road) that, when super imposed onto the fully loaded spring would be just on the verge of causing eventual fatigue failure.

(b) What will be the maximum deflection of the spring when loaded as in (a).

(c) To what value could the alternating load be increased if only 10^4 cylces of life are required.

Ref	Calculations

Let the alternating load be W, N.

$\therefore\ F_{max} = 3500 + W \qquad F_{min} = 3500$

$\therefore\quad F_m = 3500 + \dfrac{W}{2},\quad F_v = \dfrac{W}{2}$

$S_{ut} = 1200$ MPa, $S_{yt} = 1080$ MPa, $S_e = 550$ MPa

Given : $L = 1000$ mm, $n = 5$, $t = 5$ mm, $b = 100$ mm

T-VIII-4 $\quad \sigma_b = \dfrac{3\,FL}{2nbt^2}\quad ; \qquad \sigma_{bm} = \dfrac{3\times 1000}{2\times 5\times 100\times 5^2}\left(3500+\dfrac{W}{2}\right)$

$$= \frac{3}{25}\left(3500+\frac{W}{2}\right) = 420 + \frac{3}{50}W$$

$$\sigma_{bv} = \frac{3\times 1000}{2\times 5\times 100\times 5^2}\times\frac{W}{2} = \frac{3}{50}W$$

T-I-20 $\quad \therefore\ \dfrac{1}{N} = \dfrac{\sigma_{bm}}{S_{yt}} + \dfrac{\sigma_{bv}\,k_f}{S_e}\qquad$ For failure N = 1.

$$1 = \frac{1}{1080}\left(420+\frac{3}{50}W\right) + \frac{1.4\times 3\,W}{50\times 550}$$

or $\quad 1080 = 420 + .06\,W + .165\,W$

or $\quad .225\,W = 660 \qquad \therefore\ W = 2933.3$ N

$\delta_{max} = \dfrac{3\,F_{max}\,L^3}{8\,Enbt^3} \qquad E = 200\times 10^3$

$= \dfrac{3\times 6433\times 10^9}{8\times 200\times 10^3\times 5\times 100\times 5^3} \qquad F_{max} = 3500 + 2933 = 6433$ N

$= 193$ mm

To find endurance limit for 10^4 cycles

T-I-19 $\quad S_{ef} = \dfrac{10^b}{N^m} \qquad N = 10^4,$

$$b = \log_{10}\frac{(0.9\,S_{ut})^2}{S_{eb}} = \log_{10}\frac{(0.9\times 1200)^2}{550}$$

$= 3.326$

$\therefore S_{ef} = \dfrac{10^{3.326}}{\left(10^{4}\right)^{.0977}}$ $\quad m = \dfrac{1}{3} \log_{10} \dfrac{0.9\, S_{ut}}{S_{eb}} = \dfrac{1}{3} \log_{10} \dfrac{0.9 \times 1200}{550} = .0977$

$= 10^{2.935} = 861$ MPa

Let the new load be W_1 N. Therefore the new equation will be :

$S_{yt} = \sigma_{bm} + \dfrac{S_{yt}}{S_{ef}} K_f \, \sigma_{bv}$ with $N = 1$

$$1080 = \left(420 + \frac{3}{50} W_1\right) + \frac{1080}{861} \times 1.4 \times \frac{3}{50} W_1$$

$660 = .06\, W_1 + .1054\, W_1^{+} = .1654\, W'$

$\therefore W_1 = 3991$ N

⁂

Chapter 10

CYLINDERS, HEADS & COVER PLATES

10.1 Introduction :

A large number of vessels are used in chemical and mechanical industries to store fluids under pressure. They may be cylindrical or spherical in shape. They may be closed or open. The cylinders are also used in reciprocating engines, compressors, pumps etc. In all the pressure vessels one end is open to put the fluid and other material in the cylinder and a cover for the opening which is bolted to the cylinder. The cylinders are divided into two classes (1) Thin wall cylinders in which the predominant stresses are hoop stresses or transverse stresses, and (2) thick wall cylinders where both transverse as well as radial stresses are to be considered. As cylindrical vessels are more common, we will discuss the design of cylinder and their covers in this book.

10.2 Design of Cylinders and Covers :

In the design of cylinders, the inside diameter D_i is fixed from other considerations and the pressure of the fluid is also given. The type of ends whether closed or open is also given. For designing a pressure vessel we have to calculate :

1. The wall thickness or body thickness, t_b
2. The maximum thickness of the cover, t_c

To calculate the wall thickness the following factors are to be considered.

1. Whether thin wall or thick wall ?
2. Whether made of ductile material or brittle material ?
3. Whether the ends are closed or open ?

The equations to be used for a particular situation are to be selected as per T-X-1 as also the value of the design stress, S_d. The various equations to be used for thick wall cylinders are based on different theories of failure. The distribution and equations for radial and transverse stresses in cylinder-walls are given in T-I-22 of data book.

To calculate the maximum thickness of covers which depends on the following factors, use has to be made of the formulae given in T-X-7 of data book.

1. Shape of the cover.
2. Efficiency of the joint between the cylinder and the cover.

In addition to finding t_b and t_c the other important thing is the design of the bolted joint for the cover which must be opened and closed for working. To make this joint leak proof some form of soft packings called *gaskets* are used. Therefore the design of this bolted joint involves :

1. Selecting proper shape and material of the gasket as per T-X-4, T-X-5 and T-X-6.
2. Selecting the pitch circle diameter, number and diameter of bolts as per T-X-2 and T-X-3.

The design procedure will be illustrated with the help of solved problems.

SOLVED PROBLEMS

Prob. 10.1 : A cast iron drum with integral heads must be designed for an internal pressure of 3.5 MPa. The inside diameter must be 350 mm. Assume a better grade of cast iron. After the wall thickness is calculated, find the radius of the inner curvature of the heads to make their thickness equal to that of the wall.

Ref.	Calculations
	$D_i = 350$ mm $P = 3.5$ MPa Mat. C.I.
T-X-1	First using the thin cylinder equation
T-II-3	Using high grade C.I.I.S. Grade 35
	$S_{ut} = 350 \quad \therefore S_d = \frac{S_{ut}}{8} = \frac{350}{8} = 43$ MPa
	$\therefore t_b = \frac{PD_i}{2S_d} = \frac{3.5 \times 350}{2 \times 43} = 14.24$ mm.
	$\therefore \frac{t_b}{D_i} = \frac{14.24}{350} = .04, \quad \frac{P}{S_d} = \frac{3.5}{43} = .08$
	So from both considerations it is a thin cylinder. But just to compare the thickness from thick cylinder equation for brittle materials i.e. Lame's equation.
	$t_b = \frac{D_i}{2}\left[\sqrt{\frac{S_d + P}{S_d - P}} - 1\right] = \frac{350}{2}\left[\sqrt{\frac{43 + 3.5}{43 - 3.5}} - 1\right] = 14.89$ mm.
	As the difference is vary small we are justified in using thin cylinder equation. Assume $t_b = 15$ mm.
T-X-7	For cover thickness of dished ends
	$t_c = \frac{P r_c \omega}{2 S_d \eta}$, for integral heads $\eta = 100\%$ or 1 ; $D_m = 350 + 15 = 365$ mm.
F-10-2	For $t_c = t_b$ Required $r_c \omega = \frac{2 \times 43 \times 15}{3.5} = 368.57$
	For $\frac{D_m}{r_c} = 1, \ w = 1.1 \qquad \therefore r_c\omega = 365 \times 1.1 = 401.5$
	For $\frac{D_m}{r_c} = 1.2, \ w = 1.08 \qquad \therefore r_c \omega = \frac{365}{1.2} \times 1.08 = 328.5$
	For $\frac{D_m}{r_c} = 1.1 \quad \omega = 1.09 \qquad \therefore r_c \omega = \frac{365}{1.1} \times 1.09 = 361.68$, very near to 368.57
	Assume $r_c = \frac{D_m}{1.1} = \frac{365}{1.1} = 332$ mm
	$\therefore \sigma_t = \frac{P r_c \omega}{2 t_c \eta} = \frac{3.5 \times 332 \times 1.09}{2 \times 15 \times 1} = 42.22$ MPa
	As σ_t is less than S_d, design is o.k.

Prob. 10.2 : A flat cast iron cylinder head 30mm thick is fastened to the flange of a 250 mm cast iron cylinder by means of through bolts. Assuming for cast iron an allowable stress of 35 MPa, determine (a) the pressure that can be carried in the cylinder, (b) the thickness of the cylinder walls and of the cylinder flange, (c) the number and size of bolts to fasten the head and (d) the outside diameter of the head.

Ref.	Calculation
	$D_i = 250$ mm, $t_c = 30$ mm $S_d = 35$ MPa
T-X-7	For flat circular bolted cover
	$t_c = K_2 . D_p \sqrt{\frac{P}{S_d}} \qquad K_2 = 0.54$ for C.I.
T-X-2	For through bolts $D_p = D_i + 2\,(t_b + d_b + 5)$
	For trial assume $D_p = 1.25 \times D_i = 1.25 \times 250 = 312.5$ mm
	$\therefore 30 = .54 \times 312.5 \sqrt{\frac{P}{35}}$ or $P = \left(\frac{30}{.54 \times 312.5}\right)^2 \times 35 = 1.106$ MPa
	Therefore it can withstand pressure $P = 1$ MPa.
T-X-1	$t_b = \frac{PD_i}{2S_d} = \frac{1 \times 250}{2 \times 35} = 3.57$ mm $= 4$ mm
	$\therefore \frac{t_b}{D_i} = \frac{4}{250} = .016$ so thin cylinder
	As casting of $t = 4$mm will be difficult to make we increase it to 6mm $\therefore t_b = 6$ mm.
	For size and number of bolts
	External load, $F_e = \frac{\pi}{4} \times 250^2 \times 1 = 49 \times 10^3$ N

T-X-2	Assuming pitch of bolts = 5 d_b No. of bolts = n
	$\therefore n \times 5\, d_b = \pi D_p = \pi\, [D_i + 2\,(t_b + d_b + 5)]$
	$= \pi\, [250 + 22 + 2\, d_b]$
	$n \times d_b = 176.8 + 1.256\, d_b$
	For $n = 6$ $\quad d_b = 23.6$ mm
	For $n = 8$ $\quad d_b = 18.45$ mm
	$\therefore$ Assume $n = 8$, F_e per bolt $= \frac{49 \times 10^3}{8} = 6.125 \times 10^3 N$
T-X-1	$F_e = \frac{S_{yt}}{150} (A_r)^{\frac{3}{2}}$
T-VI-7	Assuming material for bolts, free cutting steel, $S_{yt} = 240$
	$\therefore A_r = \left(\frac{6125 \times 150}{240}\right)^{\frac{2}{3}} = 245.38\ mm^2$
T-VI-2	For M 20 × 1.5 bolts $A_r = 251\ mm^2$ so o.k.
	$\therefore D_p = D_i + 2\,(t_b + d_b + 5) = 250 + 2\,(6 + 20 + 5) = 312$ mm.
	So our assumed D_p and P are O.K.
	Selecting compressed asbestos as gasket material.
T-X-6	3mm thick, $m = 2 \quad y = 11$
	$P_c = P_i \left[\frac{G}{4b} + m\right]$ for full gasket $G = D_p - 2b$
	O.D. of flange $= D_p + 3\, d_b = 312 + 3 \times 20 = 372$ mm.
	$\therefore$ Width of gasket $= \frac{372 - 250}{2} = 61$ mm
T-X-4	$b_o = \frac{61}{2} = 30.5 \qquad \therefore b = 2.5\sqrt{30.5} \fallingdotseq 13.8$ mm
	$G = 312 - 2 \times 13.8 = 284.4$ mm.
	$\therefore P_c = 1\left[\frac{284.4}{4 \times 13.8} + 2\right] = 7.1\,MPa$ as $P_c < y$ assume $P_c = y$
	$\therefore$ Initial tightening load, $\frac{F_i}{n} = \frac{\pi \times G \times b \times y}{8}$
	$= \frac{\pi \times 284.4 \times 13.8 \times 11}{8} = 17 \times 10^3$ N

	$\therefore$ Total bolt load, $F_t = F_e + F_i = (6.125 \times .60 + 17) \times 10^3, \quad C = 0.60$
	$= 20.675 \times 10^3$ N
	Stress in the bolt $\sigma_t = \frac{F_t}{A_r} = \frac{20.675 \times 10^3}{251} = 82.5$ MPa
	$\therefore$ Factor of safety $= \frac{S_{yt}}{\sigma_t} = \frac{240}{82.3} = 2.916$ so o.k.
	$t_f = 1.3\, d_b = 1.3 \times 20 = 26$ mm
	Final results $P = 1$ MPa $\quad t_f = 26$mm $\quad t_b = 6$ mm
F-X-2	no. of bolts = 8, $\quad$ O.D. = 372 mm, $\quad$ Bolts = M 20 × 1.5
	$D_P = 312$ mm.

Prob. 10.3 : A cast iron steam chest on an engine cylinder has a rectangular opening 350 mm long and 250 mm wide and the steam pressure is 1.2 MPa (a) Determine the thickness of a flat cover for this opening made of a better grade of cast iron (b) Determine the number and size of studs to fasten the cover.

Ref.	**Calculations**
T-X-7	$t_c = K_4\, ab \sqrt{\frac{P}{S_d\,(a^2 + b^2)}}; \qquad K_4 = 0.75$ for C.I.
T-II-3	Using I.S. grade 20 C.I. $\quad S_{ut} = 271 \qquad a = 350,\ b = 250$
	$S_d = \frac{271}{8} \doteq 34$ MPa
	$t_c = .75 \times 350 \times 250 \sqrt{\frac{1.2}{34 \times 10^4 (3.5^2 + 2.5^2)}}$
	= 28.66 mm say 30 mm.
T-X-2	For studs $l = (1.25$ to $1.5)\ d_b$
	Selecting $l = 1.4\, d_b$
	$P = 4\, d_b$ for P = 1.2 MPa.
	No. of bolts on longer side $n_1 \qquad n_1 = \frac{350 + 2l}{4\, d_b} = \frac{350 + 2.8\, d_b}{4\, d_b}$
	For $d_b = 20$mm $\quad n_1 = 5.075$
	No. of bolts on shorter side $n_2 \qquad n_2 = \frac{250 + 2.8\, b}{4 d_b}$
	For $d_b = 20$m, $n_2 = 3.825$

The bolts and their spacing is shown in the Fig.

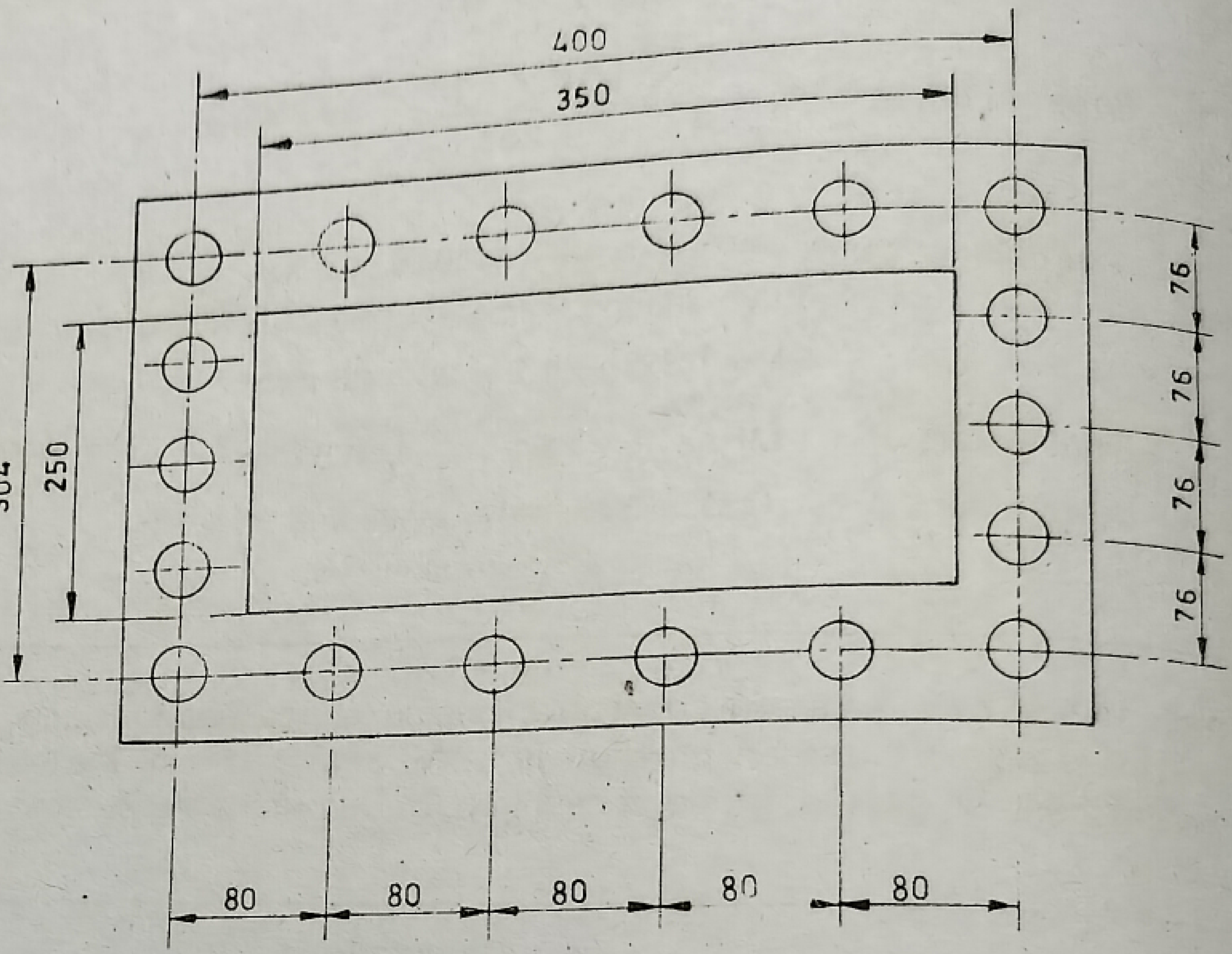


Fig.

Total force = 400 × 304 × 1.2 = 145920 N

$$\text{Load/bolt} = \frac{145920}{18} = 8106.6 \text{ N}$$

T-VI-7 Lod capacity of each bolt, $F_e = \frac{240}{150}(A_r)^{\frac{3}{2}}$

For free cutting steel $S_{yt} = 240 \quad \therefore F_e = \frac{240}{150}(251)^{\frac{3}{2}} = 6362$ N This is not sufficient

So, Using Nickel steel, SAE = 2330 forged.

$S_{yt} = 380$

$F_e = \frac{380}{150}(251)^{\frac{3}{2}} = 10074$ N so o.k.

* * *

Chapter 11

WIRE ROPES & ACCESSORIES

11.1 Introduction :

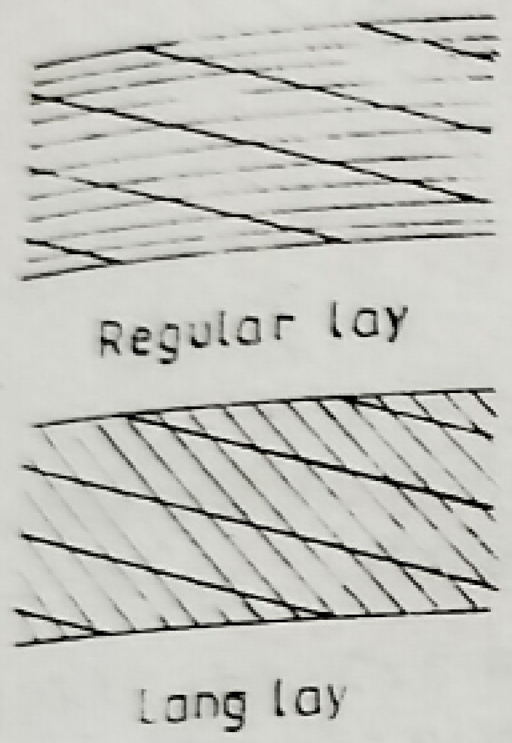


Fig.1.1

Wire ropes generally made of plow-steel are built up of strands which are made of wires twisted together. These ropes are used in hoisting and haulage equipment. Wire rope is made of two types of winding as shown in Fig. 11.1 The *regular lay*, which is the accepted standard, has the wire twisted in one direction to form the strands, and the strands twisted in the opposite direction to form the rope. In the completed rope the visible wires are approximately parallel to the axis of the rope. Regular lay ropes do not kink or untwist and are easy to handle. The *lang lay* ropes have the wires in the strands and the strands in the rope twisted in the same direction and hence the outer wires run diagonally across the axis of the rope. These ropes are more resistant to abrasive wear and failure due to fatigue than regular lay ropes, but they are more likely to kink and untwist. Tests show that lang lay rope has a life several times as lang as regular lay ropes. Hence in applications where the load is guided and constrained, long lay ropes are preferred e.g. passenger elevators. The core provides support for the strands under usual bending and loading conditions. The most commonly used core is fibre core.

11.2 Types of Construction and Properties :

The wire ropes are designated by their outer diameter, no. of strands and no. of wires in each strand. viz 6 × 19 wire rope of 20 mm diameter means there are six strands with 19 wires in each strand. There are four types of construction as given in T-IX-3 :

1. **6 × 19 :** Standard hoisting rope which is the most commonly used wire rope in mines, tramways, cranes, elevators etc.
2. **6 × 37 :** Special flexible hoisting rope which is used in steel mill ladle cranes, high speed elevators and service where smaller sheave diameter is to be used.
3. **8 × 19 :** Extra flexible rope which is used in place of 6 × 19 rope if there is restriction of space because smaller sheave diameter can be used.
4. **6 × 7 :** Standard coarse laid ropes which are used for haulage in mines, factory yards, guy wires etc.

The properties of wire ropes of these types made of plow steel with hempcore are given in T-IX-3 of data book. The other materials used for making wire ropes are given in T-IX-4 along with their ultimate strengths.

11.3 Stresses in Wire Ropes :

The following main stresses are created in a hoisting rope under load.

(a) Direct stresses due to the load hoisted and the weight of the rope. These are called static stresses, s_{st}.

(b) Stresses due to the bending of the rope about the sheave, s_b.

(c) Stress during starting which depends upon the slack in the rope before starting.

(d) Stress due to change of rope speed, including stops.

In haulage rope, δ_{st} is missing. For design purposes we calculate the forces acting on the rope during various phases of operation. The various phases of operation of a hoist are,

1. **Start from rest :** The forces acting in this phase are starting force F_{st} and the forces due to bending, F_b.
2. **Acceleration to reach the given velocity :** During this phase the forces acting are : load hoisted and the weight of the rope $(W_u + W_r)$ plus the load due to acceleration F_a which is given by,

$$Fa = \left(\frac{W_u + W_r}{g}\right).f \quad \text{where } f = \text{acceleration}$$

plus the forces due to bending, f_b.

3. **Moving with uniform velocity :** In this phase the forces acting are load hoisted and the weight of the rope $(W_u + W_r)$ and the forces due to bending F_b.
4. **Deceleration to stop :** The forces acting during this phase are same as during phase 2 except that F_a is to be calculated on the basis of deceleration and not acceleration.

To summarize the forces acting in the various phase are,

Phase 1 – $F_{st} + F_b$

Phase 2 – $(W_u + W_r) + F_b + F_a$

Phase 3 – $(W_u + W_r) + F_b$

Phase 4 – $(W_u + W_r) + F_b + F_a$

The load acting in phase 3 is called the effective load F_e and the ratio of the breaking strength of rope F_{ut} to the effective load F_e is called *safety factor,* and is given in T-IX-5 of data book for various applications.

11.4 Selection of Wire-rope diameter & Sheave Size :

The following steps are to be taken in selecting the size of wire-rope to be used in a hoist. The data given should be,

(a) Load to be hoisted, W_u, N

(b) Height of lift, m

(c) Period of acceleration and uniform velocity, sec

(d) Slack during starting, m

(e) Application

Step 1 : Select the type of construction as per T-IX-3 and as discussed in art 11.2. Calculate F_{ut} in terms of the rope diameter D_r.

Step 2 : Select the minimum safety factor N as per T-IX-3.

Step 3 : Equate the useful loa W_u to the breaking strength F_{ut} and the safety factor as given below.

$W_u \times 2 \times N = F_{ut} = K \, . \, D_r^2$ (K as per T-IX-3)

Calculate the trial diameter of wire rope D_r as per the sizes given in T-IX-3.

Step 4 : Calculate the weight of the wire rope W_r as per T-IX-3.

Step 5 : Calculate the bending load F_b as per T-IX-7 by selecting the proper sheave size as per T-IX-3.

Step 6 : Calculate the load due to acceleration F_a by calculating acceleration from the given data.

Step 7 : Calculate the load during starting with a slack, h, as per the relation given below.

$$F_{st} = (W_r + W_u)\left[1 + \sqrt{1 + \frac{2Eh}{L \, . \, \delta_{st}}}\right]$$

where δ_{st} = static stress = $\frac{W_r + W_u}{A}$; A = area of metal in the wire rope.

E = elasticity of rope = 84×10^3, MPa.

Step 8 : Calculate the actual safety factors during the three phases.

Phase 1 : $N_1 = \frac{F_{ut}}{F_{st} + F_b}$

Phase 2 : $N_2 = \frac{F_{ut}}{W_u + W_r + F_b + F_a}$

Phase 3 : $N_3 = \frac{F_{ut}}{W_u + W_r + F_b}$

Check that N_3 is greater than N selected in step 2.

Also $N_1 > \frac{N}{2}$

Then design the sheaves and drums whose diameters are already fixed as per T-IX-3. As a general rule the diameter should not be less than 500 d_w where d_w is the dia. of the wire in the wire rope. Other dimensions of sheave and drum should be calculated as per T-IX-8. The number of cycles to failure can be found from the relation.

$$K = \frac{2\,(W_u + W_r)}{S_{ut}\, D_r\, D_s}$$

where $S_{ut} = \frac{F_{ut}}{A}$

For this value of K we can read the cycles to failure from F-9.2 of data book.

These steps will be illustrated in the solved problems.

SOLVED PROBLEMS

Prob. 11.1 : Determine the size of 6 × 19 standard plow steel rope to be used with a drum hoist to lift 20 kN from a depth of 60 m. Assume a rope speed of 300 m/min and an acceleration of $2 m/sec^2$, when starting with no slack. (b) Find the influence of slack of 0.6 m when starting (c) Find the necessary drum diameter and the length (d) Also design the sheave for the hoist. (e) Determine the number of cycles of failure.

Ref.	Calculations	
	Given : 6 × 19 construction with height of lift = 60 m.	
	$W_u = 25$ kN Acceleration $f = 2 m/sec^2$ velocity = 300 m/min.	
T-IX-3	For 6 × 19 rope $F_{ut} = 1.2 D_r^2$ kN, Average sheave size = 45 D_r mm	
T-IX-5	N = 5 for miscellaneous hoists.	
	$\therefore 1.2\, D_r^2 = 25 \times 5 \times 2$	
	$D_r = 14.43$ mm. Select standard $D_r = 16$ mm	
	$\therefore F_{ut} = 1.2 \times 16^2 = 307.2$ kN	
	Weight of wire rope, $W_r = .37 \times 16^2 \times 10^{-3} \times 60$	
	$= 568.32$ N or .568 kN	
T-IX-7	Bending load $F_b = 84 \times 10^3 \times A \times \frac{dw}{D_s}$	$dw = .63\, D_r$
	$= 84 \times 10^3 \times .38 \times 16^2 \times \frac{.063}{45}$	$D_s = 45\, D_r$
	$= 11.44 \times 10^3$N or 11.44 kN	$A = .38\, D_r^2$
	Starting load with h = 0, $F_{st} = (W_u + W_r)\left[1 + \sqrt{1} + 0\right]$	
	$= 25.568 \times 2 = 51.136$ kN	
	Load for acceleration, $F_a = \frac{25.568}{9.81} \times 2 = 5.2$ kN	
	Phase 1, Total load $= F_{st} + F_b = 51.136 + 11.44 = 62.576$ KN	
	$N_1 = \frac{307.2}{62.576} = 4.9$	
	Phase 2, Total load $= W_u + W_r + F_b + F_a = 25.568 + 11.44 + 5.2 = 42.21$	
	$N_2 = \frac{307.2}{42.21} = 7.28$	

Ref.	Calculations
	Phase 3, Total load $= W_u + W_r + F_b = 25.568 + 11.44 = 37$
	$N_3 = \frac{307.2}{37} = 8.3$
	(b) With h = 0.6 m
T-IX-3	$F_{st} = (W_r + W_u)\left[\sqrt{1 + \frac{2 \times 0.6 \times 84 \times 10^3}{60 \times \delta_{st}}}\right]$
	$\delta_{st} = \frac{W_r + W_u}{A} = \frac{25.568 \times 10^3}{.38 \times 16^2} = 263$ MPa
	$F_{st} = 25.568 \times 10^3 \left[1 + \sqrt{1 + \frac{1.2 \times 84 \times 10^3}{60 \times 263}}\right] = 95.1 \times 10^3$ N or 95.1 kN.
	$\therefore$ Total load during starting = 95.1 + 11.44 = 106.54 kN
	Safety factor $N_1 = \frac{307}{106.54} = 2.88$
	Which is greater than $\frac{N}{2}$ i.e. 2.5.
	Thus we see that a slack of 1 percent reduces the safety factor from 4.9 to 2.88.
	(c) Drum diameter = 45 × 16 = 720 mm
	check : D > 500 dw = 500 × .063 × 16 = 504 so o.k.
	To find the length of the drum
	Rope per turn $= \pi d = \pi \times .72 = 2.26$ m
	$\therefore$ No. of turns of rope required $= \frac{60}{2.26} = 26.53$
	Generally there must be 2 turns left on either end of the drum with the load in lowest position.
	$\therefore$ Total no. of turns on the drum = 26.53 + 4 = 30
T-IX-8	Pitch between grooves $= 1.15\, D_r = 1.15 \times 16 = 18.4$ mm
	$\therefore$ total length = 18.4 × 20 = 368 mm
	The sheave diameter = Drum diameter = 720 mm.
	(d) For design of groove in the sheave
	r = radius of groove = .53 × 16 = 8.48 mm
	c = height of groove = 1.4 × 16 = 22.4 mm
	$\alpha = 22°$
T-IX-7	(e) No. of cycles to failure
	$S_{ut} = \frac{F_{ut}}{A} = \frac{1.2}{.38} \times 10^3 = 3.16 \times 10^3$ MPa

T-IX-4 $K = \frac{2(W_u + W_r)}{S_{ut} D_r D_s} = \frac{2 \times 25.568 \times 10^3}{3.16 \times 16 \times 45 \times 16 \times 10^3} = .0014$

F. 9.2. For this K No. of cycle to failure = 8×10^6 cycles.

Prob. 11.2 : Determine the size of special flexible 6 × 37 wire rope and the drum diameter required for an elevator in a building 120 m tall for a load of 20 kN. Assume a maximum rope speed of 300 m/min and an acceleration of $2 m/sec^2$ when starting with no slack. According to law not less than four ropes must be used. If there is a slack of 1 percent, what is the safety factor during starting ?

Ref. **Calculations**

$W_u = 20$ kN, Height = 120 m, $f = 2 m/sec^2$

6 × 37 rope, 4 rope are to be used.

$\therefore$ Load / rope $= \frac{20}{4} = 5$ kN

T-IX-3 For 6 × 37 rope

$F_{ut} = 1.12\ D_r^2$, $D_s = 27\ D_r$ weight $= 35\ D_r^2 \times 10^{-3}$ N/m

$dw = .045\ D_r$, $A = .38\ D_r^2$

T-IX-5 N = 8 for hoist with height less than 150 m.

$\therefore 1.12\ D_r^2 = 5 \times 8 \times 2 = 80$

$\therefore D_r = 8.45$ mm

Select a standard rope of $D_r = 10$mm. Which can absorb some amount of shock if there is a slack.

$W_r = 35 \times 10^2 \times 10^{-3} \times 120 = .42 \times 10^3$ N or .42 kN

$F_{ut} = 1.12 \times 10^2 = 112$ kN

T-IX-7 $F_b = 84 \times 10^3 \times .38 \times 10^2 \times \frac{.045}{27} = 5.32 \times 10^3$ N or 5.32 kN.

$F_a = \frac{W_u + W_r}{g} \times f = \frac{5 + .42}{9.81} \times 2 = 1.1$ kN

F_{st} with h = 0 ; $F_{st} = (W_u + W_r) \times 2 = 5.42 \times 2 = 10.84$ kN

$\therefore$ Loads in different phases and safety factors.

Phase	Total load	Safety factor
1.	$F_{st} + F_b = 10.84 + 5.32 = 16.16$ kN	$\frac{112}{16.6} = 6.7 > \frac{N}{2}$ i.e. 4, so. o.k.
2.	$W_u + W_r + F_b + F_a = 5 + .42 + 5.32 + 1.1 = 11.84$ kN	$\frac{112}{11.84} = 9.46 > N$ so o.k.
3.	$W_u + W_r + F_b = 5.42 + 5.32 = 10.74$	$\frac{112}{10.74} = 10.47 > N$ so o.k.

Drum diameter = 27 × 10 = 270 mm

Slack h = .01 × 120 = 1.2 m

$$\therefore F_{st} = (W_u + W_r)\left[1 + \sqrt{1 + \frac{2Eh}{L\delta_{st}}}\right] \qquad \delta_{st} = \frac{5.42 \times 10^3}{.38 \times 10^2} = 142 \text{ MPa}$$

$$= 5.42\left[1 + \sqrt{1 + \frac{2 \times 84 \times 10^3 \times 1.2}{120 \times 142}}\right] \qquad E = 84 \times 10^3$$

$= 24.8$ kN

$\therefore$ Total load during starting = 24.8 + 5.32 = 30.12 kN

$\therefore$ Safety factor $= \frac{112}{30.12} = 3.7$ which is less than N/2 i.e. 4.

⁂

Chapter 12

SHAFTS, KEYS & SPLINES

12.1 Introduction :

The term *shaft* usually refers to a relatively long member of round cross section that rotates and transmits power. However a shaft can have a non-round cross section and it need not necessarily rotate. It can be stationary and serve to support a rotating member. One or more members such as gears, sprockets, pulley, flywheels are usually attached to the shaft by means of pins, keys, splines and other devices. Though shaft is a generic term, there are other terms used for the special purpose shafts. An *axle* is a machine part which is loaded chiefly in bending and carries such rotating parts as wheels, gears or pulleys. An axle may be stationary (automobile wheel axles) or it may rotate (idler gear or idler pulley). Short shafts and axles in machines are called *spindles.* A *line shaft* often called *transmission shaft* is a comparatively long shaft which receives power from a prime mover and transmit it to various machines. A *counter shaft or jack shaft* is a shaft short placed between a prime mover and a line shaft or a driven machine.

Rotating shaft carrying gears, pulleys, sprockets, and so on must be supported by bearings to restrict its radial and axial movements. If two bearings can provide sufficient radial support to limit shaft bending and deflection to acceptable values this is highly desirable and simplifies manufacturing. If three or more bearings must be used to provide adequate support and rigidity (multi cylinder crackshaft) precise alignments of bearings in the supporting structure must be maintained. Shaft axial positioning and provision for carrying axial or thrust loads normally require that one and only one bearing take thrust in each direction. The portion of the shaft where the bearings are located is called *journal.*

12.2 Materials & Manufacturing :

Normally the shaft are made of plain carbon steels with a carbon content of 0.15 to .40 percent. When greater strength is required, as in high speed machinery, shafts are made of alloy steels, the most common being nickel, nickel-chrominum and chrome-vanadium steels. For selection of proper steel for a shaft refer T-II-6 of data book. When resistance to corrosion is desired, the shafts are made of some copper alloys.

Most commercial shafting after being hot rolled is turned and polished or ground. Steel shafting is sometimes finished by cold rolling or cold drawing. This produces a some what stronger shaft than hot rolling. However cold finished shafts have several disadvantages. The diameter tolerances are not vary close, the shaft does not come precisely straight and there are residual stresses on the surface which cause distortion of the shaft after machining. Shafts for special purposes such as hollow shafts with integral flanges, are forged.

12.3 Design :

The shafts are subjected to radial loads and axial loads. The radial loads produce torque and bending moment while the axial loads produce tension or compression. The most general case is the combination of torsion and bending moment acting at various sections of the shaft. The loads also produce transverse deflections and torsional deflections. When either the transverse or torsional deflection must be held to close limits, the shaft should be sized on the basis of deflection before analyzing the stresses. The maximum allowable transverse deflection of a shaft is usually determined by critical speed, gear or bearing requirements. (Refer T-XI-4 of data book).

The shaft made of ductile materials are designed on the basis of maximum shear stress theory. For the most general case when a shaft section is subjected to : torsion T, bending moment, M and an axial load F, the maximum shear stress induced is given by the equation.

$$\tau_{(max.)} = \frac{16}{\pi D_o^3 \left(1-k^4\right)} \sqrt{T^2 + \left[M + \frac{F D_0 \left(1+k^2\right)}{8}\right]^2}$$

where $k = \frac{D_0}{D_i}$

This equation is modified by A.S.M.E. by introducing shock, fatigue and column factors to give the following equation (T-XI-I).

$$\tau_{(max.)} = \frac{16}{\pi D_0^3 \left(1-k^4\right)} \sqrt{(K_t T)^2 + \left(K_b M + \frac{\alpha F D_o}{8}\right)^2}$$

Another approach to take care of the variation of transmitted torque which depends on the prime mover as well as the driven machine is to make use of load factor, K_1 which is the ratio of maximum torque to the average or nominal torque (T-XI-5).

In that case the maximum shear stress induced is given by the equation.

$$\tau_{(max.)} = \frac{16}{\pi D_0^3 \left(1-k^4\right)} \sqrt{T_d^2 + \left(M + \frac{\alpha F D_o}{8}\right)^2}$$

The general procedure is to draw the bending moment and torque diagrams for the entire length of the shaft and then determine the critical section by calculating the equivalent torque T_e given by the equation below, at various sections.

$$T_e = \sqrt{T^2 + M^2} \cdot$$

The section where the equivalent torque is maximum is the critical section because the induced shear stress will be maximum at that section. Now the induced shear stress is equated to allowable shear stress and the minimum diameter is calculated. The A.S.M.E. code defines the permissible or design shear stress S_{ds} :

$S_{ds} < .30\, S_{ut}$ or $.18\, S_{yt}$ (T-XI-1 of data book)

The code also states that these stresses should be reduced by 25 percent if stress concentration due to shoulder fillet or key-way is present.

In practice the magnitude of the bending moment is seldom known exactly. Therefore it is customary to determine the shaft diameter as function of torque alone by using a low value of allowable shear stress, S_{ds}. For these calculations use must be made of load factors to calculate the design torque T_d and value of S_{ds} should be selected between 20 MPa to 45 MPa, the normal value being 30 MPa. Values for S_{ds} are to be selected on the basis of the severity of shock.

A few general principles that should be kept in mind are --

1. Keep shafts as short as possible, with bearings close to the applied loads. This reduces deflection and bending moments and increase critical speeds.
2. Place necessary stress raisers away from highly stressed shaft regions if possible. If not possible, use generous radii and good surface finishes.
3. Use inexpensive steels for deflection critical shafts, as all steels have essentially the same elastic modulus.
4. When weight is critical, consider hollow shafts.

The design procedure will be illustrated with the help of examples worked out. The examples solved at the end of the chapter are for the normal case when the torques are constant, the bending moment is completely reversed on any section, the effect of which is taken care of by the A.S.M.E. code. If the torques and bending moments are both varying the problems can be solved by Soderberg approach illustrated in chapter - 4.

12.4 Critical Speed of Shafts :

All rotating shafts, particularly those that run at high speeds, must be designed to avoid operation near the critical speed. (T-XI-4). This usually means providing sufficient lateral rigidity to place the lowest critical speed significantly above the operating range. When torsional fluctuations are encountered (as with camshafts, engine and compressor crank shafts etc.) an additional dynamic requirement is imposed : the torsional natural frequencies of the shaft must be well removed from the frequency of torsional input. The natural frequency of the shaft in bending is very nearly the same as the critical speed and is usually taken as the same thing. There is a difference usually quite small, due to the gyroscopic action of the masses. For a multimass system approximate values are calculated on the basis of the Dunkerley equation (T-XI-1).

$$N_C = \frac{945}{\sqrt{\delta_1 + \delta_2 + \delta_3 + - - - - + \frac{\delta_5}{1.27}}} \text{ rpm.}$$

where δ_1 = Deflection at F_1 , mm due to F_1 , mm

δ_2 = Deflection at F_2 due to F_2 , mm.

δ_s = Maximum deflection of shaft due to its own weight, mm.

For calculating critical speed do not consider forces which remain constant irrespective of deflection such as belt tensions, gear tooth loads etc. Consider dead weights, magnetic pull etc. which depend on or affect deflection.

For uniform diameter shafts, the formulae for calculating deflection are given in T-I-2. However for calculating deflections in varying diameter shafts, Area-Moment method discussed in chapter-2 should be used. The procedure for designing shafts from this criterion are illustrated in the problems solved at the end of this chapter.

12.5 Forces on Shafts in Drivers :

The power transmission shafts are part of a drive and therefore we must analyse the forces acting on the shafts and bearings in various types of drives to design the shafts and bearings. In general the types of drives are (1) Gear drives (2) Belt drives and (3) Chain drives. We will discuss the force analysis for these three types of drives.

A Gear Drives : There are four types of gears used in the drives depending on the orientation of shaft. The types are,

1. Spur gears for parallel shafts.
2. Helical gears for parallel shafts.
3. Bevel gears for intersecting shafts.
4. Worm gears and spiral gears for non-parallel, non-intersecting shafts.

All the gears are designed on the basis of torque transmission capacity. Hence the starting point of design is the torque to be transmitted.

12.5.1 Spur Gears :

In spur gears the teeth are parallel to the axis. A pair of spur gears with pressure angle ϕ and the forces acting on them are shown in Fig. 12.1.

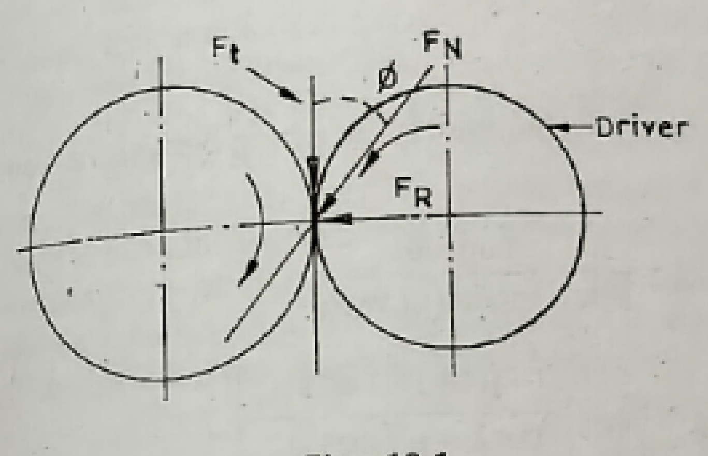


Fig. 12.1

F_N = Normal force exerted by the driver on the driven gear.

F_t = Tangential force

$$= \frac{\text{Torque}}{\text{Pitch radius of gear}}$$

F_R = Radial or seperating force.

$= F_t \tan \phi$

12.5.2 Helical Gears :

In these the teeth are cut at an helic angle ψ to the shaft axis and it is either left handed or right handed. The pressure angle is standard either in the transverse plane ϕ, or the plane normal to the helix, ϕ_n. A pair of helical gears with the forces acting on them is shown in Fig. 12.2.

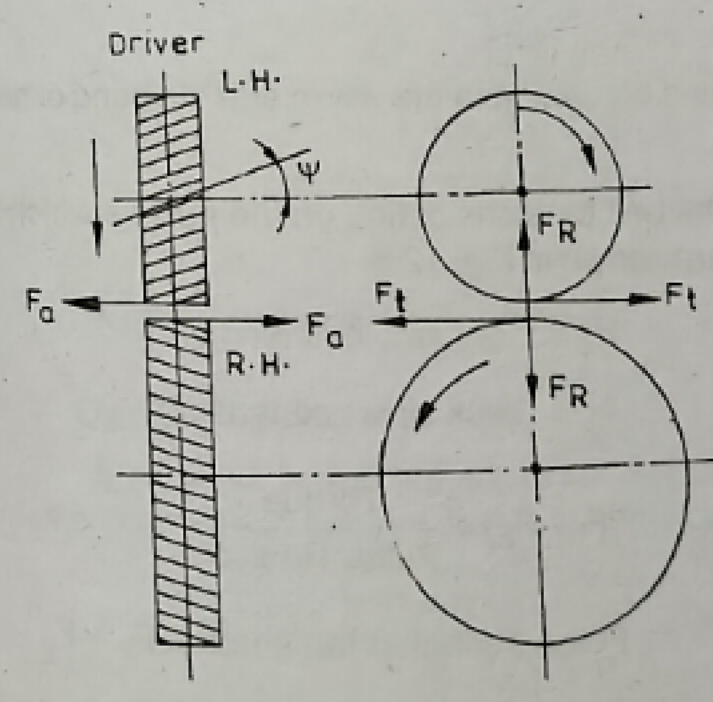


Fig. 12.2

$$F_t = \frac{\text{Torque}}{\text{Pitch circle radius}}$$

$F_R = F_t \tan \phi$

$F_a = F_t \tan \psi$

If ϕ_n is standard then use the relation.

$$\tan\phi = \frac{\tan\phi_n}{\cos \psi}$$

The direction of the thrust force depends on the direction of rotation and the hand of the gear teeth.

12.5.3 Straight Tooth Bevel Gear :

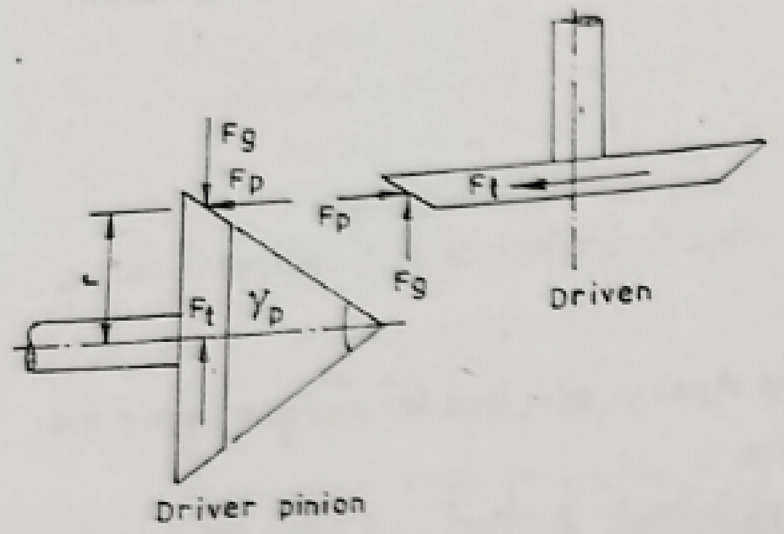


Fig. 12.3

The force acting on a pair of bevel gears for 90° shaft angle are shown in Fig. 12.3

$$F_t = \frac{\text{Torque}}{\text{Mean Pinion Pitch Radius}}$$

$F_p = F_t \tan\phi \sin\gamma_p$

$F_g = F_t \tan\phi \cos\gamma_p$

γ_p = Pinion pitch cone angle

γ_g = Gear pitch cone angle.

12.5.4 Worm Gearing :

The three mutually perpendicular components of the resultant force acting between a worm and gear are shwon in Fig 12.4.

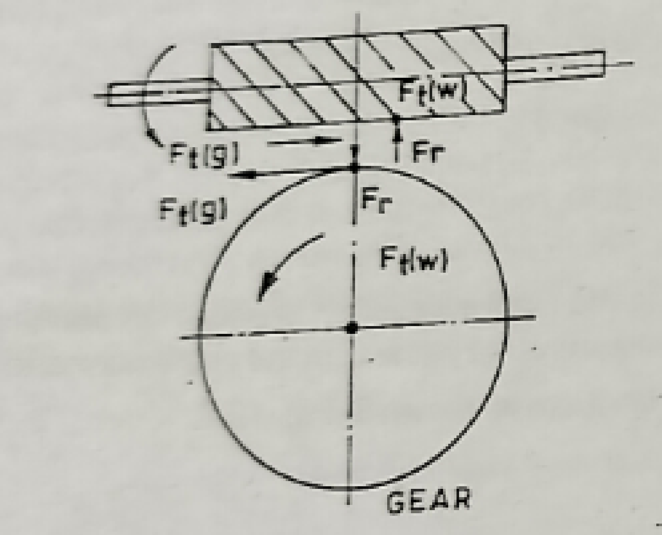


Fig. 12.4

$$F_{t(w)} = \frac{\text{Torque}}{\text{Pitch radius of worm}}$$

$$F_{t(g)} = F_{t(w)}\left[\frac{1-\mu\tan\lambda/\cos\phi_n}{\tan\lambda+\mu/\cos\phi_n}\right]$$

$$F_r = F_{t(w)}\left[\frac{\sin\phi_n}{\cos\phi_n\sin\lambda+\mu\cos\lambda}\right]$$

where λ = Lead angle of worm.

ϕ_n = Normal pressure angle.

The direction of forces depend on the direction of rotation of the worm and the hand of helix.

12.5.5 Forces in Belt Drives :

The forces acting in belt drives are due to the belt tensions acting on the pulleys which are transferred to the shaft and bearings. These forces are shown in Fig. 12.5.

F_1 = Tight side belt tension

F_2 = Slack side belt tension

$$(F_1 - F_2) = \frac{\text{Torque}}{\text{Pulley Radius}}$$

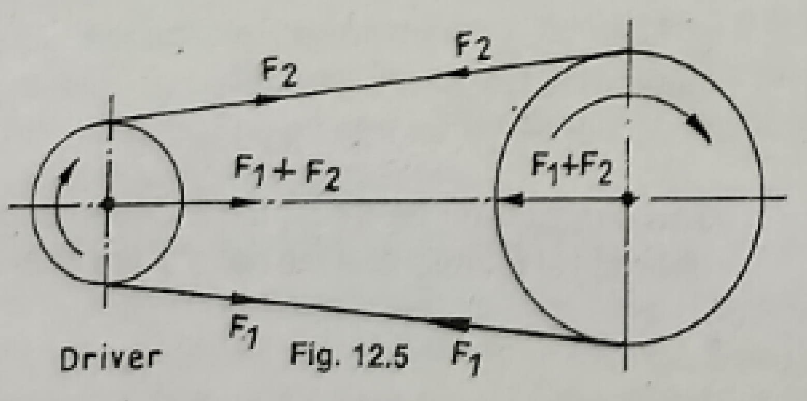


Fig. 12.5

Force acting on the shaft $= F_1 + F_2$

The belt tension ratio F_1/F_2 is to be calculated as given in chapter 18. The same forces act in a flat belt, V belt or a rope drive.

12.5.6 Chain Drives :

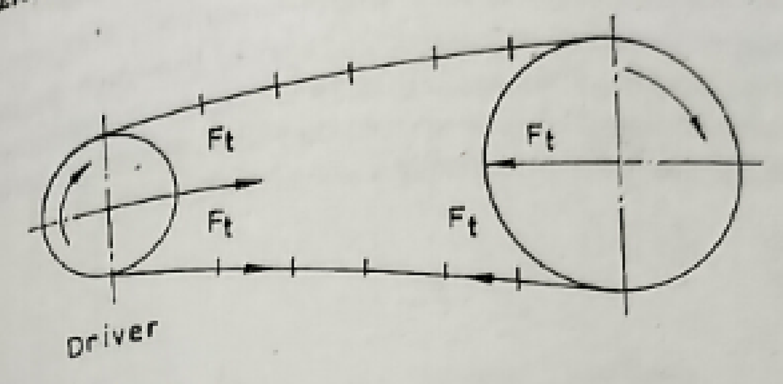


Fig. 12.6

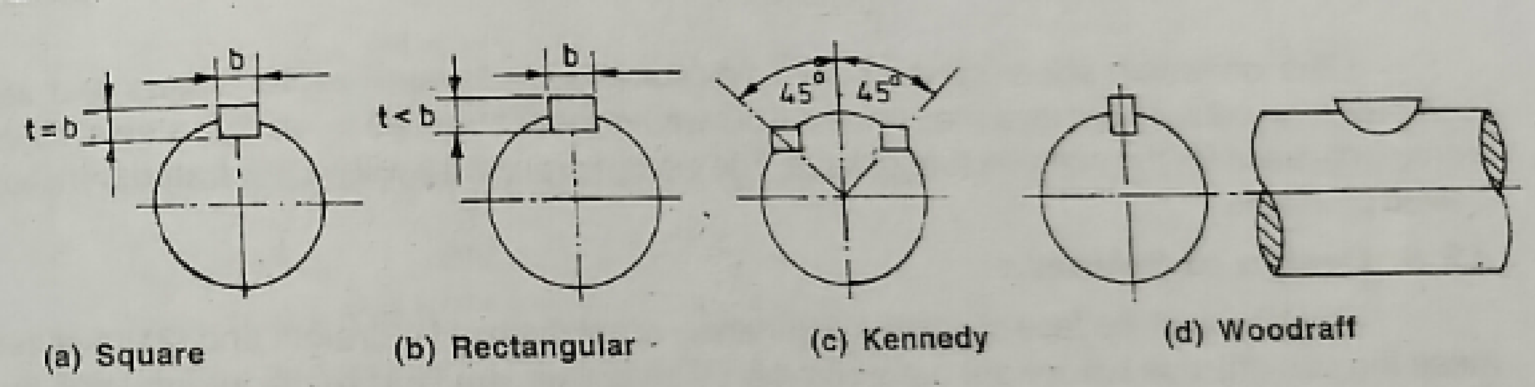
The chain drive differs from the belt drive in that there is no tension on the slack side of chain. The forces acting in a chain drive are shown in Fig. 12.6.

$$F_t = \frac{\text{Torque}}{\text{Pitch Radius of sprocket}}$$

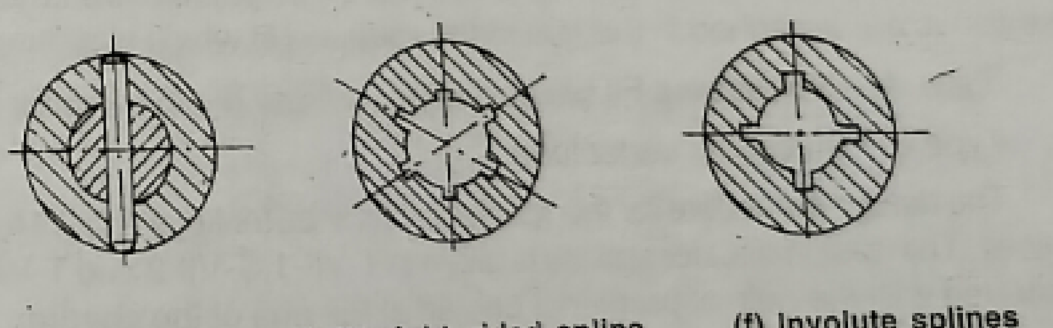
The force acting on the shaft $= F_t$

12.6 Mounting Parts Onto Rotating Shafts ;

In most of the cases the members which receive or deliver power from/to shaft such as pulleys, sprockets, gears, flywheels etc. are mounted on the shafts and attached to the shaft through keys, pins or splines. Only in special circumstances they are made integral with the shaft. The member which rotates with the shaft will be hereafter called a *rotor*. Every rotor has a portion in contact with the shaft and is called *hub*. For normal loads when no axial motion of rotor is required, keys are used to connect the hub to the shaft. A simpler attachment for transmitting relatively light loads is provided by pins. The stronger provision for torque transmission is usually by means of mating splines in the shaft and hub. The splines are generally used where axial motion motion of rotor is required. Various types of keys, pins and splines are shown in Fig. 12.7. The design of common types of keys and splines are discussed below.



(a) Square (b) Rectangular (c) Kennedy (d) Woodraff



(e) Round pin straight or tapped (f) Straight sided spline (f) Involute splines

Fig. 12.7

12.7 Design of Keys :

Sunk keys with a square or rectangular cross section are preferred for normal rotors. The stress concentration caused due to keyway cut in the shaft is given by the relation.

$$K_t = 1 + \frac{0.2b + 1.1h}{D}$$

where b = width of keyway

h = depth of keyway into the shaft

D = shaft diameter.

From this expression it will be clear that a rectangular key with width greater than thickness will give a lesser value of K_t than a square key. Therefore for larger shaft diameters rectangular keys are preferred. The normal design procedure of keys is to select the cross section of the keys as per T-VII-4 or T-VII-5 and calculate the length of the hub based on the shear and the compressive stresses induced while transmitting the required torque. The calculated lengths should be within the range given in T-VII-5. The loading of key and the hub as well as the induced stresses are shown in Fig. 12.8.

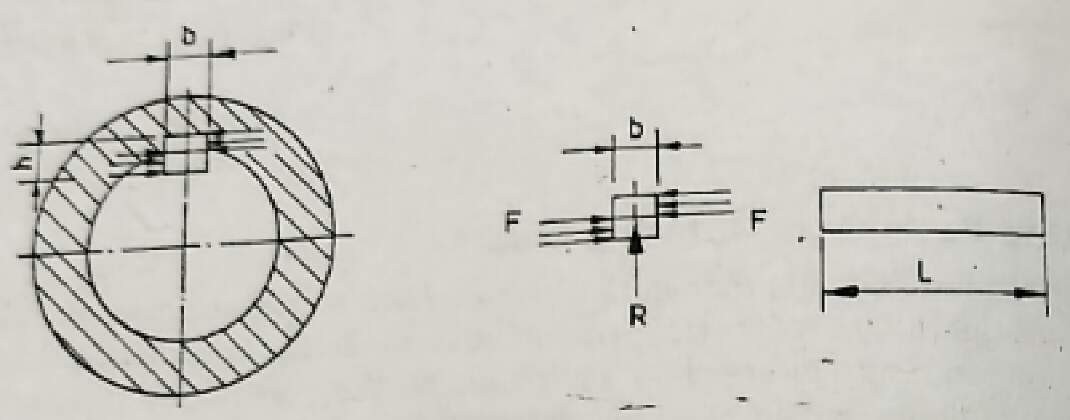


Fig. 12.8

The resultant of the distributed force acting on the two halves of the key are assumed to act at the shaft radius R. With these assumptions the induced shear and compressive or crushing stresses for transmitting a torque T are given by,

$$\tau = \frac{T}{b \times L \times R} \quad \& \quad \sigma_c = \frac{T}{\frac{h}{2} \times L \times R}$$

The allowable shear stresses for keys are assumed same as for shafts and allowable compressive stresses are assumed twice the allowable shear stresses for ductile steels, the material commonly used for the manufacture of keys. The design procedure will be illustrated with the help of solved problems.

12.8 Design of Splines :

The shape of the face of splines is generally of two types (1) Straight and (2) Involute. Out of these the straight side splines are more common. The splines are keys made integral with the shaft. The keyways are broached in the hub to the required fit which is of three types :

Type A -- Permanent Fit i.e. no sliding , Type B -- Sliding Fit under no load

Type C -- Sliding Fit under load.

The design procedure for the splines is same as for keys except for allowable pressure between the splines. The design data for splines is given in T-VII-1, T-VII-2 and T-VII-3. The design procedure will be illustrated with the help of problems solved at the end of the chapter.

SOLVED PROBLEMS

Prob. 12.1 : A pulley P of 600 mm. diameter receives 20 kW at 360 rpm through a belt drive as shown in the Fig. The gear G_1 having PCD of 450 mm. delivers 40 percent of the power to another gear located at an angle of 30° below the horizontal. The remaining power is delivered by gear G_2 to another gear located directly below it. The PCD of gear G_2 is 300 mm. Both gears have 20° involute teeth. The shaft rotates in anticlock wise direction viewing it towards gear G_2. The load is applied with heavy shock and is for continuous duty. The shaft is supported in ball bearings. Suggest suitable diameter for the shaft. Select proper material for the shaft.

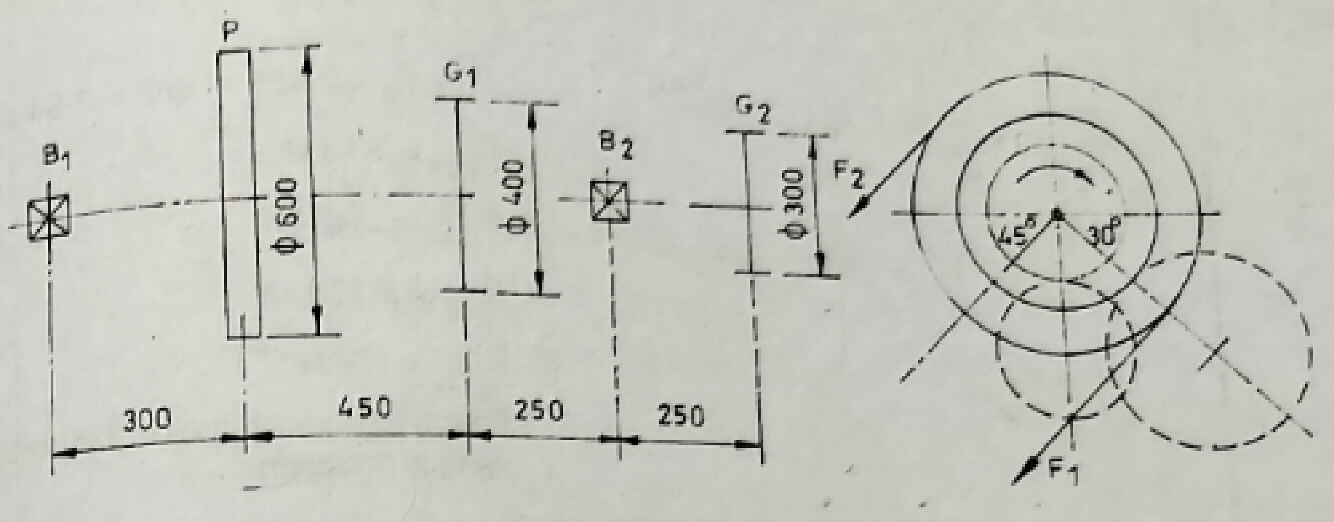


Fig.

Ref. **Calculations**

Power Received, $P_R = 20$ kW at $N = 360$ rpm

Let the torque on pulley P be T_p N - m

$$\therefore \frac{2\pi N T_p}{60} = 20 \times 10^3$$

$$T_p = \frac{20 \times 10^3 \times 60}{2\pi \times 360} = 530.8 \text{ N-m.}$$

let the torque on gear G_1 and G_2 be T_{G1} and T_{G2}

$T_{G1} = .4\, T_p$ and $T_{G2} = .6\, T_p$

$= .4 \times 530.8$ $\quad$ $= .6 \times 530.8$

$= 212.3$ N-m $\quad$ $= 318.5$ N-m

So the torque diagram is as shown in the Fig.

To determine the belt tensions for the drive assume the tension ration 3 : 1.

$$T_p = (F_1 - F_2)\,R_p \qquad \therefore F_1 - F_2 = \frac{530.8}{300} = 1.77 \text{ kN}$$

$$\frac{F_1}{F_2} = 3 \qquad \therefore F_2 = .885 \text{ kN} \ \& \ F_1 = 2.655 \text{ kN}$$

To determine the tangential and radial forces acting on gears G_1 and G_2.

$F_{tG1} \times R_{G1} = T_{G1}$ $\quad$ $F_{tG2} \times R_{G2} = T_{G2}$

$\therefore F_{tG1} = \frac{212.3}{225} = .943$ kN $\quad$ $F_{tG2} = \frac{318.5}{150} = 2.123$ kN

$\therefore F_{RG1} = F_{tG1} \tan 20°$ $\quad$ $F_{RG2} = F_{tG2} \tan 20°$

$= .943 \tan 20°$ $\quad$ $= 2.123 \tan 20°$

$= .343$ kN $\quad$ $= .773$ kN

All these forces are shown in the diagram below.

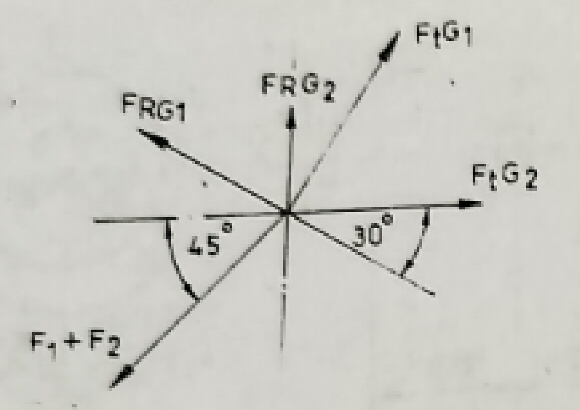

FtG1
FRG2
FRG1
FtG2
45°
30°
F1 + F2

Fig.

$F_1 + F_2 = 2.655 + .885 = 3.54$ kN

$F_{tG1} = .943$ kN

$F_{RG1} = .343$ kN

$F_{tG2} = 2.123$ kN

$F_{RG2} = .773$ kN

Resolving these forces in the horizontal and vertical direction.

Horizontal forces are,

i) $F_{tG2} = 2.123$ kN

ii) $(F_1 + F_2) \cos 45° = 3.54 \cos 45° = 2.50$ kN

iii) $F_{RG1} \cos 30° = .343 \cos 30° = .297$ kN

iv) $F_{tG1} \cos 60° = .943 \cos 60° = .47$ kN

Vertical forces are,

i) $F_{RG2} = .773$ kN

ii) $F_{RG1} \sin 30° = .343 \sin 30° = .172$ kN

iii) $(F_1 + F_2) \sin 45° = 3.54 \sin 45° = 2.501$ kN

iv) $F_{tG1} \sin 60° = .943 \sin 60° = .82$ kN

The forces are shown in the force diagram below.

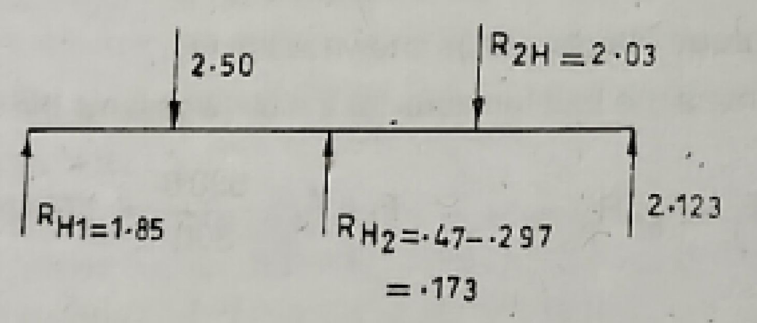
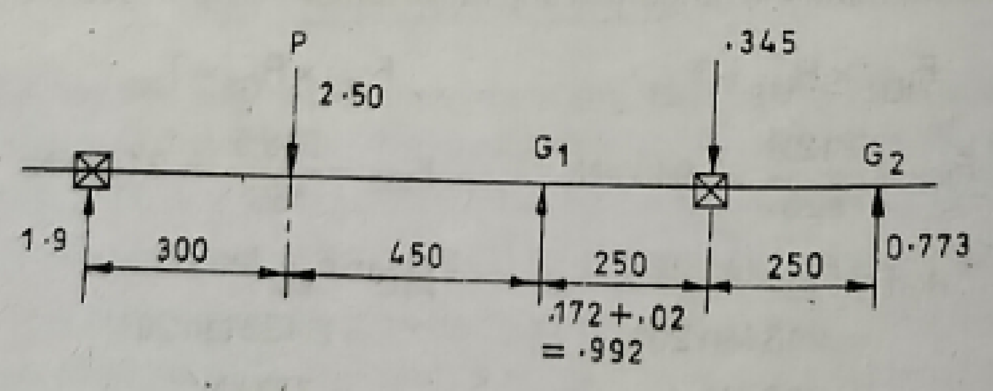

2·50
R2H = 2·03
RH1=1·85
RH2=·47−·297
=·173
2·123

Fig.

Fig.


P
2·50
·345
G1
G2
1·9
300
450
250
250
0·773
·172+·02
= ·992

To calculate bearing reaction $R_{1H}, R_{2H}, R_{1V}, R_{2V}$ take moment about B_1 and B_2 in both planes. For horizontal forces we get,

$$R_{1H} \times 1000 - 2.5 \times 700 + .173 \times 250 - 2.123 \times 250 = 0$$

$$\therefore R_{1H} = (2.5 \times 7 - 1.73 \times .25 + 2.123 \times .25) = 2.23 \text{ kN}$$

$$R_{2H} \times 1000 - 2.5 \times 300 + 1.73 \times 750 + 2.123 \times 1250 = 0$$

$$\therefore R_{2H} = (2.5 \times .3 - 1.73 \times .75 - 2.123 \times 1.25) = -2.03 \text{ kN}$$

Check : $2.23 - 2.5 + .173 - 2.03 + 2.123 = 0$ so o.k.

These are given in the H.F.D.

For vertical forces we get,

$$R_{1V} \times 1000 - 2.5 \times 700 + .172 \times 250 - .773 \times 250 = 0$$

$$\therefore R_{1V} = 2.5 \times .7 - .172 \times .25 + .773 \times .25 = 1.9 \text{ kN}$$

$$R_{2V} \times 1000 - 2.5 \times .300 + .172 \times .75 - .773 \times 1.25 = 0$$

$$\therefore R_{2v} = 2.5 \times .3 - .172 \times .172 \times .75 - .773 \times 1.25 = .345 \text{ KN.}$$

Check : $1.9 - 2.5 + .172 - .345 + .773 = 0$ so o.k. These are shown in V.F.D.

Total reactions at bearings B_1 and B_2 are :

$$R_1 = \sqrt{R_{1H^2} + R_{1V^2}} = \sqrt{2.23^2 + 1.9^2} \approx 3 \text{ kN}$$

$$R_2 = \sqrt{R_{2H^2} + R_{2V^2}} = \sqrt{1.681^2 + .345^2} = 1.72 \text{ kN}$$

Based on these loading diagrams, the bending moment diagrams in the two planes and the torque diagram are shown below.

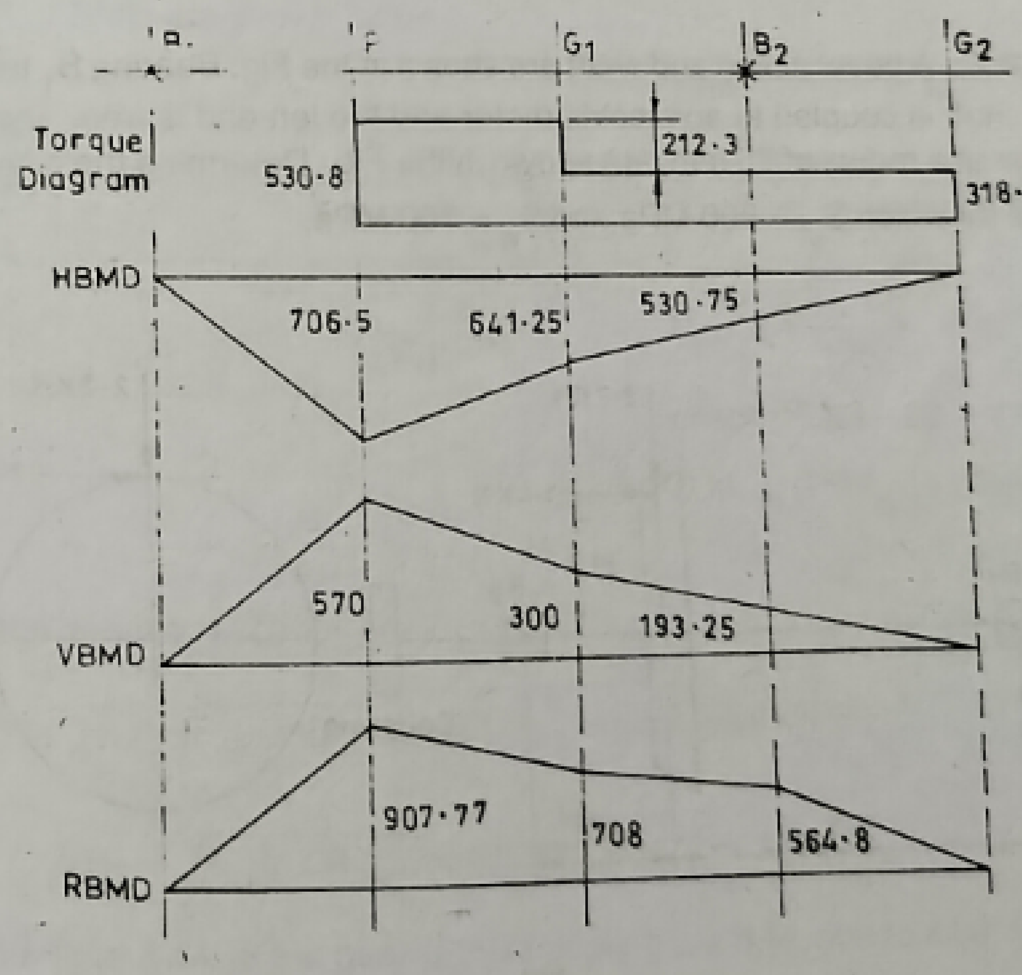

B1
F
G1
B2
G2
Torque Diagram
530·8
212·3
318·5
HBMD
706·5
641·25
530·75
VBMD
570
300
193·25
RBMD
907·77
708
564·8

Fig.

The critical section is at the pulley P because the bending moment and torque are both maximum at this section.

Selecting SAE 1030 steel for the shaft for which

T-II-7 $S_{ut} = 527$ MPa and $S_{yt} = 296$ MPa

T-XI-1 Therefore the design shear stress should be

$$S_{ds} = .30 \times 296 \text{ or } .18 \times 527$$

$$= 88.8 \text{ MPa or } 94.86 \text{ MPa}$$

Now because there is keyway at the critical section the S_{ds} should be reduced by 25 percent.

$$\therefore S_{ds} = 88.8 \times .75 = 66.6 \text{ MPa}$$

T-XI-3 $$\tau_{max.} = \frac{16 \times 10^3}{\pi D^3} \sqrt{(K_t T)^2 + (K_b M)^2}$$ Assume $K_b = 2.5$ & $K_t = 2.5$

Putting $\tau_{max.} = 66.6$ For Heavy shock

$$66.6 = \frac{16 \times 10^3}{\pi D^3} \sqrt{(2.5 \times 530.8)^2 + (2.5 \times 907.7)^2}$$

$D = 58.79$ Assume D = 63 mm.

T-XI-4 Which is the next standard size commercially available. This is the maximum diameter from which the steps will be provided to locate various components on the shaft.

Prob. 12.2 : A bevel pinion and shaft are shown in the Fig. Bearing B_1 takes the thrust. The right end of the shaft is coupled to an electric motor and the left end is free. The load components acting on the gear at a radius of 75 mm. are shown in the Fig. Determine the diameter of shaft if it is made of the steel for which $S_{ut} = 900$ MPa and $S_{yt} = 700$ MPa.

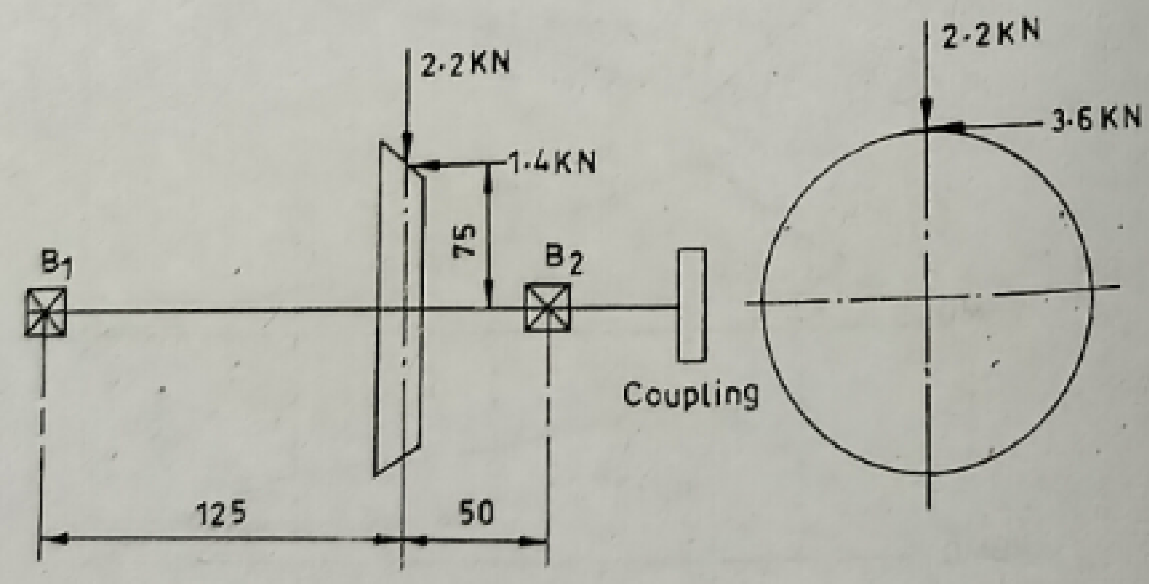
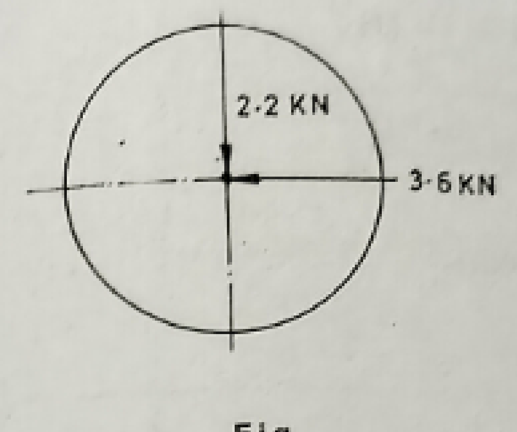


Fig.

Ref. **Calculations**



For finding the reactions of the bearing all the transverse loads acting on the shaft are shown below.

From this load diagram the bearing reactions in the two planes and the total reactions are calculated. The bending moment diagrams in the two planes and the resultant bending moment diagram is also plotted.

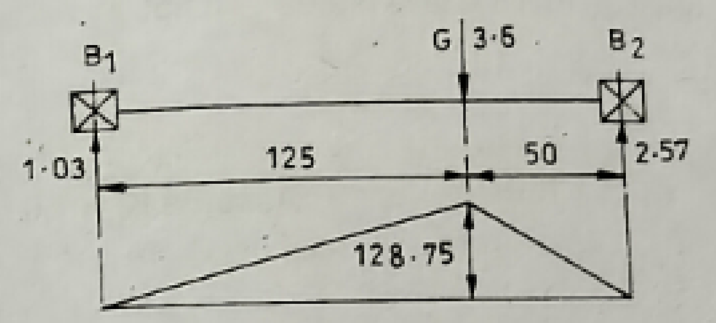
Fig.

The torque acting on the shaft : $T = 3.6 \times 75 = 270$ N-m.

Horizontal force diagram & Bending moment diagram are shown below :

Taking moments at B_2 and B_1.

$$R_{1H} = \frac{3.6 \times 50}{175} = 1.03 \text{ kN}$$



$$R_{2H} = \frac{3.6 \times 125}{175} = 2.57 \text{ kN}$$

check : $1.03 - 3.6 + 2.57 = 0$ so o.k.

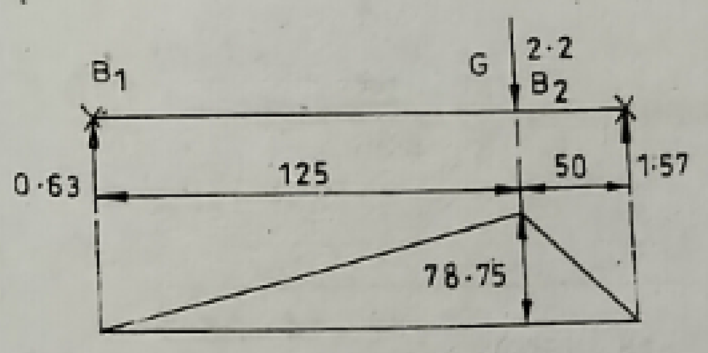
B.M. at G : $M_{GH} = 1.03 \times 125 = 128.75$ N-m

Fig.

VFD + BMD are shown below :

Taking moments at B_2 & B_1

$$R_{1V} = \frac{2.2 \times 50}{175} = .63 \text{ kN}$$



$$R_{2V} = \frac{2.2 \times 125}{175} = 1.57 \text{ kN}$$

check : $0.63 - 22 + 1.57 = 0$ so o.k.

B.M. at G : $M_{GV} = 0.63 \times 125 = 78.75$ N-m.

Fig.

Total bearing reactions are ,

$$R_1 = \sqrt{R_{1H^2} + R_{1V^2}} = \sqrt{1.03^2 + 0.63^2} = 1.2 \text{ kN}$$

$$R_2 = \sqrt{R_{2H^2} + R_{2V^2}} = \sqrt{2.57^2 + 1.57^2} = 3.01 \text{ kN}$$

The total B.M. at the gear G : (Neglecting the B.M. due to axial load)

$M_G = \sqrt{128.75^2 + 78.75^2} = 150.92\ \text{N} - \text{m}$

The other loading at G are,

$T = 270\ \text{N} - \text{m}$ & Axial force $F = 1.4\ \text{kN}$

$$\tau_{max.} = \frac{16 \times 10^3}{\pi D^3} \sqrt{(K_t\, T)^2 + \left(K_b M + \frac{\alpha F D}{8}\right)^2}$$

(D should be in m under the square root)

$S_{ds} = .18 \times 900$ or $.3 \times 700$

$= 162$ or 210 MPa.

As there is keyway at the gear

$S_{ds} = 162 \times .75 = 121.5$ MPa

As the equation for $\tau_{max.}$ contains D under the square root, first neglect this term and solve for D. Calculate $\tau_{(max.)}$ and see that it is less than S_{ds}. If not, increase D to get the suitable value for $\tau_{(max.)}$

T-XI-3 $$121.5 = \frac{16 \times 10^3}{\pi D^3} \sqrt{(1 \times 270)^2 + (1.5 \times 150.92)^2}$$ Assume $K_t = 1.0$

$D = 22.5$ mm $Kb = 1.5$

T-XI-4 Assuming the nearest standard value of 25 mm check the value for induced $\tau_{max.}$

For calculating the value of α calculate the value $\frac{L}{k}$

T-XI-2 $L = 125$ mm, $K = \frac{D}{4} = \frac{25}{4} = 6.25$ m, So $\frac{L}{k} = \frac{125}{6.25} = 20$

$\therefore\ \alpha = 1.12$

$$\tau_{max.} = \frac{16 \times 10^3}{\pi \times 25^3} \sqrt{(1 \times 270)^2 + \left(1.5 \times 150.92 + \frac{1.12 \times 1.4 \times 25}{8}\right)^2}$$

$= 115.9$ MPa

Because this is less than $S_{ds} = 121.5$ so o.k.

Select $D = 25$ mm.

Prob. 12.3 : The shaft shown in the Fig. below is to be made of SAE 1030 steel. Determine a safe diameter to insure that the first critical speed be no less than 3600 r.p.m. Consider the shaft weight also.

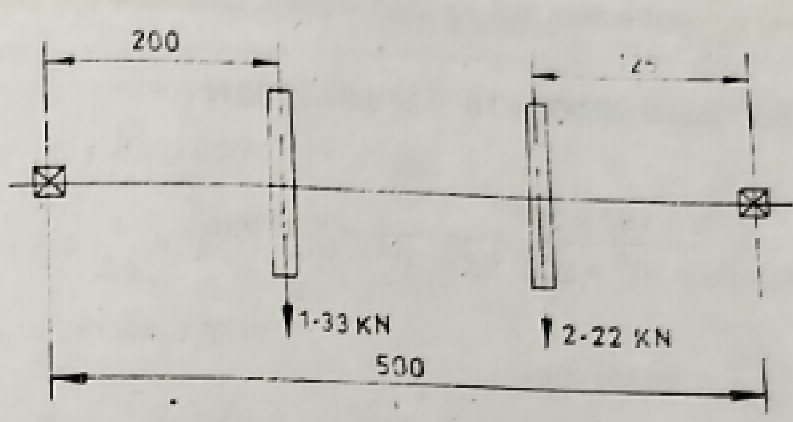


Fig.

Ref.	Calculations

T-I-2

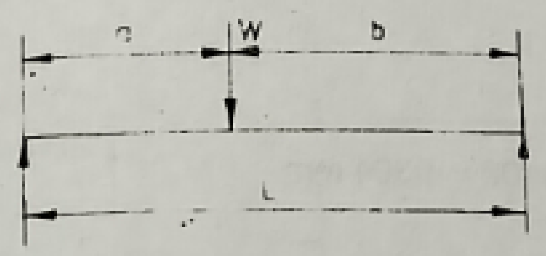


Fig.

To calculate deflection δ_1 and δ_2 under the two rotors, the formula will be

$$\delta \text{ at } W = \frac{W a^2 b^2}{3\,EIL}$$

T-II-7 For SAE 1030 steel $E = 79 \times 10^3$ MPa

T-I-1 $I = \frac{\pi D^4}{64}$, $L = 500$ mm.

For δ_1 : $a = 200$ mm $b = 300$ mm

For δ_2 : $a = 375$ mm $b = 125$ mm

$$\therefore\ \delta_1 = \frac{1.33 \times 10^3 \times 200^2 \times 300^2 \times 64}{3 \times 79 \times 10^3 \times \pi \times d^4 \times 500} = \frac{82.35 \times 10^4}{d^4}$$

$$\delta_2 = \frac{2.22 \times 10^3 \times 375^2 \times 125^2 \times 64}{3 \times 79 \times 10^3 \times \pi \times d^4 \times 500} = \frac{83.9 \times 10^4}{d^4}$$

T-XI-1 $$N_c = \frac{945}{\sqrt{\delta_1 + \delta_2}} \quad \therefore\ 3600 = \frac{945 \times d^2}{10^2\sqrt{82.35 + 83.9}}$$

$\therefore\ d = 70$ mm

To consider the effect of shaft weight assume a higher diameter. Assume $d = 80$ mm

$$I = \frac{\pi}{64} \times 80^4 = 200.96 \times 10^4$$

$$\therefore\ \delta_1 = \frac{82.35 \times 10^4}{80^4} = .02\ \text{mm} \quad ; \quad \delta_2 = \frac{83.9 \times 10^4}{80^4} = .02\text{mm.}$$

For finding δ_s, deflection due to shaft weight, calculate the shaft weight W_s.

$$W_s = \frac{\pi}{4} 80^2 \times 500 \times 7800 \times 10^{-9} \times 9.81 = 192 \text{ N}$$

T-I-2

$$\delta_s = \frac{5 \times 192 \times 500^3}{384 \times 79 \times 10^3 \times 200.96 \times 10^4} = .002 \text{ mm}$$

$$\therefore \frac{\delta_s}{1.27} = \frac{.002}{1.27} = .00157 \text{ mm}$$

$$N_c = \frac{945}{\sqrt{\delta_1 + \delta_2 + \frac{\delta_s}{1.27}}} = \frac{945}{\sqrt{.02 + .02 + .00157}} = 4636.9 \text{ rpm.}$$

very high

Using $d = 72$ mm

$$\delta_1 = \left(\frac{80}{72}\right)^4 \times .02 = 1.524 \times .02 = .0304 \text{ mm}$$

$$\delta_2 = \delta_1 = .0304 \text{ mm}$$

$$\delta_s = \left(\frac{80}{72}\right)^2 \times .002 = .0025 \text{ mm}$$

$$\therefore \sqrt{\delta_1 + \delta_2 + \frac{\delta_s}{1.27}} = \sqrt{.0304 + .0304 + \frac{.0025}{1.27}} = .25$$

$$N_c = \frac{945}{.25} = 3772 \text{ rpm so o.k.}$$

Prob. 12.4 : Determine the critical speed of the steel shaft shown in the figure below.

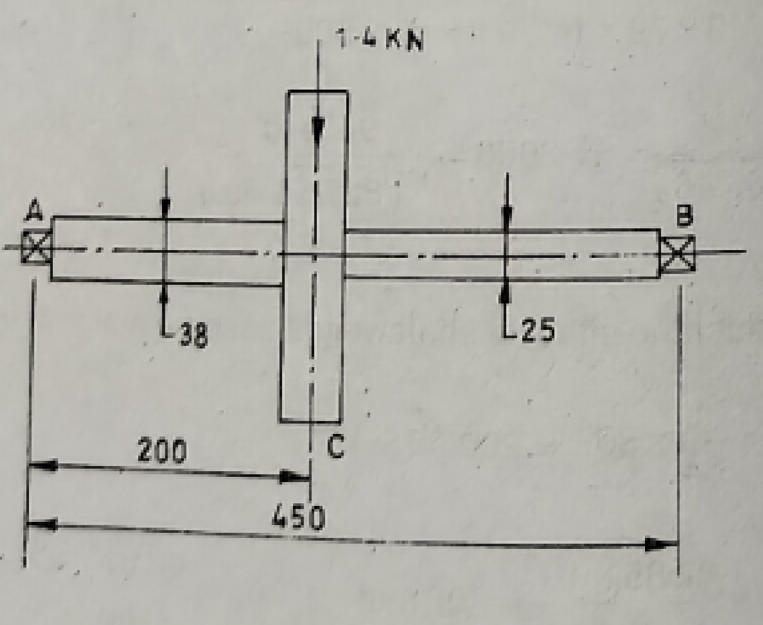


Fig.

Ref.

Calculations

This is stepped shaft having different I along the length of the shaft.

I_1 = Moment of inertia from

$$\text{A to C} = \frac{\pi}{64} \times 38^4 = 10.23 \times 10^4, \text{ mm}^4$$

I_2 = Moment of inertia from

$$\text{C to B} = \frac{\pi}{64} \times 25^4 = 1.9 \times 10^4 \text{ mm}^4$$

To draw the B.M.D. calculate R_A and R_B.

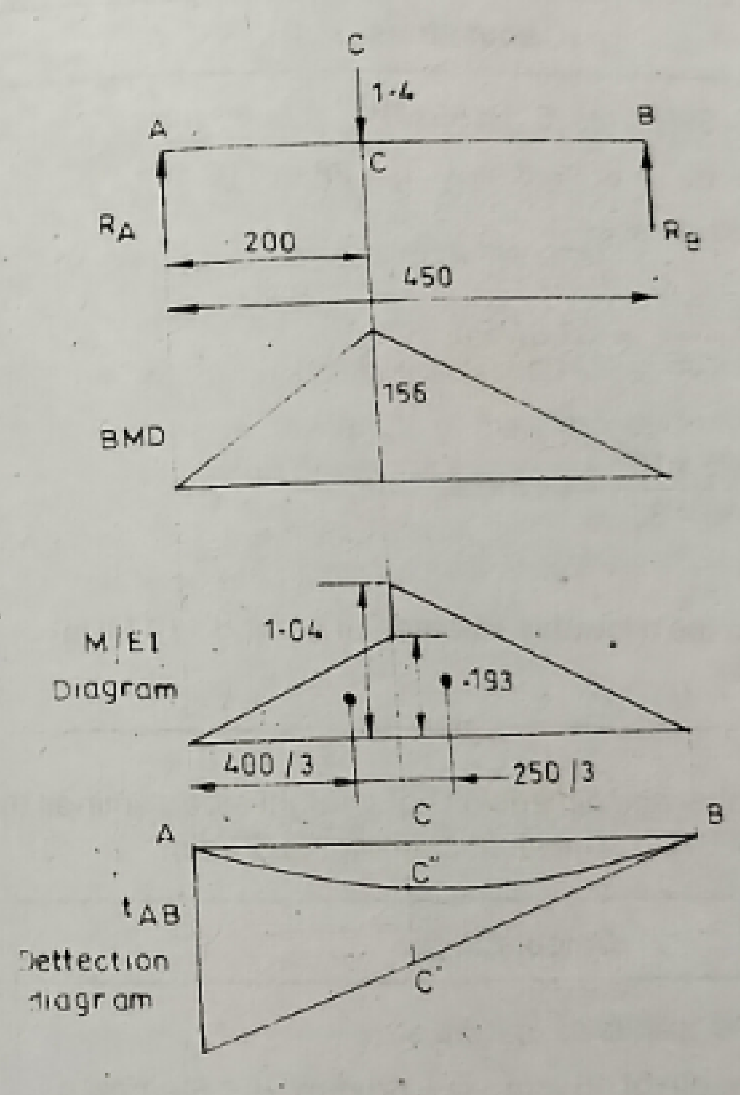


Fig.

$$R_A = \frac{1.4 \times 250}{450} = .78 \text{ kN}$$

$$R_B = \frac{1.4 \times 200}{450} = .62 \text{ kN}$$

B.N. at C = .78 × 200 = 156 N-m.

Taking $E = 79 \times 10^3$ MPa

The $\frac{M}{EI}$ diagram is plotted.

At C there will be two values of $\frac{M}{EI}$ for two values of I.

$$\frac{M}{EI_1} = \frac{156 \times 10^3}{79 \times 10^3 \times 10.23 \times 10^4}$$

$$= .193 \times 10^{-4}$$

$$\frac{M}{EI_2} = \frac{156 \times 10^3}{79 \times 10^3 \times 1.9 \times 10^4}$$

$$= 1.04 \times 10^{-4}$$

Applying second Area-Moment theorem :

t_{AB} = Moment of $\frac{M}{EI}$ diagram about A

$$= \frac{250}{2} \times .193 \times 10^{-4} \times \frac{400}{3} + \frac{1}{2} \times 1.04 \times 10^{-4} \times 250 \times \frac{850}{3} = 3.94$$

$$t_{CB} = \frac{1}{2} \times 1.04 \times 10^{-4} \times 250 \times \frac{250}{3} = 1.08 = C''C'$$

$$C\,C' = \frac{t_{AB}}{450} \times 250 = \frac{3.94}{450} \times 250 = 2.19$$

$$\therefore \delta_c = CC'' = C\,C' - C''\,C' = 2.19 - 1.08 = 1.11 \text{ mm}$$

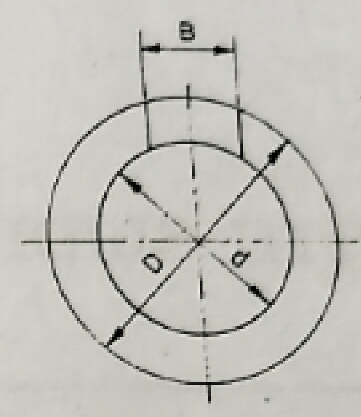
$$\therefore N_e = \frac{945}{\sqrt{\delta_c}} = \frac{945}{\sqrt{1.11}} = 897 \text{ rpm.}$$

Prob. 12.5 : Design a key to be used for mounting a gear with a hub length of 60 mm on a 35 mm. diameter steel shaft to transmit a torque of 395 N-m. The key and shaft are made of same material with an allowable shear stress of 55 MPa.

Ref.	Calculations
T-VII-5	Given $D = 35$ mm, $T = 395$ N-m, $S_{ds} = 55$ MPa, $L = 60$ mm. Key cross section : $b = 10$, h or $t = 8$ mm, $L = 22$ to 110 mm Checking for the induced stresses $\tau = \frac{2T}{b \times L \times D} = \frac{2 \times 395 \times 10^3}{10 \times 60 \times 35} = 37.6$ MPa $\sigma_c = \frac{2T}{\frac{h}{2} \times L \times D} = \frac{2 \times 395 \times 10^3}{4 \times 60 \times 35} = 94$ MPa As τ_c and σ_c are less than the allowable stresses of 55 and 110 MPa so our design is o.k.

Prob. 12.6 : Determine the number and dimensions of straight sided splines that wil permit sliding under load to transmit 75 kW at 800 rpm. The shaft diameter is 60 mm.

Ref.	Calculations
T-VII-2	Choose medium duty series splines as follows : $8 \times 52 \times 60$ These have a width of 10 mm. $D = 60$ mm, $d = 52$ mm, $n = 8$, $B = 10$ Power $P = 75$ kW, $N = 800$ rpm $2\pi \times \frac{800}{60} \times T = 75 \times 10^3$ $\therefore T = \frac{75 \times 10^3 \times 60}{2\pi \times 800} = 895.7$ N-m Shear stress induced in the shaft due to this torque :

Ref.	Calculations
	 Fig.
T-VII-3	$\tau = \frac{16\,T}{\pi D^3} \times K_{ts} = \frac{16 \times 895.7 \times 10^3}{\pi \times 60^3} \times 1.8 = 38$ MPa
T-II-7	For normal shaft material of SAE 1030 steel for which $S_{ut} = 527$, $S_{yt} = 296$ The design shear stress for the shaft $S_{ds} = .18 \times 527$ or $.3 \times 296$ $= 94.86$ or 88.8 so select 88 MPa.
T-VII-3	To determine the length of the hub, select the bearing pressure $P = 7$ MPa because there is sliding under load. $\therefore L \times \left(\frac{D-d}{2}\right) \times$ no. of splines $\times P \times \left(\frac{D+d}{2}\right) = 895.7 \times 10^3$ $L = \frac{895.7 \times 10^3}{4 \times 8 \times 7 \times 56} = 71.4$ mm $= 72$ mm To calculate shear stress induced : $\therefore \tau = \frac{T}{B \times L \times \text{no. of splines} \times D_m} = \frac{895.7 \times 10^3}{10 \times 72 \times 8 \times 56} = 2.8$ MPa Hence the design is o.k. Select the splines with specifications $8 \times 52 \times 60$ with a length of 72 mm.

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Chapter 13

BEARINGS WITH SLIDING CONTACT

13.1 Introduction :

A bearing is a machine part which supports a moving part and confines its motion. *Sliding bearings* involve direct sliding of the load carrying member on its support as distinguished from *rolling contact bearings* where balls or rollers are interposed between the moving surfaces. Sliding bearings are of two types (1) Journal or sleeve bearings which are cylindrical and support radial loads (i.e. loads perpendicular to shaft axis) and (2) Thrust bearings which are generally flat and support loads in the direction of the shaft axis. The part of the shaft which rotates in a bearing is called *journal*. When relative motion occurs between the surfaces it is usually desirable to minimize friction and wear. Any interposed substance that reduces friction and wear is a *lubricant*. Lubricants are usually liquid, but can be semisolid (greases) or gas (pressurized air).

There are many types of lubrication. It is classified according to the degree with which the lubricant separates the sliding surfaces. The three basic types are :

1. Hydrodynamic or Thick Film or Fluid Film Lubrication : It involves complete separation of the surfaces by the lubricant film. The load tending to bring the surfaces together is entirely supported by fluid pressure generated by the motion of the surfaces. Typical film thickness at the thinnest point are 0.008 to 0.020 mm and typical values of coefficient of friction are 0.002 to .010.

2. Mixed Film Lubrication : It involves intermittent contact of surface peaks, and partial hydrodynamic support. Coefficients of friction commonly range from 0.004 to 0.010.

3. Boundary or Thin Film Lubrication : It involves continuous and extensive surface contact, but the lubricant is continuously smeared over the surfaces and provides a continuously renewed adsorbed surface film which reduces friction and wear. Typical values of coefficient of frictions are 0.05 to 0.20.

Complete surface separation can also be achieved by *hydrostatic lubrication*. This involves introducing a highly pressurized fluid (as air, oil or water) into the load bearing area. Since the fluid is pressurized by external means, full surface separation can be obtained irrespective of whether or not there is relative motion between the surfaces. The principal advantage is extremely low friction at all times, including starting and low speed operation. Disadvantages are the cost and complication.

The most desirable type of lubrication is obviously thick film or hydrodynamic lubrication and this will be treated in greater detail beginning with the next section. Boundary lubrication and hydrostatic lubrication are treated in later sections.

13.2 Hydrodynamic Lubrication :

The three basic parameters which affect the type of lubrication and the resulting coefficient of friction is shown in the figure below. These three basic parameters are combined into a single variable $\frac{Zn}{P}$ called *bearing modulus*.

Z = absolute viscosity, mPa – s

n = speed of journal, rps

P = Unit load on the projected area, MPa.

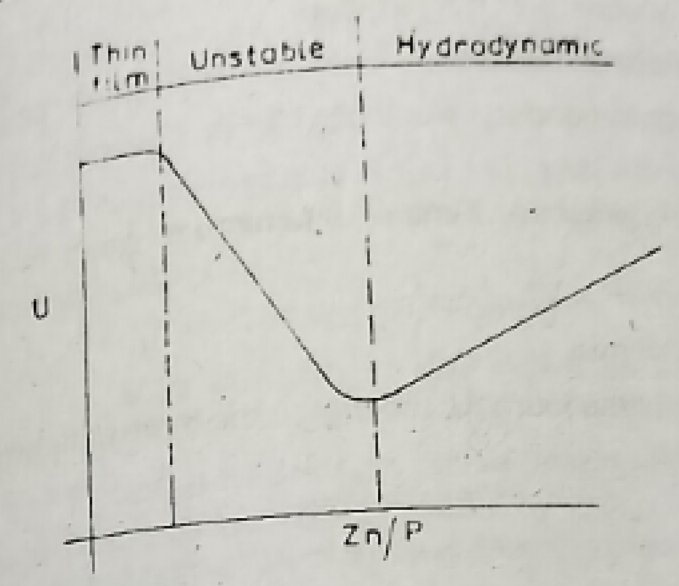


Fig. 13.1

1. **Viscosity Z :** The higher the viscosity, the lower the rotating speed needed to float the journal at a given load. Increases in viscosity beyond that necessary to establish thick film or hydrodynamic lubrication increase bearing friction by increasing the forces required to shear the lubricant film.

2. **Rotating speed n :** The higher the rotating speed, the lower the viscosity needed to float the journal at a given load. Once hydrodynamic lubrication is achieved, further increases in rotating speed increase bearing friction.

The achievement of hydrodynamic lubrication requires three things :

1. Continuous relative motion of the surface to be separated.
2. Wedging action as provided by the eccentricity and
3. Supply of sufficient quantity of suitable fluid.

In addition to the three parameters mentioned above, other two parameters which affect the bearing performance are :

1. Length to diameter ratio of the journal, $\frac{L}{D}$
2. Clearance ratio $\frac{C}{R}$ where C is the radial clearance and R is the radius of the journal.

All these parameters are combined into a dimensionless variable called Sommerfield number S as defined below.

$$S = \left(\frac{Zn}{P}\right)\left(\frac{R}{C}\right)^2$$

Raymondi and Boyd have plotted extensive performance charts for hydrodynamic bearings which are widely used in the design of these bearings as discussed in the next section.

13.3 Hydrodynamic Bearing Design :

Normally the data which is provided for the design of bearings is,

1. Diameter of journal 2. Load and 3. Speed of rotation.

The design involves the selection of following parameters so that the thick film is maintained with the temperature of the lubricant within acceptable range.

1. Length of journal 2. Clearance and 3. Viscosity of the lubricant.

The steps to be followed in bearing design are as follows :

Step 1 : Referring T-XIII-4 note down the recommended values of;

(a) Range of $\frac{L}{D}$ ratio (b) Maximum bearing pressure, Pmax (c) Minimum $\frac{Zn}{P}$

(d) Suitable viscosity Z and (e) The range of (P × V).

Step 2 : Calculate the rubbing velocity V in m/min.

Step 3 : Choose suitable value for the length of the journal L, so that all the three criteria given below are satisfied.

(a) the $\frac{L}{D}$ ratio is within the given range.

(b) the bearing pressure is within the recommended range.

(c) $P \times V$ value is within the given range.

Step 4 : Select suitable bearing material as per T-XIII-6.

Step 5 : Calculate the suitable radial clearance C as per T-XIII-1.

Step 6 : Select suitable $\frac{Zn}{P}$ value which should be between 3 to 10 times the recommended minimum value of $\frac{Zn}{P}$ depending upon the variability of load and speed. Calculate the value of Z and suggest suitable lubricating oil as per Fig. 13.2.

Step 7 : Calculate the dimensionless value of the Sommerfeld no as T-XIII-1.

Step 8 : Read the coefficient of friction variable $\left(\frac{R}{C}\mu\right)$ from Fig. 13.3. Calculate the coefficient of friction μ.

Step 9 : Calculate the heat generated Hg as per T-XIII-1.

Step 10 : Calculate the temperature rise of the oil by equating Hg to the heat dissipating capacity, H_d as per T-XIII-1. Calculate the oil film temperature t_{oil} and see that it is as per T-XIII-5. If it is not within the given limits, it means that external cooling by pressure feed of the lubricant is needed.

Step 11 : Calculate minimum film thickness h_{min} with the help of Fig. 13.5. Suggest suitable surface finish for the journal and the bushing.

Step 12 : Calculate the amount of side flow of the lubricant with the help of Fig. 13.6 and Fig. 13.7. This is quantity of oil to be supplied to the bearing to maintain the fluid film.

The procedure will be illustrated with the help of solved examples.

13.4 Thin Film or Marginal Lubrication :

When the bearing modulus $\frac{Zn}{P}$ is too low, or when the supply of lubricant to the bearing is insufficient to maintain fluid film lubrication, thin film lubrication and metal to metal contact will exist as

shown in Fig. 13.1. The coefficient of friction in this case can be estimated as per T-XIII-1. It the load varies, the average pressure is used for P but it must not be less than one half the maximum pressure. At low and moderate speeds and low loading these bearings are quite satisfactory. The bearing clearance in such bearings should be slightly greater than that in bearings with thick film lubrication. A journal in sintered porous bearings is an example of thin film lubrication.

13.5 Thrust Bearings :

The shafts are also subjected to axial loads which are called thrust loads. The bearings which support these loads are called thrust bearings. A horizontal shaft is often subjected to a thrust and a vertical shaft always.

The types of thrust bearings are :

1. **Step Bearings :** This is the simplest thrust bearing which is used for vertical shafts. Their design procedure is same as collar bearings.

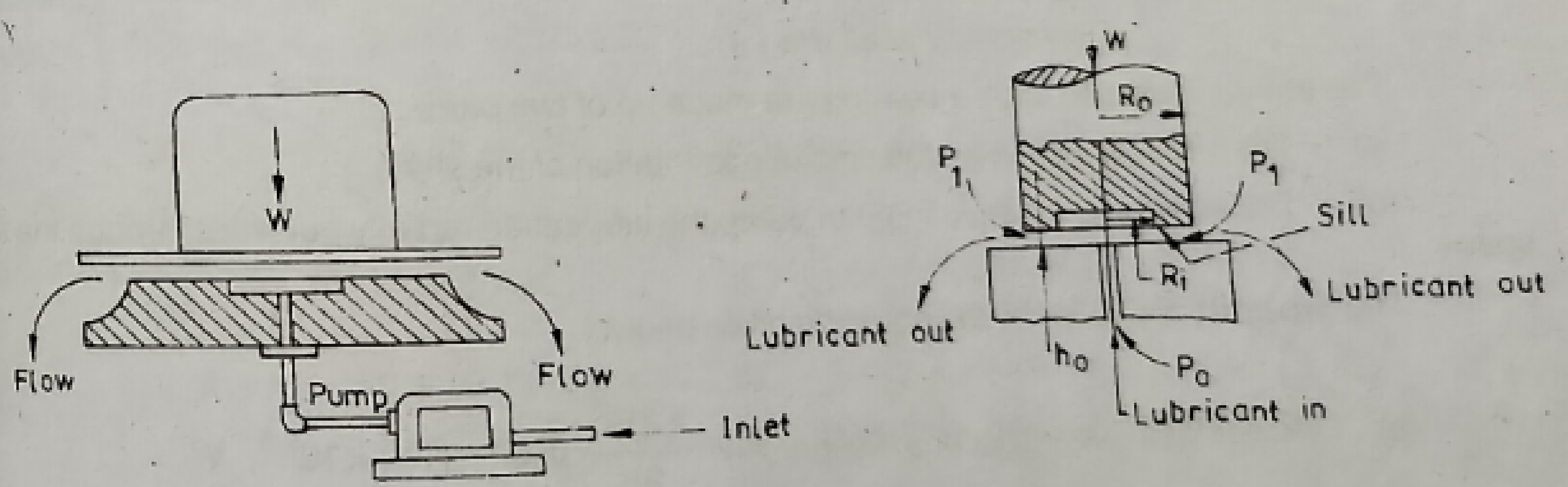
2. **Collar Bearings :** These bearings are used chiefly on a horizontal shaft which must carry a large axial load such as is created by a ship propeller. If feasible, the collars should be placed near the point of application of the axial load in order to avoid a column action upon the shaft.

3. **Tilted Pad or Kings bury Bearings :** These bearings consists of several stationary pivoted segments or pads against which is pressed a thrust collar fastened to the rotating shaft. The collar runs in an oil bath, and its rotation draws an oil film into the spaces between it and the pads, which tilt automatically so that a fluid wedge is formed at the back edge. These bearings are made in types suitable for vertical or horizontal shafts, and for carrying thrust in both axial directions or in only one direction.

The design procedure for these bearings is as per T-XIII-9 and T-XIII-11 and will be illustrated with the help of solved examples.

13.6 Hydrostatic Thrust Bearings :

Hydrostatic lubrication provides a convenient way for designing into a particular bearing quantitative and predetermined performance characteristics. With hydrostatic lubrication a bearing design can be easily optimized in terms of flow, pressure, load carrying capacity, film rigidity, friction & pumping power. In its simplest form, a hydrostatic bearing consists of a flat circular disc, either with or without a recess (pocket) connected to a positive displacement pump as shown in Fig. (13.2[a]).



(a) Simple hydrostatic bearing

(b) Schematic diagram of hydrostatic step bearing

Fig. 13.2

The recess may be either in the bottom member or in the top member. The bearing shown in Fig. (13.2[b]) is often called a step bearing. A load W is acting on the top member and there is initially metal to metal contact. Since the pump, being positive displacement, must discharge a fixed volume of lubricant, pressure will build up in the recess until the top member and its load lift and establish a film thickness on the sill of the bearing through which the discharge volume of the pump passes.

A steady state equilibrium condition will be established in which the summation of the pressures acting upward from the recess and from the sill will be just equal and opposite to the downward external load. A representation of a typical pressure distribution is shown in Fig. (13.3).

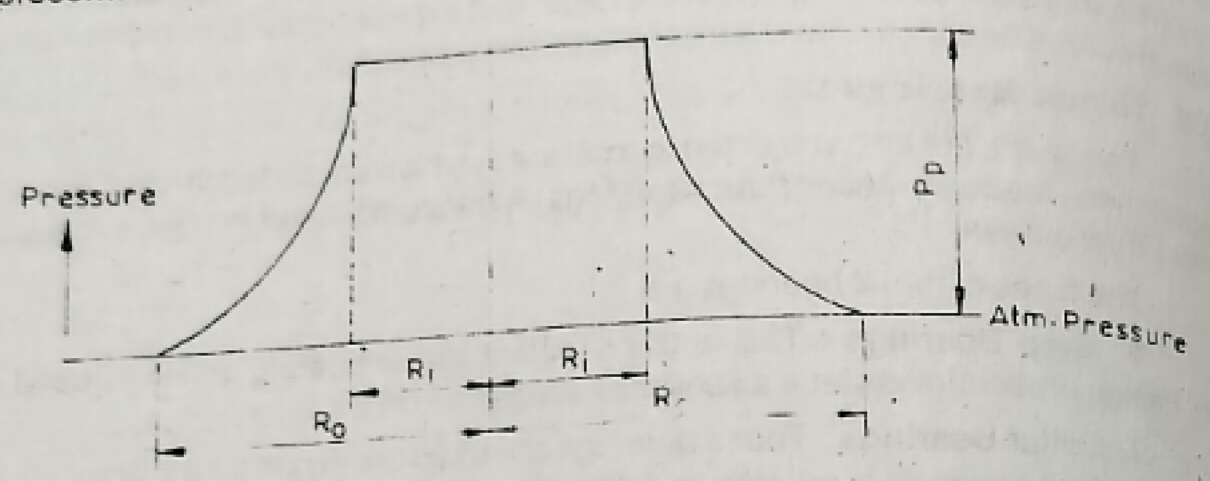


Fig. 13.3

The equations that may be used to evaluate the significant parameters of this type of bearing are given in T-XIII-12. They are as follows :

1. Pressure in the recess, $$P_p = \frac{2\,W \ln\left(\frac{Ro}{Ri}\right)}{\pi\left(Ro^2 - Ri^2\right)}, \text{ MPa}$$

where W = load on the bearing, N

Ro & Ri = Radii of the recess, mm.

2. Oil flow from the recess, $$Q = \frac{\pi P_p h^3}{6\,Z \ln\left(\frac{Ro}{Ri}\right)} \times 10^6, \text{ mm}^3/\text{sec.}$$

where h = film thickness, mm.

Z = viscosity of oil, Pa - s

The energy losses in such a bearing are made up of two parts :

(a) The viscous friction on the step due to rotation of the shaft.

(b) The energy required to force or pump the lubrication radially out ward through the film space.

The two parts are given by the equation given below :

(a) Power loss due to viscous friction, $$P_1 = \frac{\pi Z V^2}{2h}\left(Ro^2 - Ri^2\right) \times 10^{-3}, \text{ W}$$

where V = mean peripheral velocity, m/sec.

(b) Pumping power $P_2 = Q\,P_p \times 10^{-3}$, W

The total power loss will be the sum of the two parts, $P_1 + P_2$. Out of this P_1 is proportional to the rotational speed of the shaft and vanishes at standstill. The second part is constant and does not depend on the shaft speed. The use of these equations will be illustrated with the help of solved problems.

SOLVED PROBLEMS

Prob. 13.1 : The shaft for a centrifugal blower is to be mounted on ring oiled babitt bearings. The blower speed is 600 rpm and the minimum journal diameter based on strength is 50 mm. The bearing loads due to weight of the rotor and belt forces have been determined to be 560 N and 2.8 kN. However only one size of bearing will be used for both ends of the shaft. The blower must operate continuously without the oil temp. exceeding 80°C, when the ambient temp. varies between 15°C and 40°C. Design the bearing, specify the lubricating oil and determine the power loss and side flow in the bearing.

Ref.	Calculations
	For design we will take $W = 2.8$ kN, $d = 50$ mm, $N = 600$ rpm, $t_{oil} < 80°C$, $ta = 40°C$
T-XIII-4	For centrifugal pumps $P = 0.7 - 1.4$ MPa, $Z = 25$, $\left(\frac{Zn}{p}\right)$ min $= 46$ $\frac{l}{d} = 1 \text{ to } 2, \quad P \times V = 100 - 180$ Selecting $\frac{l}{d} = 1, \quad P = \frac{2.8 \times 10^3}{50 \times 50} = 1.12 \text{ MPa so o.k.}$ $V = \pi \times \frac{50}{1000} \times 600 = 94.2 \text{ m/min} \quad \therefore P \times V = 1.12 \times 94.2 = 105.5 \text{ so o.k.}$ $= 1.57$ m/sec.
T-XIII-6	Selecting tin base babitt $K = .0005$, $S_{min} = .06$
T-XIII-1	$C = (KR + .002) = .0005 \times 25 + .002 = .0145$ mm $\therefore \frac{R}{C} = \frac{25}{.0145} = 1724$ $t_{oil} = 80°C, \; ta = 40°C \quad \therefore (t_b - t_a) = \frac{1}{2}(80 - 40) = 20°C$ $A = 50 \times 50 \times 10^{-6} = 2.5 \times 10^{-3} \text{ m}^2$ $H_d = KA\,(t_b - t_a + 18)^2 = 3.67 \times 2.5 \times 10^{-3}\,(20 + 18)^2 = 13.25 \text{ W}$ $\therefore Hg = \mu WV = 13.25$ $\therefore \mu = \frac{13.25}{2800 \times 1.57} = .003 \quad \therefore \frac{R}{C}\mu = 1724 \times .003 = 5.2$ For this value of $\left(\frac{R}{C}\mu\right)$, $S = 0.28$

F-13.3	$S = .28 = \left(\frac{Zn}{P}\right)\left(\frac{R}{C}\right)^2 \times 10^{-9} \therefore \frac{Zn}{P} = \frac{.28 \times 10^9}{(1724)^2} = .094 \times 10^3 = 94$
	$\therefore Z = \frac{94 \times 1.12}{10} = 10.5$
F-13.2	$\therefore$ For Z = 10.5 at 80°C use SAE 20 oil
F-13-6	$\frac{Q}{RCnL} = 3.6 \qquad \therefore Q = 3.6 \times 25 \times 0.145 \times 10 \times 50 = 625.5\ mm^3/sec.$
F-13.7	$\frac{Q_S}{Q} = 0.5 \qquad \therefore Q_S = 0.5 \times 652.5 = 326.25\ mm^3/sec.$

Prob. 13.2 : A journal bearing with the following data operates at a satisfactory temperature. Journal dia. = 50 mm. Length = 100 mm, Speed = 400 rpm, Radial clearance C = .025 mm, $W = 25\ kN$, $Z = 50\ mPa^{-5}$.

Due to change in operating conditions, the speed had to be increased to 500 rpm and after reconditioning of bearing, the radial clearance obtained was 0.030 mm. Suggest suitable viscosity of oil which will keep the bearing temp same as before.

Ref.	Calculations
	Given : $d = 50$ mm, $L = 100$ mm, $N = 400$ rpm, $C = .025$ mm, $W = 25$ kN, $Z = 50\ mPa^{-5}$
	$P = \frac{W}{L \times d} = \frac{25 \times 10^3}{50 \times 100} = 5MPa, \quad \frac{Zn}{P} = \frac{50 \times 400}{60 \times 5} = 66.6$
T-XIII-1	$\frac{R}{C} = \frac{25}{.025} = 1000 \quad \therefore S = \left(\frac{Zn}{P}\right)\left(\frac{R}{C}\right)^2 \times 10^{-9}$ $= 66.6 \times 10^6 \times 10^{-9} = .0666$
	$\frac{L}{d} = \frac{100}{50} = 2$
F-13.3	$\frac{R}{C}\mu = 1.8 \quad \therefore \mu = 1.8 \times 10^{-3} = .0018, \quad V = \pi \times \frac{50}{1000} \times \frac{400}{60} = 1.406\ m/sec.$
T-XIII-1	$\therefore Hg = \mu W V = .0018 \times 25 \times 10^3 \times 1.046 = 47\ W$
	In new operating conditions
	New $V' = \pi \times \frac{50}{1000} \times \frac{500}{60} = 1.308\ m/sec$
	If the same temp is to obtained i.e. (Hg) should be constant

	$\therefore \mu' W V' = 47 \quad ; \quad \mu' = \frac{47}{25 \times 10^3 \times 1.308} = 1.44 \times 10^{-3}$
	New $\frac{R}{C} = \frac{25}{.030} = 833.3$
	New Friction variable $\frac{R}{C}\mu = 833.3 \times 1.44 \times 10^{-3} = 1.199$
F-13.3	For this value of $\left(\frac{R}{C}\mu\right)$, $S = .02 \quad ; \quad S = .02 = \left(\frac{Zn}{P}\right)\left(\frac{R}{C}\right)^2 \times 10^{-9}$
	$\frac{Zn}{P} = \frac{.02 \times 10^9}{833.3^2} = 28.8 \ ; \quad Z = \frac{28.8 \times 5 \times 60}{500} = 17.3\ mPa - s$

Prob. 13.3 : A 75 mm dia. full journal bearing supports a radial load of 3.6 kN. The bearing is 75 mm long and the shaft operates at 400 rpm. The permissible minimum oil film thickness is 0.02 mm and the diametric clearance is 0.1 mm. Suggest suitable viscosity of the lubricating oil for the bearing. Calculate the coefficient of friction, the heat generated and the amount of oil which should be supplied to the bearing.

Ref.	Calculations
	Given : $W = 3.6$ kN, $d = 75$ mm, $L = 75$ mm, $N = 400$ rpm, $C_d = 0.1$ mm, $h_m = .02$ mm
F-13.5	$C = \frac{C_d}{2} = \frac{0.1}{2} = .05\ mm \quad \therefore \frac{h_m}{C} = \frac{.02}{.05} = .4, \ \frac{L}{d} = 1$
	For this value of $\frac{h_m}{C}$, $S = .105$
	$S = .105, \qquad P = \frac{3600}{75 \times 75} = .64\ MPa, \quad n = \frac{400}{60}, \ \frac{R}{C} = \frac{37.5}{.05} = 750$
	$S = \left(\frac{Zn}{P}\right)\left(\frac{R}{C}\right)^2 \times 10^{-9} = \frac{Z \times 400}{60 \times .64}(750)^2 \times 10^{-9} = .58 \times 10^{-2} Z = .105$
	$\therefore Z = \frac{.105 \times 100}{.58} = 18.1 mPa - s$
F-13.3	$\frac{R}{C}\mu = 2.9 \quad \therefore \mu = \frac{2.9}{750} = .004$
	$V = \pi \times \frac{75}{1000} \times \frac{400}{60} = 1.57\ m/sec.$
T-XIII-1	$\therefore Hg = \mu W V = .004 \times 3600 \times 1.57 = 226\ W$

F-13.6	$\frac{Q}{RCnL} = 4.1$ $\quad \therefore Q = 4.1 \times 37.5 \times .05 \times \frac{400}{60} \times 75$ $= 3843.75$ mm^3/sec.
F-13.7	$\frac{Q_s}{Q} = .71$ $\quad \therefore Q_s = .71 \times 3843.75 = 2729$ mm^3/sec.

Prob. 13.4 : A journal bearing has journal diameter of 25 mm and the length to diameter ratio of one. It is used to support a turbine rotor that rotates at 12000 rpm. The lubricating oil is SAE 10 and the average film temp is 65° C. The minimum oil film thickness can be as low as 0.0075 mm.

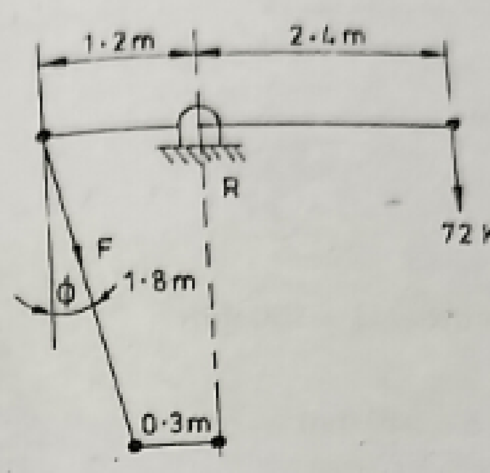
(a) What diametric clearance will give the bearing its greatest load carrying capacity which occurs for a value of 0.52 for the minimum film thickness ratio $\left(\frac{hm}{C}\right)$?

(b) Using this clearance what maximum load can be carried ?

(c) With this load and clearance what would be the power loss due to friction ?

Ref.	Calculations
	Given : d = 50 mm, l = 50 mm, N = 12000, $t_{oil} = 65°C$, Oil SAE 10, $h_m = .0075$ mm
F-13.2	$Z = 10$ mPa-s, $n = \frac{12000}{60} = 200$ rps
F-13.5	For $\frac{hm}{C} = 0.52$, $S = 0.19$ $\therefore C = \frac{.0075}{0.52} = .0144$ mm, $C_d = 2\,C = .0288$ mm For $S = .19 = \left(\frac{Zn}{P}\right)\left(\frac{R}{C}\right)^2 \times 10^{-9}$, $\frac{R}{C} = \frac{12.5}{.0144} = 866.66$ $\therefore \frac{Zn}{P} = \frac{.19 \times 10^9}{866.66^2} = 252$ $\quad \therefore P = \frac{10 \times 200}{252} = 7.9$ MPa $\therefore$ Load $W = 7.9 \times 25 \times 25 = 4960$ N $= 4.96$ kN
F-13.3	$\frac{R}{C}\mu = 4.2$ $\therefore \mu = \frac{4.2}{866.66} = .0048$ $\quad V = \pi \times \frac{25}{1000} \times 200 = 15.7$ m/sec.
T-XIII-1	$Hg = \mu W V = .0048 \times 4960 \times 15.7 = 374$ W

Prob. 13.5 : A walking beam is 3.6 m long and the bearing fulcrum is 1.2 m from one end and 2.4 m from the other. The long end pulls a vertical load of 72 kN. The short end is connected by a 1.8 m rod to a crank pin with a 300 mm radius of rotation. The diameter of the fulcrum pin is 150 mm. Assuming that thin film lubrication exists, determine (a) the length of the fulcrum pin necessary to support the load and (b) the work lost in friction when the crank pin turns at 30 rpm.

Ref.	Calculations
	 Fig. Force F acting on the short end : $F \cos\varphi \times 1.2 = 72 \times 2.4$ $\therefore F \cos\varphi = 144$ kN For φ : $1.8 \sin\varphi = (1.2 - 0.3)$ $\varphi = 30°$ $\therefore F = \frac{144}{\cos 30} = 166.28$ kN $\therefore$ Reaction R at the fulcrum pin $R = \sqrt{(72 + F\cos\varphi)^2 + (F\sin\varphi)^2}$ $= \sqrt{(72+144)^2 + 6912.26}$ $= 231.45$ kN
T-XIII-4	Assuming bearing pressure of 60 MPa as recommended for crank pin of punching and shearing m/c, $\frac{1}{d} = 1.0$ to 2.5 $\cdot 1 \times d \times P_b = 231.45 \times 10^3$ $l \times 150 \times 60 = 231.45 \times 10^3$ $\quad \therefore l = 25.7$ mm $\frac{l}{d} = 1$ gives $l = 150$ mm so select length of fulcrum pin = 150 mm. $\therefore P = \frac{231.45 \times 10^3}{150 \times 150} = 10.2$ MPa $\quad V = \pi \times \frac{150}{1000} \times \frac{30}{60} = .235$ m/sec
T-XIII-1	Coefficient of friction $\mu = \frac{C_1 C_2}{306}\sqrt{\frac{P}{V}}$ $\quad C_1 = 2$ for ordinary conditions $C_2 = 1$ for crank pins
T-XIII-2	$= \frac{2 \times 1}{306}\sqrt{\frac{10.2}{.2355}} = .043$
T-XIII-3	$Hg = \mu W V = .043 \times 231.45 \times 10^3 \times .2355 = 2344.5$ $W = 2.34$ kW

Prob. 13.6 : Determine the main dimensions of multicolar thrust bearing for a propeller shaft of a 450 kW marine oil engine. The engine makes 220 rpm, the shaft diameter is 150 mm. The ship speed is 400 m/min. Determine the power lost in friction.

Ref. | **Calculations**

Let the axial thrust be F_a, N

$$\frac{400}{60} = 450 \times 10^3$$

$$\therefore F_a = 67.5 \times 10^3 \text{ N}$$

Design of thrust bearing

T-XIII-9 Shaft dia = 150 mm. Assume dia. for collar $D_i = 160$ mm

$\frac{Do}{Di} = 1.5$ $\quad \therefore Do = 160 \times 1.5 = 240$ mm

$$\therefore Dm = \frac{240 + 160}{2} = 200 \text{ mm}$$

$$\therefore Vm = \pi \times \frac{200}{1000} \times \frac{220}{60} = 2.3 \text{ m/sec.}$$

P = 0.7 MPa for watercooled multicollar thrust bearing

$$\therefore \text{Load capacity/collar} = \frac{\pi}{4}(240^2 - 160^2) \times 0.8 \times 0.7 = 14067 \text{ N}$$

$$= 14.067 \text{ kN}$$

$$\text{No. of collars} = \frac{67.5 \times 10^3}{14.067 \times 10^3} = 4.8 \text{ say 5 collars}$$

$$\text{Supporting area/collar} = \frac{\pi}{4}\left(240^2 - 160^2\right) \times 0.8 = 20 \times 10^3 \text{ mm}^2$$

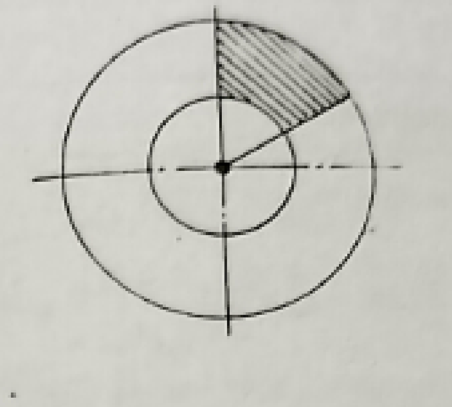
$$\therefore \text{Actual pressure} = \frac{67.5 \times 10^3}{5 \times 20 \times 10^3} = .675 \text{ MPa}$$

$$\therefore \mu = \frac{.008\sqrt{Vm}}{(P)^{0.66}} = \frac{.008\sqrt{2.3}}{(.675)^{0.66}} = .0155$$

$$P_f = \mu \,.\, W \,.\, V_m = .0155 \times 67.5 \times 10^3 \times 2.3 = 2406 \text{ W or } 2.4 \text{ kW.}$$

Prob. 13.7 : The runner of a vertical thrust bearing is supported by four pivoted shoes, each subtending an angle of 80° at the centre. The bearing has an O.D. of 300 mm and I.D. of 150 mm. The expected mean viscosity of the lubricant is 40-mPa-s at the operating temp. Find (a) the speed at which this bearing must operate in order to support an axial load of 45 kN if the permissible maximum oil film thickness is 0.055 mm (b) coefficient of friction and (c) power loss in the bearing.

Ref. | **Calculations**



Shoe dimensions : OD = 300 mm, ID = 150 mm,
$D_m = 225$ mm, $r_m = 112.5$ mm

Subtended angle = 80°, h = 0.055 mm, W = 45 kN,
No. of shoes = 4, Z = 40 mPa-s

Fig.

T-XIII-11 Load carrying capacity

$$W = \frac{6\,ZULB^2}{h_2^2} \times 10^{-6} \times K_w \times \eta_w$$

$$B = \text{Length in direction of motion} = r_m \,.\, \theta = 112.5 \times \frac{80}{180} \times \pi = 157 \text{ mm}$$

L = Length in direction perpendicular to motion = 150 − 75 = 75 mm.

$$\frac{B}{L} = \frac{157}{75} = 2.1$$

F-13.9 For minimum friction i.e. minimum K_f, m = 1.4, K_f = 4.4

$\therefore$ at m = 1.4, K_w = .026

$$\text{at } \frac{B}{L} = 2.1, \;\; \eta_f = 0.2, \;\; \eta_w = .19 \;\; ; \;\; \text{Load/shoe} = \frac{45 \times 10^3}{4} = 11.25 \times 10^3$$

$$\therefore 11.25 \times 10^3 = \frac{6 \times 40 \times U \times 75 \times 157^2}{.055^2} \times 10^{-6} \times .026 \times .19$$

$$\therefore U = \frac{11.25 \times .005^2 \times 10^9}{240 \times 75 \times 157^2 \times .026 \times .19} = 15.5 \text{ m/sec.}$$

$$U = \pi \,.\, D_m \,.\, n = 15.5 \qquad \therefore n = \frac{15.5 \times 10^3}{\pi \times 225} = 22 \text{ rps}$$

$$\therefore N = 60 \times 22 = 1320 \text{ rpm}$$

T-XIII-11 Coefficient of friction, $\mu = \frac{h_2}{n_f B} K_f = \frac{.055 \times 4.4}{157 \times 0.2} = .0077$

Power loss in the bearing μ W U = .0077 × 45 × 10^3 × 15.5 = 5.37 kW

Prob. 13.8 : A slider bearing with a rectangular pivoted shoe has the following specifications : Length = 75 mm, Width = 112.5 mm, Velocity of shoe = 2 m/sec.

Viscosity at the operating temp. = 28 mPa-s, Min. film thickness = .025 mm. Assume that the inclination of the bearing surfaces corresponds to max. load carrying capacity. Determine (a) Load carrying capacity and (b) power loss in bearing.

Ref.	Calculations
F-13.9	For max. Kw, m = 1.2 and $K_f = 4.25$, L = 75 mm, B = 112.5, $K_w = .0265$
	$\frac{B}{L} = \frac{112.5}{75} = 1.5 \qquad \eta_w = .29, \quad \eta_f = .31$
T-XIII-9	$W = \frac{6\,ZULB^2}{h_2^2} \times 10^{-6} \times K_w \times \eta_w$
	$\mu = \frac{h_2 K_f}{\eta_f B} = \frac{.025 \times 4.25}{.31 \times 112.5} = .003$
	$\therefore$ Power loss in bearing = μ W U = .003 × 3922 × 2 = 23.9 w

Prob. 13.9 : A hydrostatic thrust bearing has shaft dia. of 100 mm and recess dia. of 35 mm. The axial load on the shaft is 22.5 kN. The speed of the shaft is 85 rpm. The desirable film thickness is 0.1 mm. The mean operating temp. is 40°C and the viscosity at this temp. is 105 mPa-s. Determine the required inlet pressure, the oil flow and power loss in the bearing.

Ref.	Calculations
	Given : W = 22.5 kN, Do = 100 mm, Di = 35 mm, N = 85 rpm, Z = 105 mPa-s = .105 Pa-s, h = 0.1 mm
T-XIII-12	Inlet pressure $P_p = \frac{2W \ln\left(\frac{Ro}{Ri}\right)}{\pi\left(Ro^2 - Ri^2\right)}$; $Ro = \frac{100}{2} = 50$ mm
	$Ri = \frac{35}{2} = 17.5$ mm
	$= \frac{2 \times 22.5 \times 10^3 \times \ln\left(\frac{50}{17.5}\right)}{\pi\left(50^2 - 17.5^2\right)} = 6.86$ MPa
	Oil flow $Q = \frac{\pi P_p h^3 \times 10^6}{6\,Z \ln\left(\frac{Ro}{Ri}\right)} = \frac{\pi \times 6.86 \times 0.1^3 \times 10^6}{6 \times .105 \times \ln\left(\frac{50}{17.5}\right)} = 32.563\ mm^3/sec.$
	$= 0.032 \times 10^{-3}\ m^3/sec.$

Ref.	Calculations
	Powerloss due to friction :
	$P_f = \frac{\pi\,ZV}{2h}\left(Ro^2 - Ri^2\right) \times 10^{-3}$ W, $\qquad \dot{V} = \pi \times \left(\frac{100+35}{2}\right) \times \frac{85}{60} \times 10^{-3}$ = 0.3 m/sec.
	$P_f = \frac{\pi \times .105 \times 0.3}{2 \times 0.1}\left(50^2 - 17.5^2\right) \times 10^{-3} = 1.085$ W
	Power loss due to puming :
	$P_{pump} = Q\,P_p = .0326 \times 10^{-3} \times 6.86 \times 10^6$ = 223.6 W
	$\therefore$ Total power loss = 1.085 + 223.6 = 224.685 W.

* * *

Chapter 14

BEARINGS WITH ROLLING CONTACT

14.1 Introduction :

Rolling element bearings involve separation of the shaft and outer member by balls or rollers and thus substitutes rolling friction for sliding friction. Since the contact areas are small and the stresses high, the loaded parts of rolling element bearings are normally made of hard, high strength materials, superior to those of the shaft and outer member. These parts include inner and outer rings (also called races) and the balls or rollers. An additional component of the bearing is usually a retainer or separator, which keeps the balls or rollers separated.

Both sliding and rolling element bearings have their place in modern machinery. A major advantage of rolling element bearings is low starting friction. Sliding bearings can achieve comparably low friction only after the full film is established which is not there while starting. Rolling bearings are ideally suited for applications involving high starting loads. On the other hand fluid film bearings are well suited for high rotating speeds with impact and momentary overloads. The higher the rotating speed the more effective the hydrodynamic pumping action. Also the fluid film effectively cushions impact. High rotating speeds are generally disadvantageous to rolling bearings because of the rapid accumulation of fatigue cycles and the high centrifugal force on the rolling elements.

Rolling element bearings take up more radial space around the shaft, but fluid film bearings require greater axial space. Rolling element bearings generate and transmit a certain amount of noise where as fluid film bearings do not normally generate noise and may dampen noise from other sources. Sliding bearings are cheaper than rolling element bearings for simple applications where minimum lubrication provision is required. Another advantage of rolling element bearings is that they can be preloaded. This is important in applications requiring precise positioning of the rotating member. The rolling element bearings are also called antifriction bearings which is a misnomer because these bearings do not always provide a lower friction than fluid film bearings. The typical values for coefficient of friction are given in T-XIII-16 of data book.

14.2 Types of Rolling Element Bearings :

Based on the shape of the rolling element they are classifed in two types.

1. Ball bearings which are preferred for higher speed.
2. Roller bearings which are preferred for greater loads.

Both types are further subdivided into three based on the type of load which they can support.

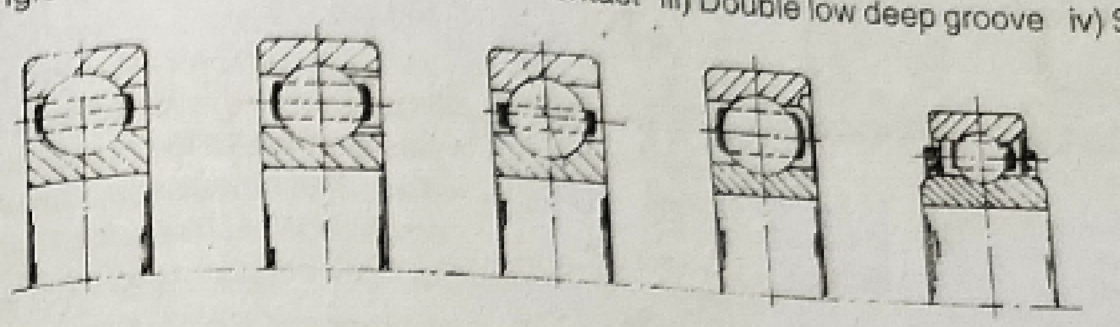
1. Radial load
2. Axial or Thrust loads.
3. Cominbed Radial and axial loads or angular contact bearings.

14.2.1 Ball bearings :

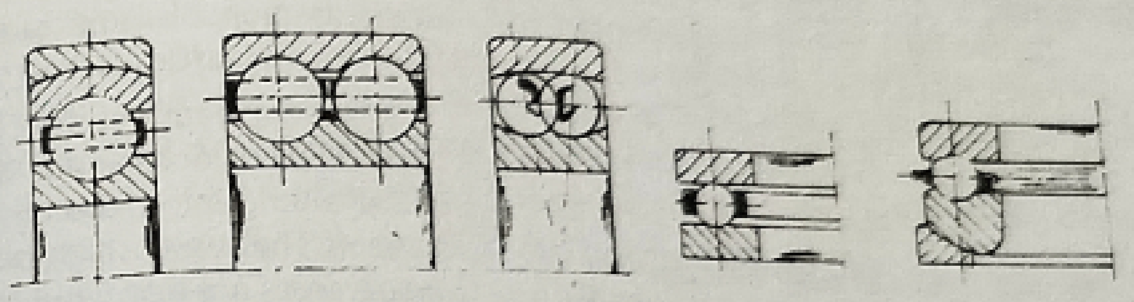
There are two types of ball bearings :

(A) Radial load ball bearings which can be take a small amount of thrust. They are predominantly of four types :

i) Single row deep grovve ii) Angular contact iii) Double low deep groove iv) Self aligning



(a) Deep grovve (b) Filling notch (c) Angular contact (d) Shielded (e) Sealed



(f) External self aligning (g) Double row (h) Self aligning (i) Thrust (j) Self aligning thrust

Fig. 14.1 Various types of ball bearings

These four types are shown in Fig. 14.1 Each bearing consist of four elements :

1) Inner race grooved on its outer surface, 2) Outer race grooved on its inner surface, 3) Balls 4) Retainer or cage for spacing the balls so that they do not touch each other.

These components are shown in Fig. 14.2.

(B) Thrust ball bearings :

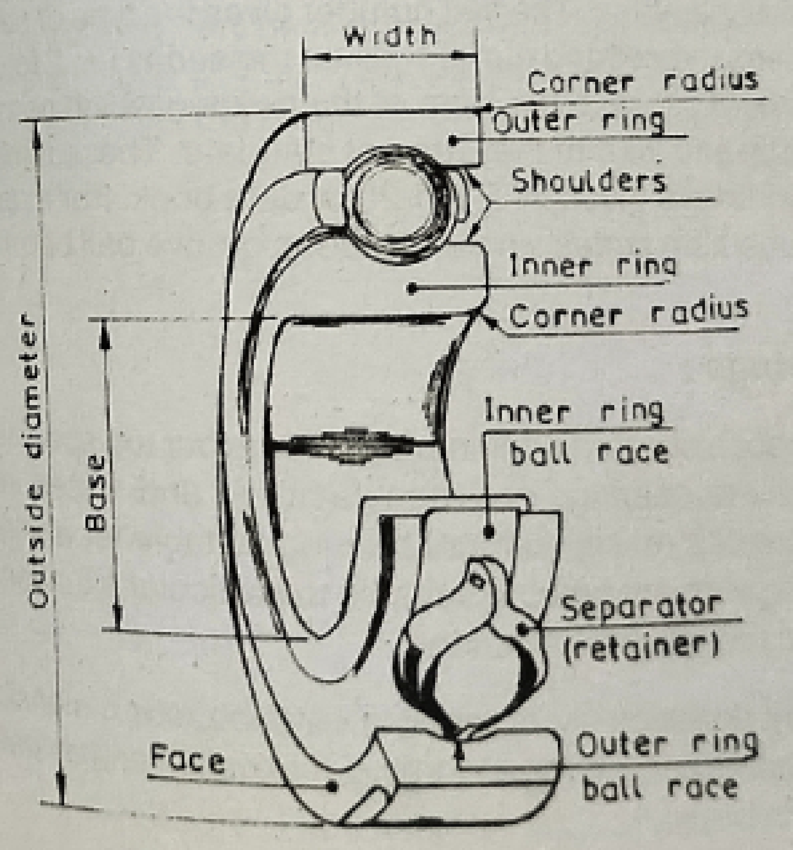


Fig. 14.2 Nomenclatur of a ball bearing

These bearings are designed to carry only axial loads. They are of three types as shown in Fig. 14.1 (i) & (j).

1. One direction thrust bearing with parallel seats.
2. Self aligning one direction thrust bearing.
3. Two direction thrust bearing.

14.2.2 Roller Bearings :

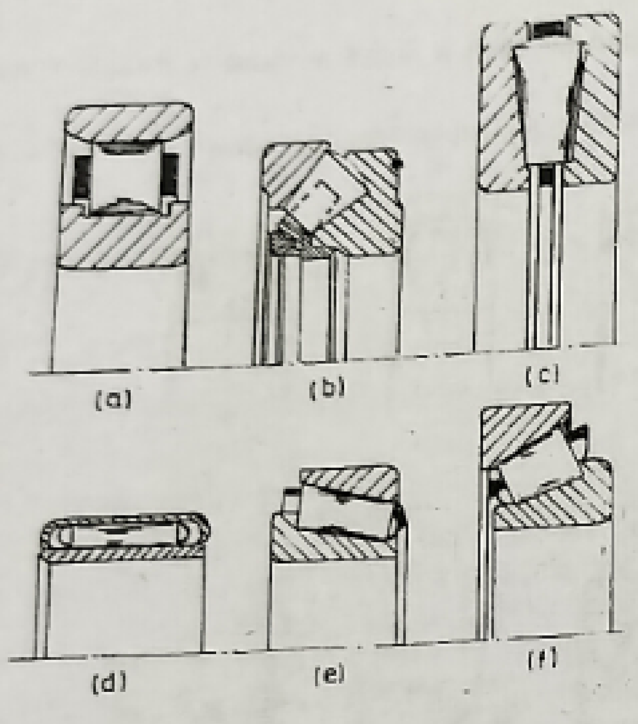


Fig. 14.3 Types of roller bearings -
(a) straight roller, (b) spherical roller thrust;
(c) tapered roller thrust, (d) needle, (e) tapered roller,
(f) steepangle tapered roller.

Based on the shape of the rollers they are of four types as shown in Fig. 14.3.

(a) Straight or cylindrical roller bearings which are meant for pure radial loads Fig. (a).

(b) Spherical roller bearings are used where misalignment is likely to occur. They can take large radial loads plus very small thrust load. The spherical elements have the advantage of increasing their contact area as the load is increased, Fig. (b).

(c) Tapered Roller bearings combine the advantages of ball and straight roller bearings since they can take any combination of radial and thrust loads Fig. (c), (e) & (f).

(d) Needle bearings are special case of cylindrical roller bearings where the rollers have a length to diameter ratio of four or greater. They have a high load capacity when separators are used, but may be obtained without separators. They are available both with and without races. Fig. (d)

In addition we have spherical roller and tapered roller thrust bearings. The bearings described above represent only a small portion of the many available for selection. Many special purpose bearings are also made for special classes of machinery.

14.3 Standardization :

The manufacturers of radial ball bearings have adopted the international standard dimensions according to which the bearings are mainly divided into three series called the *light*, *medium* and *heavy* series. All bearings are designated by (XXXX) four numbers. The first number gives the type of bearing i.e. single row deep groove or angular contact etc. the second number gives the series i.e. 2 for light, 3 for medium and 4 for heavy series, the last two digits give the bore of the bearing which comes in multiples of 5 mm above 20 mm. Between 10 mm and 20 mm they are not standard. These numbers and corresponding size namely bore, O.D. & width are given in T-XIII-20 of data book. For example 6308 bearing number of SKF company is for a medium series single row deep groove ball bearing of $8 \times 5 = 40$ mm bore.

14.4 Design of Rolling Element Bearings :

The detail design of these bearings is a sophisticated engineering endeavour too specialized to warrant extensive treatment here. Further these bearings are manufactured and tested as an assembled unit which precludes the manufacture of a rolling element bearing suitable for a specific situation. Hence, the selection and use of a rolling element bearing suitable to particular situation is a concern to all engineers and will be discussed in the following sections.

Bearing manufacturers catalogs identify bearings by number, give complete dimensional information, list rated load capacities and furnish details concerning mounting, lubrication and operation

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14.5 Load Capacity :

The load capacity of bearings is of two types :

(a) Static capacity and (b) Dynamic capacity

(a) Static Capacity : The static load is defined as a load acting on a non-rotating bearing. Permanent deformation appear in balls and race ways under static load of moderate magnitude and increase gradually with increasing load. The permissible static load is therefore dependent upon the permissible magnitude of permanent deformation. Experience shows that a total permanent deformation of 0.0001 of the ball diameter occuring at the most heavily loaded ball and race contact, can be tolerated in most bearing applications without impairment of bearing operation. The basic static load rating is given in manufacturers catalog. As these load ratings are for vary slow speed of rotation, they are not given in the tables of data book.

(b) Dynamic Load : The dynamic capacity of a bearing is based on the fatigue life of the material, constrasted with the static capacity which is based on permanent deformation or brinelling. Under ideal conditions the fatigue failure will consist of spalling of the load carrying or surfaces. As the individual lif of a bearing varies these bearings are tested in a group and the basic load rating or specific dynamic capacity is established. The AFBMA has established a standard load rating for bearings in which speed is not a consideration. The basic load rating or *specific dynamic capacity*, C is defined as the constant radial load which 90 percent of a group of identical bearings can endure for a rating life of one million revolutions of the inner ring (Stationary load and stationary outer ring). This life is called L_{10} life while the average life is that life which 50 percent of the bearings can endure. Generally it is about five times the L_{10} life. The use of the specific dynamic capacity and bearing life for selection of bearings will be illustrated in the solved problems. The effect of reliability on the design is taken care of by the factor K_{rel} which are tabulated in T-XIII-15 of data book. For other reliabilities above 90 percent we can use the relationship :

$$K_{rel} = 6.84\left[\ln\left(\frac{1}{R}\right)\right]^{1.17}$$

14.6 Design Procedure :

The design consists of selecting the proper type and size of a bearing which can be fitted on the given shaft and will endure the given load for the specified life with the specified reliability. The bearing may be subjected to a pure radial, pure axial or a combined load which may be constant in magnitude or vary in a cyclic manner. For selecting a bearing subjected to a variable cyclic load, we first calculate the mean load as per T-XIII-13 with the help of formulae :

$$F_m = \left[\frac{F_1^3\,t_1 + F_2^3\,t_2 + \text{-------} + F_n^3\,t_n}{\Sigma t}\right]^{\frac{1}{3}} \qquad \text{OR}$$

$$F_m = \left[\frac{F_1^3 N_1 + F_2^3 N_2 + \text{---------} + F_n^3 N_n}{\Sigma N}\right]^{\frac{1}{3}}$$

where F_1, F_2 etc. are loads acting for t_1, t_2, units of time or N_1, N_2 revolutions.

Having calculated the mean loads which can be treated as constant load the procedure is as follows :

Step 1 : Calculate the equivalent radial load as per T-XIII-13.

Step 2 : If the life of the bearing is specified, calculate the required specific dynamic capacity C, of the bearing as per T-XIII-13. If the life of the bearing is not specified, select life of the bearing as per T-XIII-17 and then calculate C.

Step 3 : When the shaft size is specified select the proper series of bearing which has a C greater than the calculated C, as per T-XIII-21.

The procedure will be illustrated with the help of solved examples.

14.7 Needle Bearings :

The use of needle bearings is rapidly increasing because of their comparatively small radial dimensions and their exceptionally high load capacity, particularly at low peripheral speeds. However, the coefficient of friction in needle bearings is three to four times that of deep groove ball or cylindrical roller bearings as shown in T-XIII-16 of data book. Generally the needle bearings are used without races. The design procedure for these bearings is as follows :

Step 1 : Select the needle diameter as per T-XIII-19. Calculate the diameter of the revolving race.

Step 2 : Calculate the allowable pressure as per T-XIII-18.

Step 3 : Calculate hardness factor K_h and life load factor K_l as per T-XIII-18.

Step 4 : From the required load capacity, calculate the length of the needles as per T-XIII-18. Select the standard length available as per T-XIII-19.

The procedure will be illustrated with the help of solved problems.

SOLVED PROBLEMS

Prob. 14.1 : A 6306 single row deep groove ball bearing with the inner ring rotating has a 10 seconds work cycle as given below :

Item	For 2 Secs.	For 8 Secs.
Radial load F_r	3600 N	2700 N
Axial load F_a	1800 N	0 N
Speed rpm	900	1200
Type of load	Light shock	Steady

Find the expected average life of the bearing in hours of operation.

Ref.	Calculations
	Assuming a cycle time of 1 minute. As the speed is varying we will have to calculate the number of revolutions for each load.
	$N_1 = \frac{900}{60} \times 2 \times \frac{60}{10} = 180$ rev.
	$N_2 = \frac{1200}{60} \times 8 \times \frac{60}{10} = 960$ rev.
	$\therefore\ \Sigma N = 180 + 960 = 1140$ rev. in 1 minute. To find the mean loads.
T-XIII-13	Mean radial load, $F_{rm} = \left[\frac{(F_{r1} \times K_{s1})^3 N_1 + (F_{r2} \times K_{s2})^3 N_2}{\Sigma N}\right]^{\frac{1}{3}}$
	(It is necessary to include the service factor Ks in this step because the type of load is also different in each part of cycle. If the type of load remains same during the whole cycle, we can consider it while calculating the equivalent load).
T-XIII-15	$F_{rm} = \left[\frac{(3.16 \times 1.5)^3 \times 180 + (2.7 \times 1)^3 \times 960}{1140}\right]^{\frac{1}{3}} = 3.456$ kN
	Mean axial load, $F_{am} = \left[\frac{(Fa_1 \times Ks_1)^3 N_1 + (Fa_2 \times Ks_2)^3 N_2}{\Sigma N}\right]^{\frac{1}{3}}$
	$= \left[\frac{(1.8 \times 1.5)^3 \times 180 + 0}{1140}\right]^{\frac{1}{3}} = 1.46$ kN
	To find equivalent load Fe.
T-XIII-14	$Fe = [X\,F_{rm} + Y\,F_{am}]\,K_o\,K_p\,K_r$
T-XIII-15	$\frac{F_{am}}{F_{rm}} = \frac{1.46}{3.456} = .422$ which is greater than e for deep groove ball bearing.
	$\therefore\ X = .56,\ Y = 1.6,\ K_o = 1,\ K_r = 1$
	$\therefore\ Fe = [.56 \times 3.456 + 1.6 \times 1.46] = 4.2$ kN
T-XIII-21	C for a 6306 bearing = 22 kN
T-XIII-13	$L = \left(\frac{C}{Fe}\right)^3 \times K_{rel}$ $\quad K_{rel} = 5$ for average life
	$= \left(\frac{22}{4.27}\right)^3 \times 5 = 683.84$ millions of rev.
	$\therefore$ Life of hours $L_h = \frac{683.84 \times 10^6}{1140 \times 60} = 9997$ hrs.

Prob. 14.2 : A cross section through an industrial car and the loading for one axle are shown schematically in the figure. Based on the loading conditions of the left bearing find the expected average life a 6310 bearing at a speed of 300 rpm. Assume that the bearing is subjected to a moderate shock loading.

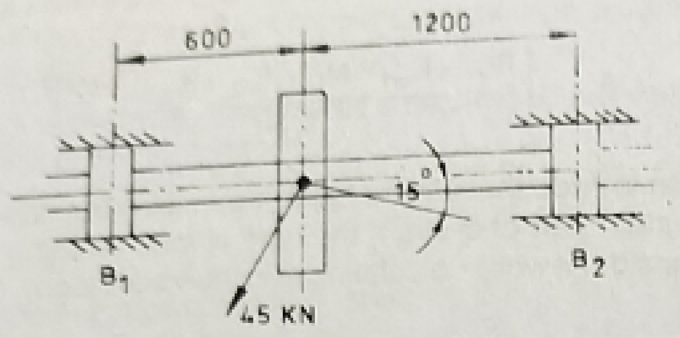


Fig.

Ref.	Calculations
	Resolving the load into two components :
	1. Along the shaft or axial load,
	$Fa = 45 \sin 15° = 1.165$ kN
	2. Perpendicular to the shaft or radial load,
	$Fr = 4.5 \cos 15° = 4.35$ kN
	The radial load will produce reactions at the two bearings B_1 and B_2 "
	$\therefore R_1 = \frac{4.35 \times 1200}{1800} = 2.9$ kN
	$R_2 = \frac{4.35 \times 600}{1800} = 1.45$ kN
	Now the axial load will be taken by the left bearing B_1. Hence B_1 is the most heavily loaded bearing. Therefore we will find the life of B_1. For this bearing the loads are ;
	$Fr = R_1 = 2.9$ kN, $Fa = 1.165$ kN
	$\therefore \frac{Fa}{Fr} = \frac{1.165}{2.9} = 0.4$
T-XIII-13	$Fe = [X\,Fr + Y\,Fa]\,Ks\,.\,Kp\,.\,Kr$
T-XIII-15	$Ko = Kp = 1$, $Ks = 2$, $Kr = 1.5$ (Outer race rotating)
T-XIII-14	For $\frac{Fa}{Fr} > e$ i.e. $> .25$, $X = .56$ and $Y = 1.6$
	$\therefore Fe = [.56 \times 2.9 + 1.6 \times 1.165] \times 2 \times 1.5 = 10.464$ kN
T-XIII-21	For 310 bearing, $C = 48$ kN, $K_{rel} = 5$ for average life
	$\therefore L = \left(\frac{C}{Fe}\right)^3 \times K_{rel} = \left(\frac{48}{10.464}\right)^3 \times 5 = 482.61$ millions
	$\therefore$ Life in hours $= L_h = \frac{482.61 \times 10^6}{300 \times 60} = 26811.66$ hrs.

Prob. 14.3 : A bearing supporting a 1000 rpm shaft carries a 3 kN radial load and a 1 kN thrust load. The loading can be assumed as moderate shock load. The required life is 5000 hr with only 2 percent probability of failure. Select a 300 series ball bearing for this application.

Ref.	Calculations
	$Fr = 3$ kN, $Fa = 1$ kN, $N = 1000$ rpm
	Life in revolutions $= 1000 \times 60 \times 5000 = 300$ millions :
	$R = (1 - .02) = .98$
	$K_{rel} = 6.84 \left[\ln \frac{1}{R}\right]^{1.17} = 6.84 \left[\ln \left(\frac{1}{.98}\right)\right]^{1.17} = .071$
T-XIII-13	$Fe = [X\,Fr + Y\,Fa]\,Ks\,.\,Ko\,.\,Kp\,.\,Kr$
T-XIII-15	$Ks = 2$, $Ko = Kp = Kr = 1$
T-XIII-14	$\frac{Fa}{Fr} = \frac{1}{3} = .333 > c$ so $X = .56$ and $Y = 1.6$
	$\therefore Fe = [.56 \times 3 + 1.6 \times 1]\,2 = 6.56$ kN ; $L = \left(\frac{C}{Fe}\right)^3 \times K_{rel}$
	$\therefore C = \left(\frac{L}{K_{rel}}\right)^{\frac{1}{3}} \times Fe = \left(\frac{300}{.071}\right)^{\frac{1}{3}} \times 6.56 = 105.75$ kN
T-XIII-21	For 317 bearing $C = 104$ kN, 318 bearing $C = 112$ kN
	So a 318 bearing should be used.

Prob. 14.4 : Design the wrist pin and the needle bearing for a 250 mm oil engine piston. The maximum pressure is 5 MPa and the engine speed is 300 rpm. Give all dimensions with the necessary tolerance.

Ref.	Calculations
	Load capacity of the bearing $F = \frac{\pi}{4} \times 250^2 \times 5 = 245.3$ kN. The wrist pin diameter is generally greater than 0.33 times the cylinder bore.
	$d > .33 \times 250 > 82.5$ mm. Assume $d = 85$ mm.
T-XIII-19	For shaft diameters 75 – 125 mm, needle dia. = 4 mm.
T-XIII-18	Allowable pressure in oscillating bearings = 35 MPa
	Assuming a desirable life of 10,000 hrs
	Life of factor $K_1 = \frac{14}{\sqrt[3]{10{,}000}} = .65$

Hardness factor $K_h = 0.58$ for $Rc = 60$

$$F = K_h \times K_l \times P \times L \times D$$

$$\therefore L = \frac{F}{K_h \times K_1 \times P \times D} = \frac{245.3 \times 10^3}{0.65 \times .58 \times 35 \times 85}$$

$$= 129.4 = 130 \text{ mm.}$$

So use 4 mm × 130 mm needles on 85 mm dia. wrist pin.

Diameter of revolving race $= 85 + 2 \times 4 = 93$ mm.

Radial clearance $= .05$ mm.

* * *

Chapter 15

ROTORS & FLYWHEELS

15.1 Introduction :

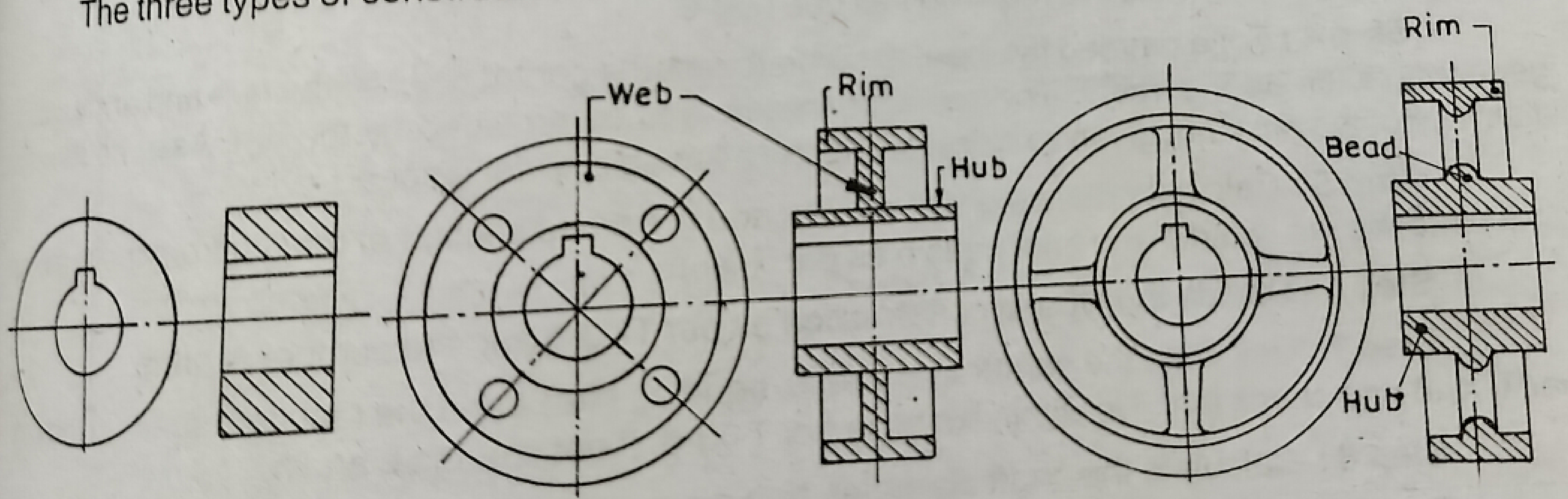
Any member which rotates with the shaft and transmits power or receives power is called a rotor. Generally the rotors are keyed to the shafts. The examples of rotors are gears, pulleys, sprockets, flywheels etc. Every rotor consists of three parts :

1. Hub or boss which receives/delivers power from/to the shaft through the key.
2. Rim which receives/delivers power from/to the hub through the connecting member.
3. Member connecting hub to rim. When the difference between hub diameter and rim diameter is small the connecting member cannot be seen separately. This type of construction is called *solid construction.* When the difference is of medium order, *web* is used to connect. For still larger difference, *arms* are used to connect. As a normal rule rotors have the following types of construction.

Dia. < 150 mm -- Web construction

Dia. > 150 mm -- Arm construction

The three types of construction are shown in Fig. 15.1



(a) Solid construction (b) Web construction (c) Arm construction

Fig. 15.1

The design of gears, pulleys and sprockets is given along with the design of the particular type of drive. In this chapter we will only discuss the design of flywheels.

15.2 Flywheels :

The purpose of a flywheel is to keep the variation of the speed in a given cycle within limits, while the machine is receiving energy or delivering energy at a variable rate. A flywheel stores up energy by increase in speed and gives up energy by decrease in speed. The allowable fluctuation in energy by increase in speed depends on the condition of the problem. The variation in the energy stored in a flywheel and variation in speed are given by the relationship.

$$\Delta E = \frac{1}{2} I\left(\omega_{max}^2 - \omega_{min}^2\right) = I K_s \, \omega_m^2$$

where ΔE = variation in the energy stored, J

ω_{max} = maximum angular velocity, rad/sec.

ω_{min} = minimum angular velocity, rad/sec.

ω_m = mean angular velocity $= \frac{\omega_{max} + \omega_{min}}{2}$

I = mass moment of inertia of the flywheel, $Kg - m^2$

K_s = Coefficient of fluctuation of speed $= \frac{\omega_{max} - \omega_{min}}{\omega_m}$

The variation in energy ΔE can be found graphically from the T-θ diagram or by analytical methods. Another method of defining the variation of energy is the term coefficient of fluctuation of energy K_E.

$K_E = \frac{\Delta E}{E}$ where E = Net energy required in one revolution or cycle.

15.3 Design of Flywheels :

Generally the flywheels are made of cast iron. However the selection of material depends on the surface speed and the power of the machine as given in T-XI-6. Having selected the material,the steps in design are as follows :

Step 1 : Calculate the fluctuation of energy, ΔE, from the given data.

Step 2 : If permissible speed fluctuation is not given, select it as per T-XI-7.

Step 3 : Calculate the required I as per T-XI-6.

Step 4 : For a rimmed flywheel assume the inertia contributed by the hub and arms which is generally taken as 5 percent. The remainig inertia is to be provided by the rim. Assume the mean diameter of the rim if not given by considering the allowable surface speed.

Step 5 : Calculate the weight of the rim and the cross section of the rim from the required I. Calculate the rim width b and thickness h as per T-XI-8.

Step 6 : Decide the type of construction as per T-XI-8 and the number of arms.

Step 7 : Calculate the centrifugal stress, bending stress and the resultant stress in the rim as per T-XI-6 and check that it is within limits as per T-XI-6. If not change b and h.

Step 8 : Calculate the shaft diameter, hub diameter and arm dimensions as per T-XI-8.

The procedure will be illustrated with the help of solved problems.

SOLVED PROBLEMS

Prob. 15.1 : Suggest suitable weight and cross section for the rim of a C.I. flywheel to be used on a 4 stroke I.C. engine developes 40 kW at 600 rpm. The total fluctuation of speed should be limited to 5 percent of the mean speed. The work done during the power storke may be assumed to be 1.4 times the work done during the whole cycle. The flywheel diameter should not be greater than 1.2m.

Ref. **Calculations**

Flywheel for 4 stroke I.C. engine

$N = 600$ rpm, $P = 40$ kW, $K_s = .05$, $D_o < 1.2$ m

Work done in power storke = 1.4 work done in cycle.

$$\omega_{mean} = \frac{2\pi \times N}{60} = \frac{2\pi \times 600}{60} = 62.8 \text{ rad/sec.}$$

$$\text{No. of cycle/sec.} = \frac{600}{60 \times 2} = 5$$

$$\text{Work done in one cycle} = \frac{40 \times 10^3}{5} = 8 \times 10^3 \, N - m \text{ or } J$$

$\therefore$ Work done in power stroke $= 1.4 \times 8 \times 10^3 = 11.2 \times 10^3$ N-m or J.

Assume the T-θ diagram as follows :

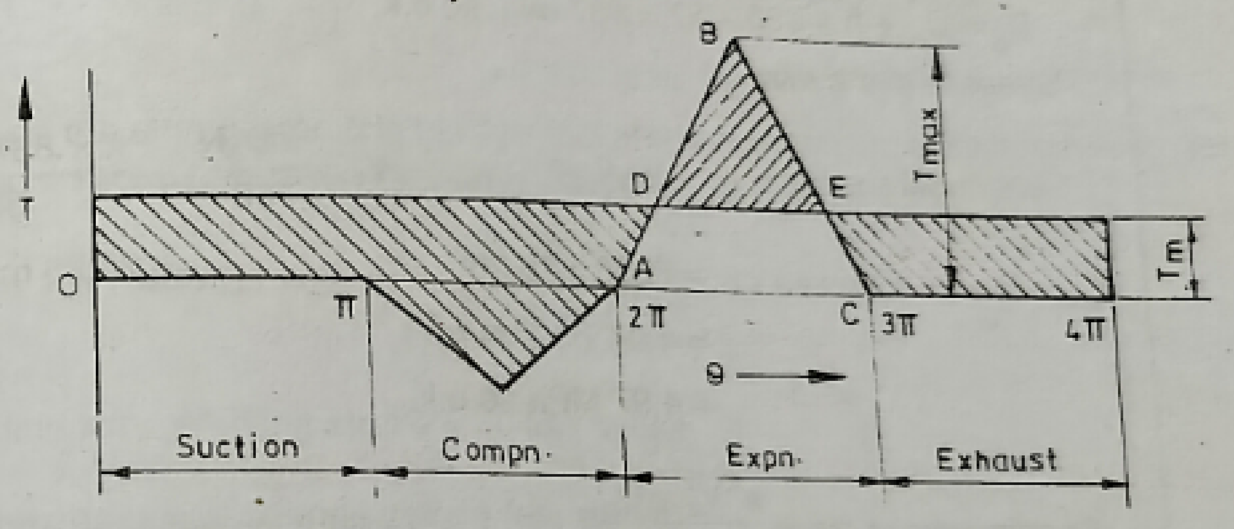


Fig.

Work done in power stroke $\Delta ABC = \frac{1}{2} T_{max} \times \pi = 11.2 \times 10^3$

$\therefore T_{max} = 7.13 \times 10^3$ N-m

Work done in one cycle $= T_m \times 4\pi = 8 \times 10^3$

$\therefore T_m = 637$ N-m

$$\therefore \text{Fluctuation of energy, } \Delta BDE = \left[\frac{T_{max} - T_m}{T_{max}}\right]^2 \times \Delta ABC$$

$$= \left[\frac{7130 - 637}{7130}\right]^2 \times 8000 = 6634.4 \, N - m \text{ or } J$$

T-XI-6 $\Delta E = I K_s \, \omega_m^2$ $\therefore I = \frac{\Delta E}{K_s \, \omega_m^2} = \frac{6634.4}{.05 \times 62.8^2} = 33.64 \text{ kg m}^2$

$V_s < 1600$ m/min $= \pi D_o N$ $\therefore D_o = \frac{1600}{\pi \times 600} = .849$ m

Assume $D_m = .8$ m $\therefore$ K 0.4 m.

Assume that the rim provides 95 percent of the required I

$\therefore M K^2 = .95 I = .95 \times 33.64$

$$\therefore M = \frac{.95 \times 33.64}{.4^2} = 199.73 \text{ kg.}$$

Let the rim cross section be A, m^2

T-XI-8 $\therefore \pi \times D_m \times A \times \rho = M$ $\rho = 7200$ kg/m^3

T-II-2

$$\therefore A = \frac{199.73}{\pi \times .8 \times 7200} = .0110 \text{ m}^2$$

If $b = 2h$, $A = 2b^2 = .0110$

$\therefore b = .074$ m or 74 mm, $h = 37$ mm

$D_o = D_m + h = 800 + 37 = 837$ mm, so o.k.

Stresses in the min.

Centrifugal stresses, $\sigma_1 = \rho V_s^2$, $V_s = \frac{\pi D_o N}{60} = \frac{\pi \times .837 \times 600}{60}$

$= 26.28$ m/sec.

$= 7200 \times 26.28^2$

$= 4.97 \times 10^6$ N/m^2

$= 4.97$ MPa so o.k.

T-XI-6 Bending stress $\sigma_2 = \frac{\pi^2 V_s^2 \rho D_o}{i^2 . h}$ $i = 6$ for Do between 400 – 3000

$$= \frac{\pi^2 \times 26.28^2 \times 7200 \times .837}{6^2 \times .037} = 30.97 \times 10^6 \text{ N/m}^2$$

$= 30.97$ MPa

$\sigma_3 = .75\,\sigma_1 + .25\,\sigma_2$

$= .75 \times 4.97 + .25 \times 30.97$

$= 11.47$ MPa so o.k.

Prob. 15.2 : Design a C.I. flywheel for a punch press which must be brought to rest by one punching operation if the power has been shut off. The maximum work required consists of punching a 28 mm hole in a 20 mm mild steel plate. The punch capacity is 24 holes per minute and the speed ratio of the driving to eccentric shaft is 9 : 1. Due to space limitations the wheel diameter cannot be larger than 1.2 m. The flywheel is keyed to the driving shaft. Determine the coefficient of fluctuation of speed for the press if the duration of the working storke is one fourth of the idle storke. Draw a neat sketch of the flywheel.

Ref. **Calculations**

To calculate mean speed :

Speed of eccentric shaft = 24 rpm

Speed of driving shaft = 24 × 9 = 216 rpm.

$$\omega_{mean} = \frac{2\pi \times 216}{60} = 22.6 \text{ rad/sec.}$$

For steel $\frac{S_{us}}{S_{ut}} = 0.65 - 0.75$ Assume $S_{us} = 0.75\, S_{ut}$

T-II-6A For SAE 1020 steel $S_{ut} = 435$ $\therefore S_{us} = .7 \times 435 = 304.5$ MPa

T-II-7

Work done in punching one hole $= \pi dt \times S_{us} \times \frac{2}{3}$ t (factor commonly used in punching)

$$= \frac{2}{3} \times \pi \times 28 \times 20 \times 304.5 \times 20$$

$$= 7.139 \times 10^6 \text{ N-mm} = 7139 \text{ N-m}$$

This is the energy stored in the flywheel at mean speed because it comes to rest in punching one hole when the power is cut off.

$\therefore \frac{1}{2} I \omega_m^2 = 7139$ $\therefore I = \frac{7139 \times 2}{22.6^2} = 27.95$ kg m^2

To find Ks : Working stroke $= \frac{1}{4}$ Idle stroke

Total stroke = Working stroke + Idle stroke

= 5 × Working stroke

$\therefore$ Working stroke $= \frac{1}{5}$ Total stroke

$$\therefore \Delta E = \frac{4}{5} \times 7139 = 5711.2 \text{ N-m}$$

$\Delta E = I \,.\, K_s\, \omega_m^2$ $\therefore K_s = \frac{5711.2}{27.95 \times 22.6^2} = 0.4$

One hole is punched in 9 rev. of the driving shaft

$\therefore$ Working stroke $= \frac{1}{5} \times 9 = 1.8$ rev.

If T_{max} is the max. torque on the driving shaft

$T_{max} \times 1.8 \times 2\pi = 7139$ $\therefore T_{max} = 631.54$ N-m

Assume $D_m = 1$m $\therefore K = 0.5$ m

Assume rim supplies 95% of total I

$\therefore\ M K^2 = .95 \times 27.95$;

$$M = \frac{.95 \times 27.95}{.5^2} = 106.21\,\text{kg}$$

T-XI-6 $M = \pi D_m A \times \rho$;

$$A = \frac{106.21}{\pi \times 1 \times 7200} = 4.7 \times 10^{-3}\,\text{m}^2$$

Assume $b = h$, $b^2 = 4.7 \times 10^{-3}$

$\therefore\ b = .068$ or 68 mm, $h = 68$ mm.

$D_o = D_m + h = 1 + .068 = 1.068$ m,

$$V_s = \pi \times 1.068 \times \frac{216}{60} = 12.08\ \text{m/sec.}$$

To calculate stresses in rim

$$\sigma_1 = \rho V_s^2 = 7200 \times 12.08^2 = 1.05 \times 10^6\ \text{N/m}^2 = 1.05\ \text{MPa}$$

$$\sigma_2 = \frac{\pi^2 \rho V_s^2 D_o}{i^2 h}$$ $i = 6$ for Do between 400 to 3000 mm, $h = .068$

$$= \frac{\pi^2 \times 1.05 \times 10^6 \times 1}{6^2 \times .068} = 4.2 \times 10^6\ \text{N/m}^2 = 4.2\ \text{MPa}$$

$\therefore\ \sigma_3 = .75\,\sigma_1 + .25\,\sigma_2 = .75 \times 1.05 + .25 \times 4.2$

$= 1.84$ MPa, so o.k.

T-XI-8 To design the hub and arms, we must first calculate the shaft dia. based on T_{max}.

Assume allowable shear stress = 40 MPa

$$\tau = \frac{16\,T}{\pi d^3} \text{ or } d = \sqrt[3]{\frac{16 \times 631.54 \times 10^3}{\pi \times 40}} = 43\ \text{mm say 45 mm.}$$

D_h = hub diameter = $2 \times d = 2 \times 45 = 90$ mm

L_h = hub length = $2 \times d = 2 \times 45 = 90$ mm

$$\text{Bending moment on each arm} = \frac{T_{max}(D_m - d_h)}{6 \times D_m}$$

$$= \frac{631.5\,(1 - .09)}{6 \times 1} = 95.77\ \text{N-m}$$

T-XI-6 Assuming elliptical cross section and allowable stress for cast iron and sudden load as 8 MPa.

T-XVI-14 $Z = \frac{\pi h^3}{64}$ where h = major axis,

$\frac{h}{2}$ = minor axis.

$$\sigma_b = \frac{M}{Z} \text{ or } Z = \frac{95.77 \times 10^3}{8} = 11.97 \times 10^3\ \text{mm}^3$$

$$\frac{\pi h^3}{64} = 11.97 \times 10^3$$

$\therefore\ h = 62.37 = 62.5$ mm

$$\therefore\ \text{Bending dia.} = \frac{6 \times 62.37}{\pi} = 119\ \text{mm say 120 mm.}$$

T-VII-5 Key cross section = 14 × 9

$\therefore$ Key way depth in hub = 3.8 mm.

* * *

Chapter 16

COUPLINGS & POSITIVE CLUTCHES

16.1 Introduction :

A *coupling* is used to connect two shafts permanently; it is disconnected only for repairs or to make a change in the installation. On the other hand, a *clutch* permits easy and quick connection and disconnection of two shafts. Clutches will be treated in the later sections of this chapter and the next chapter. The couplings are classified on the basis of the orientation of the axes of the shafts,

1. **Axes of the shafts are collinear :** There are two types of couplings for this situation (i) Rigid couplings and (ii) Flexible couplings. The detail design of only these will be discussed in later sections.
2. **Axes of the shaft intersect :** A universal coupling is used for this situation.
3. **Axes of the shafts are parallel, but not collinear :** A coupling of Oldham's type is used in this situation. This type is to be avoided, where possible, with heavy loads because of friction due to sliding.

16.2 Rigid Couplings :

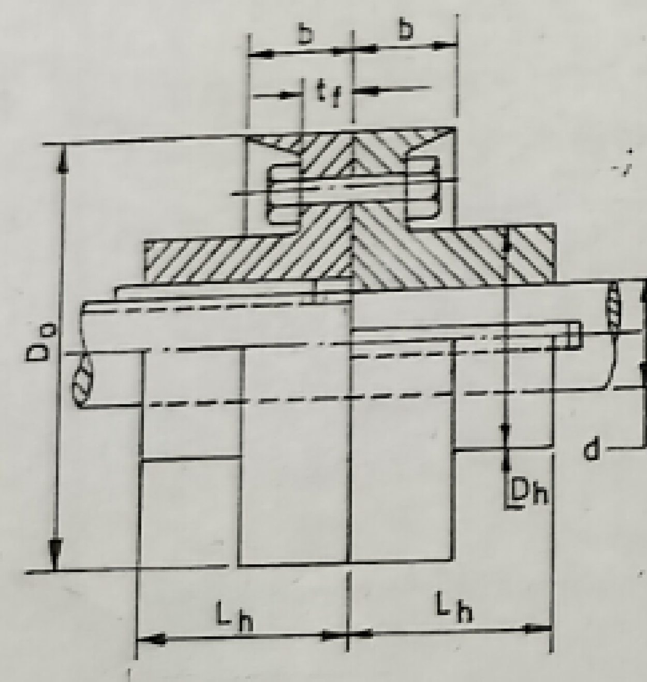


Fig. 16.1 Rigid flange coupling

This type of coupling is suitable for low speeds and accurately aligned shafts. Any misalignment subjects the shafts and coupling parts to bending stresses which can cause fatigue failure. If the shafts are accurately aligned the shafts and coupling are subjected to torsion only. Hence the rigid couling and flexible couplings are usually analyzed for torsion only.

The most commonly used rigid coupling is the protective flange coupling or flange coupling. It consists of two cast iron flanges, keyed to the shaft ends and bolted together as shown Fig. 16.1. The best practice is to press or shrink the flanges on the shaft in which case straight keys should be used. If the flanges are put on only with a push fit, taper keys must be used. A register is necessary to insure correct alignment of the two shafts. It is usually made by turning a projection on one flange and boring a slightly deeper female register in the other flange.

The other types of rigid coulings which are not so common are :

1. **Muff Coupling :** This consists of a long hollow shaft and a single long key which connects both the shafts to the same hollow shaft called muff.

2. **Clamp coupling :** This coupling is simlar to muff coupling except that the muff is split in two halves which are bolted together by a number of bolts. During the boring operation the two halves are seperated by a thin shim. A one piece square key assists in transmitting the torque.

3. **Compression Coupling :** These couplings are based on the pressure which can be created by wedge action produced by a hollow cone.

The design procedure consists of calculating all the important dimensions for a rigid flange coupling as per T-XII-1. All the dimensions are in terms of shaft diameter. Then the failure of the coupling is checked from the two considerations :

1. Failure of bolts in shear and crushing.
2. Shear failure of the flange at the hub.

The allowable shear stress for bolts is assumed much less because they are already stressed due to initial tightening. The procedure will be illustrated with the help of solved problems.

16.3 Flexible Couplings :

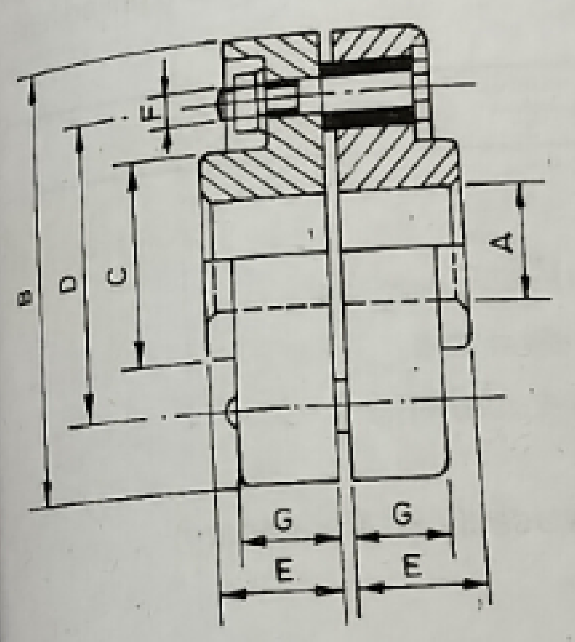


Fig. 16.2 Flexible coupling

The purpose of a flexible coupling is to allow for imperfect alignment of two joining shafts, or to absorb impact from the fluctuations of torque or of angular speed. In practice perfect alignment and constant torques and speeds are very seldom obtained. Therefore in number of cases of joining shafts which transmit power, it is necessary or at least desirable to use a flexible coupling of one type or another. There are numerous design of flexible couplings, but from the stand point of design, all flexible couplings may be divided into two classes :

1. Couplings whose flexibility is obtained kinematically by the use of rigid members in which constraint is absent in certain directions e.g. oldham's coupling.

2. Couplings which incorporate flexible members e.g. bush pin coupling, Flak flexible coupling etc. Out of these we will discuss the design of bush pin type of flexible coupling shown in Fig. 16.2. The design procedure for these couplings is same as rigid flange couplings except the additional check of the bearing strength of rubber bushes. The procedure will be illustrated with the help of the solved problems.

16.4 Positive Cluthes :

A *jaw clutch* is the commonest type of positive clutch. Jaw clutches are made either with square jaws for driving in both directions or with sprial jaws for driving in one direction only. When a clutch is used for connecting two shafts, it is called clutch coupling. In a clutch coupling, one half of the clutch, usually the driver, is keyed fast to one shaft, while the other half the follower, is attached to the other shaft by means of a feather key or spline which permits it to slide along the shaft when the shfiting lever is operated. Jaw clutches and couplings can be engaged only at low speeds, below 66 rpm. However when engaged, the shafts can run at much higher speeds. The spiral jaws are enagaged more easily and are used when engagement is frequent. They are vary convenient for presses, punches and shears.

The main dimensions of cast iron jaw clutches can be calculated as per T-XII-2A of data book and then checked for failure of jaws as was done in case of rigid flange coupling. The procedure will be illustrated with the help of the solved problem. The standard proportions are shown in Fig. (16.3).

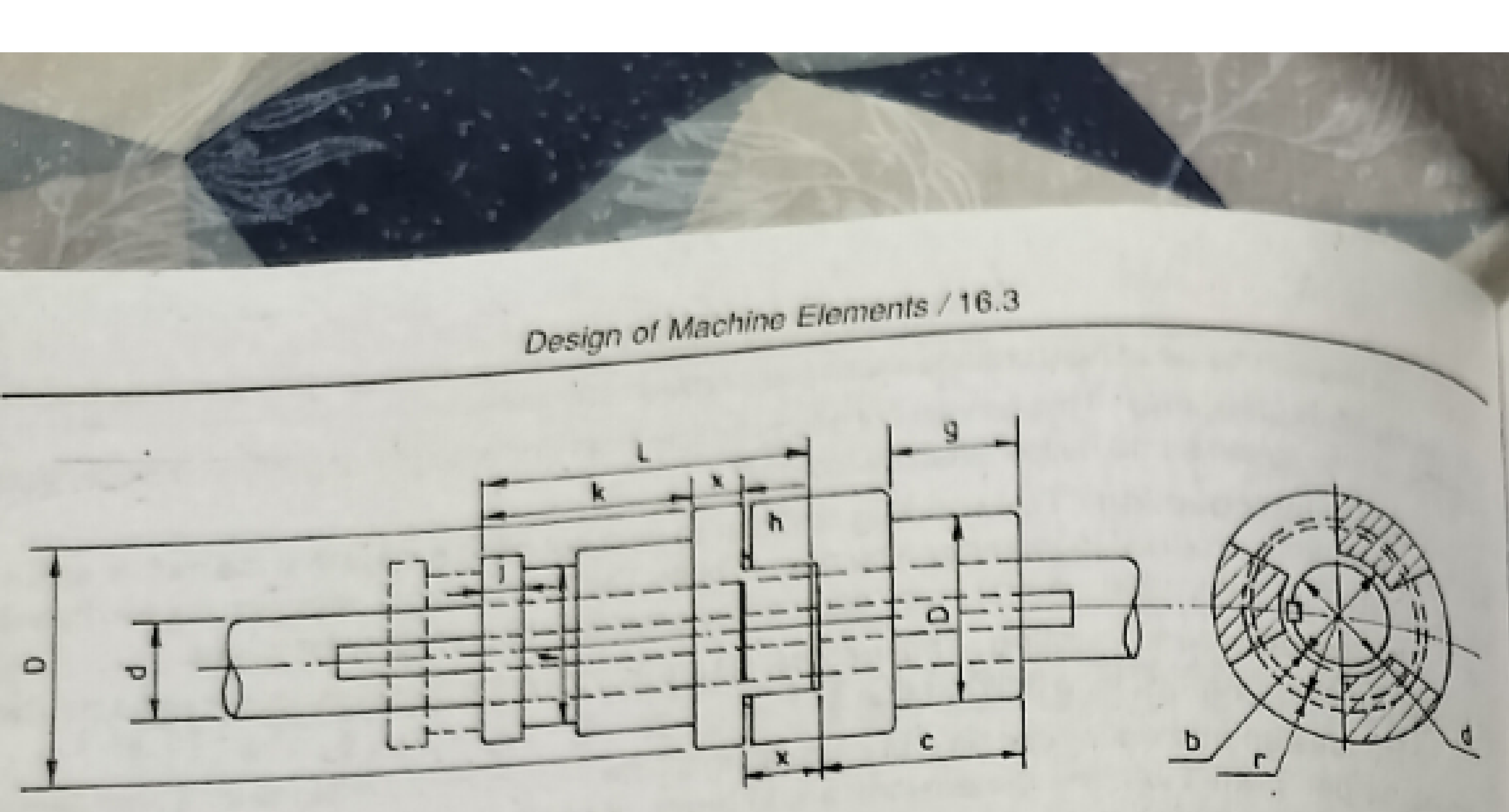


Fig. 16.3

SOLVED PROBLEMS

Prob. 16.1 : A flange coupling connects two 50 mm diameter length of commercial shafting. The shafts are made of SAE 1030 steel as also the bolts. The flanges are made of cast iron. Design the flange coupling for the same torque capacity as of the shaft. The bolts are fitted into drilled holes with clearance.

Ref.	Calculations
T-XII-1	$d = 50$ mm. Ref. Fig. (16.1) Hub diameter, $D_h = 1.5\,d + 25 = 1.5 \times 50 + 25 = 100$ mm Hub length, $L_h = 1.25\,d + 20 = 1.25 \times 50 + 20 = 82.5$ mm Number of bolts, $n = 0.02d + 3 = 0.02 \times 50 + 3 = 4$
T-VI-2	Bolt diameter, $d_b = 0.5 \frac{d}{\sqrt{n}} = \frac{0.5 \times 50}{\sqrt{4}} = 12.5$ mm Use $d_b = 14$ (standard) Flange thickness, $t_f = d_b = 14$ mm Flange protector width, $b = 2.5 \times d_b = 2.5 \times 14 = 35$ mm Pitch circle diameter, $D_p = 2d + 50 = 2 \times 50 + 50 = 150$ mm Outer diameter, $D_o = 2.5\,d + 75 = 2.5 \times 50 + 75 = 200$ mm To check the strength Properties of SAE 1030 steel
T-II-7	$S_{ut} = 527$ MPa $\quad S_{yt} = 296$ MPa
T-XI-6	As per A.S.M.E. code allowable shear stress in the shaft $\tau_{max} < .18\,S_{ut}$ or $.30\,S_{yt} = .18 \times 527$ or $.30 \times 296 = 94.86$ or 88.8 MPa $\tau_{max} < 88.8$ without keyway $< .75 \times 88.8 = 66.6$ with keyway

	$\therefore$ Torque capacity of the shaft, $T = \frac{\pi}{16} d^3 \times \tau_{max}$ $= \frac{\pi}{16} \times 50^3 \times 66.6$ $= 1634 \times 10^3$ N-mm $= 1634$ N-m 1. **Checking the bolts for shear failure :** $\frac{\pi}{4} d_b^2 \times \tau \times \frac{D_p}{2} \times n = T$ $\frac{\pi}{4} 14^2 \times \tau \times \frac{150}{2} \times 4 = 1634 \times 10^3$ $\therefore\ \tau = 35$ MPa less than allowable shear stress, so o.k. 2. **Checking the bolts for crushing failure :** $d_b \times t_f \times \frac{n}{2} \times \frac{D_p}{2} \times \sigma_{cr} = T$ (Only half the bolts are considered as they are not fitted) $14 \times 14 \times \frac{4}{2} \times \frac{150}{2} \times \sigma_{cr} = 1634 \times 10^3$ $\therefore \sigma_{cr} = 55.6$ MPa This is less than the allowable crushing stress which is assumed as, $2 \times \tau_{max}$ or $2 \times 88.8 = 177.6$ MPa 3. **Chekcing the flange for shear failure :** $\pi \times D_h \times t_f \times \tau \times \frac{D_h}{2} = T$ $\pi \times 100 \times 14 \times \tau \times \frac{100}{2} = 1634 \times 10^3$ $\therefore\ \tau = 7.43$ MPaa
T-II-3	For C.I.I.S. grade 20 $\quad S_{ut} = 271 = S_{us}$ with F.S. = 6, allowable $\tau = \frac{271}{6} = 45$ MPa so our design is ok.

Prob. 16.2 : Design bush-pin type of flexible couling for connecting the 7.5 kW electric motor running at 1440 rpm to a centrifugal pump.

Ref.	Calculations
T-XII-3	Diameter of shaft for a 7.5 kW electric motor at 1440 rpm = 38 mm. Assume pump shaft dia. also = 38 mm.

T-XII-2 Select coupling no. B_4 which is suitable for shaft dia. between 30 to 45 mm.

Other dimensions of B_4 couplings are Ref. Fig. (16.2)

Outside dia.	$B = 132$ mm
Hub dia.	$C = 55$ mm
Hub length	$E = 40$ mm
Flange length	$G = 30$ mm
Bolt recess	$H = 15$ mm
PCD of bolts,	$D_p = 90$ mm
No. of bolts,	$n = 4$
Dia. of bolt,	$F = 12$ mm
Dia. of bush	$= 25$ mm

Torque to be transmitted $T = \dfrac{7.5 \times 10^3 \times 60}{2\pi \times 1440} = 49.76 \text{ N}-\text{m}$

First chekcing the failure of pins

1. **Failure under shear :**

$$\frac{\pi}{4} \times F^2 \times n \times \tau \times \frac{D}{2} = T$$

$$\tau = \frac{49.76 \times 10^3 \times 4 \times 2}{\pi \times 12^2 \times 4 \times 90} = 2.44 \text{ MPa}$$

2. **Failure due to crushing :**

$$F(G-H) \times \frac{n}{2} \times \sigma_{cr} \times \frac{D}{2} = T \quad ; \quad \sigma_{cr} = \frac{49.76 \times 10^3 \times 2 \times 2}{12(30-15) \times 4 \times 90} = 3.07 \text{ MPa}$$

3. **Shear failure of flange :**

$$\pi \times C \times (G-H) \times \tau \times \frac{C}{2} = T$$

$$\tau = \frac{49.76 \times 10^3 \times 2}{\pi \times 55 \times 15 \times 55} = .689 \text{ MPa}$$

These stresses are very low so o.k.

For checking the bearing pressure on bushes.

Assume the length of bushes = G – Height of pin head = 30 – 5 = 25 mm.

$$\therefore\ 25 \times 25 \times 4 \times P_b \times \frac{90}{2} = 49.76 \times 10^3$$

$$\therefore\ P_b = \frac{49.76 \times 10^3}{2500 \times 45} = .44 \text{ MPa.}$$

Prob. 16.3 : Determine the main dimensions of a jaw clutch coupling for two 80 mm shafts to transmit full torque capacity of shafts. Assume the maximum allowable shear stress for shafts as 40 MPa which accounts for stress concentration due to key way.

Ref. | **Calculations**

Torque capacity of shafts $T = \dfrac{\pi}{16} d_s^3 \times \tau_{all}$

$$= \frac{\pi}{16} \times 80^3 \times 40 = 4019.2 \times 10^3 \text{ N-mm.}$$

T-XII-2A The dimensions of jaw clutch suitable for this are calculated below : Ref. Fig. (16.3)

number of jaws, $n = 3$

Width of jaws, $h = 0.3\,d + 12.5 = .3 \times 80 + 12.5 = 36.5$

O.D. of jaws, $a = (1.8 - 2.5)\,d = 2.1 \times 80 = 168$ mm.

I.D. of jaws, $b = (1.5 - 2.1)\,d = 1.8 \times 80 = 144$ mm.

The other dimensions are empirical and based on other considerations. To check the jaws for failure.

1. Shear failure :

$$\frac{(a-b)}{2} \times \pi \left(\frac{a+b}{2 \times 2}\right) \times \tau = T$$

$$\pi \times 12 \times 204^2 \times \tau = 4019.2 \times 10^3$$

$$\tau = \frac{4019.2 \times 10^3}{\pi \times 12 \times 204} = 2.563 \text{ MPa so o.k.}$$

❖ ❖ ❖

Chapter 17

FRICTION CLUTCHES & BRAKES

17.1 Introduction :

The object of a friction clutch is to connect a stationary machine part to a rotating part to bring it upto speed and to transmit the required power with a minimum of slippage. On the other hand, the function of the brake is to regulate the speed of a mechanism by transforming through friction, the energy of a moving body into heat and dissipating it. Though the functions are different, their analysis is based on the same principles and their construction is also similar. There are other types of clutches e.g. fluid clutch, magnetic cluth, but they will not be discussed in this book.

A simplified dynamic representation of a friction clutch or brake is shown in the Fig. 17.1

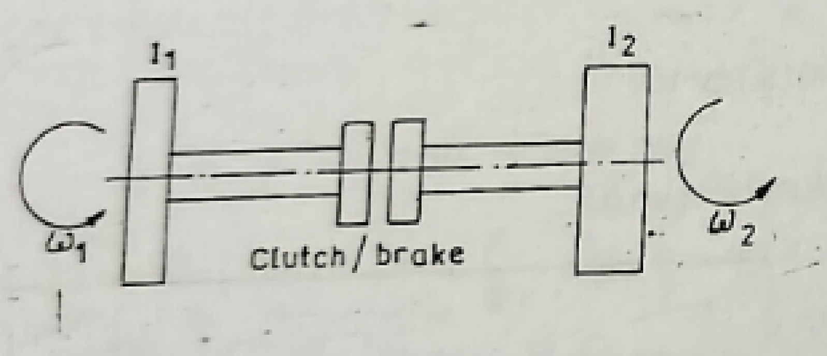


Fig. 17.1 Clutch brake

Two inertias I_1 and I_2 travelling at the respective angular velocities ω_1 and ω_2, one of which may be zero in the case of brakes, are to be brought to the same speed by engaging the clutch or brake. Slippage occurs because the two elements are running at different speeds and energy is dissipated during actuation, resulting in temperature rise. In analyzing the performance of these devices we shall be interested in :

1. The actuating force   2. The torque transmitted
3. The energy loss   4. The temperature rise.

With sliding surfaces encountered in most machine components (as bearings, cams, gears) it is desirable to minimise interface friction in order to reduce energy loss and wear. In contrast, clutches and brakes depend on friction in order to function. With these devices, one objective is to maximize the friction coefficient and at the same time to minimize wear. These devices must be designed to satisfy three basic requirements :

1. The required frictional torque must be produced by an acceptable actuating force.
2. The energy loss during the actuation converted to heat must be dissipated without producing destructively high temperatures.
3. The wear characteristics of the friction surfaces must be such as to give acceptable life.

A brake or clutch friction material should have the following characteristics, to a degree which is dependent upon the severity of service :

1. A high and uniform coefficient of friction.
2. Properties which are not affected by environmental conditions such as moisture.
3. The ability to withstand high temperatures together with good heat conductivity.
4. Good resiliency.
5. High resistance to wear, scoring and galling.

T-XII-6 of data book lists the various friction lining material along with their properties and fields of use.

17.2 Friction Clutches :

They may be divided into two main grups :

1. Axial Clutches in which the contact pressure is applied in a direction parallel to the axis of rotation. In turn these clutches can be subdivided into (a) Cone clutches, (b) Disc clutches and (c) Combined cone and disc clutches.
2. Rim Clutches in which the contact pressure is applied upon the rim in a radial direction. In turn they can be subdivided into (a) band clutches and (b) block clutches. The only type which is commonly used is the block type called *centrifugal clutch*. Band type construction is more commonly used in brakes.

The factors which must be taken into consideration in deciding the type of clutch to be used are :

1. Torque   2. Speed
3. Space limitation   4. Frequency of operation

For high speed operation disc clutches are preferred. At lower speeds and higher torque capacity cone clutches of single cone or double cone type are preferred. For high speed and high torque capacity multidisc clutches are preferred. For clutches which are in frequent or continuous operation, single disc cluthces are preferred.

17.3 Design of Friction Clutches :

For designing these clutches we require the torque capacity of the clutch. This can be calculated from the power rating of the clutch and the speed of operation. The steps to be followed are as given below.

Step 1 : Calculate the design torque as per T-XII-4.

Step 2 : Select proper material and its properties from T-XII-6. For selecting the normal pressure P check the values as per as PV limits given in T-XII-5 for brakes.

Step 3 : Assume the dimensions of the clutch in terms of the O.D., I.D. and the mean diameter. Also assume the semi cone angle α if it is a cone clutch as per T-XII-4.

Step 4 : Calculate the torque transmitting capacity of the clutch in terms of Dm – mean diameter. By equating it to the design torque, T_d calculate the mean diameter and other dimensions.

The torque capacity and the necessary axial force can be calculated on two assumptions :

(a) Uniform presssure distribution : This condition is applicable when the clutch is new. The expressions are,

$$Fa = P\pi\left(r_2^2 - r_1^2\right)$$ where r_1 and r_2 are inside and outside radii

$$T = \frac{2}{3}\frac{\mu\,Fa}{\sin\alpha}\left[\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right]$$ where α = semi cone angle, P = Normal pressure

(b) Uniform wear : This condition is applicable after the 'initial wear' has taken place. The expressions are,

$$P \times r = C \quad ; \quad Fa = 2\pi C\,(r_2 - r_1)$$

$$T = \frac{\mu\,Fa}{\sin\alpha}\left(\frac{r_2 + r_1}{2}\right)$$

For simplifying the design we use of following expressions for the design of clutches :

$$Fa = P\,.\,\pi\,\left(r_2^2 - r_1^2\right) \quad ; \quad T = \frac{\mu\,Fa}{\sin\alpha}\left(\frac{r_2 + r_1}{2}\right)$$

Step 5 : Calculate the actuating force and force for disengagement.

Step 6 : Decide the number of springs to be used to provide the actuating force in disc clutches. Also calculate the required stiffness of springs by assuming the ratio of the torque capacity when it is new to when it is worn out and the thickness of friction lining is reduced by a given amount.

Step 7 : Design the springs by assuming suitable material for the springs.

In designing friction clutches, the formulae given in data book are based on the following assumptions.

1. For torque capacity, uniform wear theory is used.
2. For the calculation of actuating force, uniform pressure distribution is assumed.

Theoretically, this approach is not absolutely correct, but it gives reasonable results and hence used for design. For design purposes use a smaller value of P than given in T-XII-6 which are maximum values. After design find the value of P_{max} from the relation.

$$P_{max} = \frac{Fa}{2\pi r_1 (r_2 - r_1)}$$

The procedure will be illustrated with the help of solved problems.

17.4 Centrifugal Clutches :

These clutches are designed to give automatic and smooth engagement of the load to the driving member whenever the driving member is running at or above some minimum speed. Since the operating centrifugal force is a function of the square of the angular velocity, the friction torque available for accelerating the load is also a function of the square of the speed of the driving member. this characteristic makes the centrifugal clutch ideal for applications requiring smooth load pick ups, as for textile and paper machinery. These clutches are particularly suitable and useful for I.C. engines which can not be started under load. These features have been responsible for wide spread use of the centrifugal clutches in motor cycels, portable chain saw etc.

The simplest form consists of a single ring of rotating shoes guided in a spider which are pressed against the cylindrical drum by centrifugal force. A variation of this construction called Rawson coupling has two rows of shoes. Also available are clutches, in which springs keep the shoes from contacting the drum until a predetermined speed is reached. this is advantageous when an engine is the prime mover, because it permits warming up and idling of the engine without friction between the shoes and the drum. The design is similar to block brakes except that the actuating force is provided by the centrifugal action instead of the operating lever. The procedure will be illustrated with the help of the solved problems.

17.5 Brakes :

These are machine elements that absorb either kinetic or potential energy in the process of slowing down or stopping a moving part. The absorbed energy is dissipated as heat. Brake capacity depends on the unit pressure between the braking surfaces, coefficient of friction and the ability of the brake to dissipate heat equivalent to the energy being absorbed. The performance of the brakes is similar to that of clutches except that clutches connect one moving part to another moving part, whereas brakes connect a moving part to a frame.

The brakes may be divided into two groups according to the direction of the acting force :

(a) Radial brakes which may be subdivided into,
  i) block brakes and ii) band brakets

Both type may be internal expanding type or external contracting type.

(b) Axial brakes which may be subdivided into cone brakes and disc brakes. Their analysis and design is similar to that of clutches. Hence will not be discussed further.

We will discuss the design of these three types of radial brakes :

(a) External shoe brakes.
(b) Internal shoe brakes.
(c) External band brakes.

17.6 External Shoe or Block Brakes :

They consists of shoes or blocks pressed against the surface of a rotating cylinder called brake drum. The shoe may be rigidly mounted to a pivoted lever or may be pivoted to the lever as shown in Fig. 17.2.

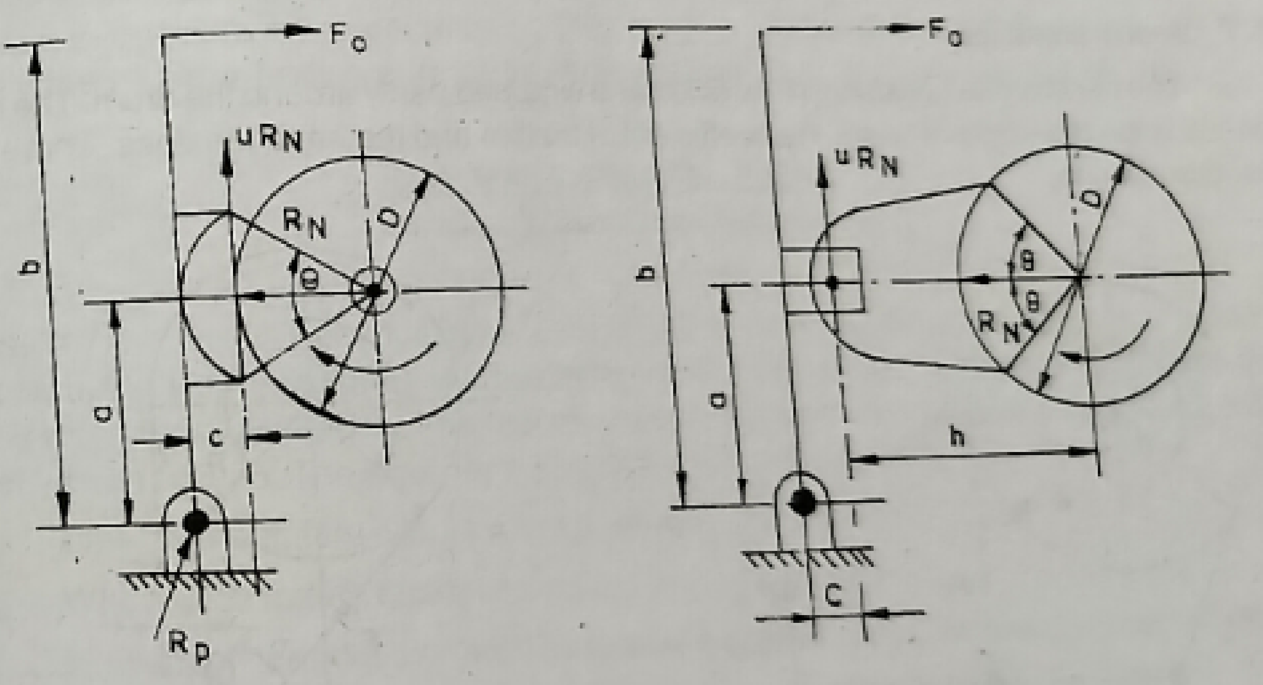


Fig. 17.2

Fig. (A) shows the single block brake with the operating lever shoe fixed to the operating lever while Fig. (B) shows the pivoted shoe. The figures also show the normal reaction R_n, the frictional force μR_n and the operating force F_o. Note the difference of point of action of the frictional force in case of fixed shoe and the pivoted shoe. This gives the difference in the braking torque as shown later. The normal reaction R_n and the frictional force μR_n may be assumed to act at the midpoint of contact of the shoe without appreciable error for angles of θ below 30°. Therefore the pivoted shoe is the more usual construction when using shoes with θ greater than 30°. By taking moments about the fulcrum pin we get,

$$F_o = \frac{R_n a - \mu R_n c}{b}$$

Braking torque for fixed shoes $T = \mu R_n \frac{D}{2}$

Braking torque for pivoted shoe $T = \mu R_n h$

$$= \mu R_n \frac{D}{2}\left[\frac{4\sin\theta}{2\theta + \sin 2\theta}\right]$$

We get the increased braking torque with pivoted shoe because the frictional force acts through the pivot which is located at a distance h from the centre of the drum. The correct value of h depends upon the shoe angle 2 θ and the drum diameter and is given by the relation :

$$h = \frac{D}{2}\left[\frac{4\sin\theta}{2\theta + \sin 2\theta}\right]$$

If the pivot is not correctly located the pressure distribution changes and the given braking torque will not be obtained. If long shoes are fixed to the operating lever the anaylsis becomes complicated and is similar to internal expanding shoe brakes which are not discussed in this book. Generally single shoe brakes are not used for larger torques ; instead double shoe brakes are commonly used in order to reduce shaft and bearing loads to obtain greater capacity, and to reduce the amount of heat generated per m^2. The analysis is similar to single shoe brakes. The operating lever and mechanism consists of usual levers, screws etc. which should be designed as per usual practice. Only the design of the shoe will be discussed here. The dimensions of the shoe depend on the allowable normal pressure as per T-XII-6 and the P-V limits as per T-XII-5. These steps will be illustrated with the help of solved problems.

17.7 Band Brakes :

The band brake consists of a flexible band wrapped partly around the drum. The brake capacity depends uopn the angle of wrap, the coefficient of friction and the band tensions. There are two types of band brakes :

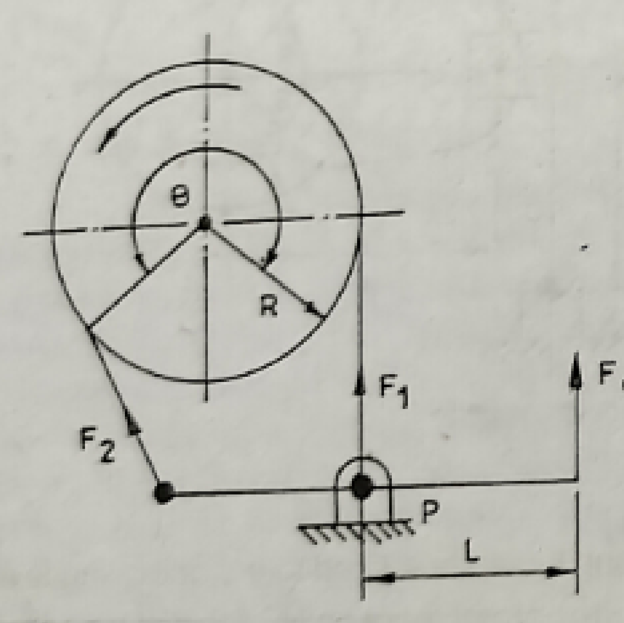


Fig. 17.3 Simple band brake

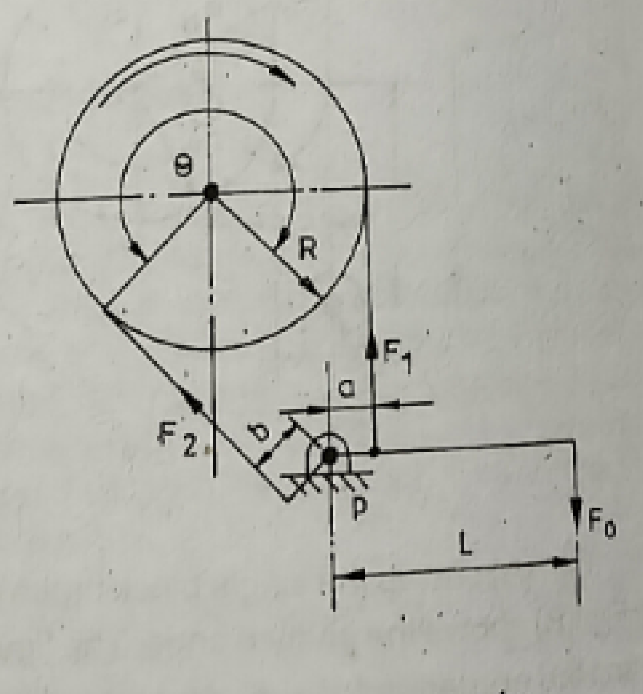


Fig. 17.4 Differential band brake

(a) Simple band brake : In this type of brake one end of the band is attached to the fulcrum pin of the operating lever as shown in Fig. 17.3, In this type of brake, the direction of rotation of the drum is such that the band anchored to the fulerun is the tight side F_1. This can be used for one direction of rotation only.

(b) Differential band brake : In this type of brake, the two ends of the band are attached on the two sides of the fulcrum pin of the operating lever at different distances, as shown in Fig. 17.4. It is usualy designed so that the direction of drum rotation permits the tight side of the band to aid in applying the brake.

Taking moments about P :

(a) For simple band brake :

$F_o \times L = F_2 \times b$ or $F_o = \frac{F_2 \times b}{L}$ where $\frac{F_1}{F_2} = e^{\mu\theta}$

Braking torque = $(F_1 - F_2)\,R$

(b) For differential bank brake :

$F_o L = F_2 a - F_1 b$ $\qquad \frac{F_1}{F_2} = e^{\mu\theta}$

$$F_o = \frac{F_2\left(a - be^{\mu\theta}\right)}{L}$$

Braking torque $T = (F_1 - F_2)\,R$

The simple band brake cannot be made self locking i.e. $F_o = o$, while the differential band brake becomes self locking when F_o is either zero or negative. The condition for self locking is,

$$a \le be^{\mu\theta} \quad \text{or} \quad \frac{a}{b} \le e^{\mu\theta}$$

It should be noted that the brake may be made self locking for one direction of rotation only. Hence it is used in belt conveyors or hoists so that it prevents rotation in one direction. The design procedure is same as block brakes and will be illustrated with the help of solved problems.

SOLVED PROBLEMS

Prob. 17.1 : An automotive type clutch has inside and outside diameters of 150 and 225 mm respectively. Clamping force is provided by nine springs, each compressed 6.5 mm to give a force of 625 N when the clutch is new. The moulded friction material provides a conservatively estimated coefficient of friction of 0.35. The maximum engine torque is 280 N-m.

(a) What is the safety with respect to slippage of a brand new clutch ?

(b) What is the safety factor after initial wear has occured ?

(c) How much wear of the friction material can take place before the clutch will slip ?

(d) Design the springs for the clutch.

Ref.	Calculations
	Given : $D_o = 225$ mm, $r_o = \frac{225}{2} = 112.5$ mm, $D_i = 150$ mm, $r_i = \frac{150}{2} = 75$mm $\quad \therefore \frac{r_i}{r_o} = \frac{75}{112.5} = .666,$

$\mu = 0.35$, $Fa = 9 \times 625 = 5.625$ kN $\qquad T_{max} = 280$ N-m.

Torque capacity of new clutch $T = \frac{2}{3} \mu\, Fa \left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \right) \times N_f$

$$= \frac{2}{3} \times .35 \times 5.625 \times 10^3 \times 112.5 \left(\frac{1 - .666^3}{1 - .666^2} \right) \times 2$$

$$= 373.86 \text{ N-m}$$

Torque capacity after initial wear $= \frac{\mu\, Fa\, Dm}{2} \times N_f$

$$= \frac{.35 \times 5.625 \times 10^3 \times \left(\frac{225 + 150}{2} \right)}{2} \times 2$$

$$= 369.14 \times 10^3 \text{ N-mm or } 369.14 \text{ N-m}$$

(a) Safety factor when new $= \frac{373.86}{280} = 1.335$

(b) Safety factor after initial wear $= \frac{369.14}{280} = 1.318$

(c) Reduction in clamping force so that slippage occurs

New clamping force, $F'_a = 5.625 \times 10^3 \times \frac{280}{369.15}$

$$= 4.27 \times 10^3 \text{ N} = 4.27 \text{ kN}$$

$\therefore$ Change in clamping force, $\Delta Fa = 5.625 - 4.27 = 1.358$ kN

Change in clamping force of each spring $= \frac{1.359}{9} = .151$ kN

Spring stiffness $= \frac{625}{6.5} = 96.15$ N/mm

$\therefore$ Required wear $= \frac{151}{96.15} = 1.57$ mm

For design of springs :

T-VIII-9 Max force $= 625 \times 1.1 = 687.5$ N [10% extra force for disengagement]

Spring stiffness = 96 N/mm

For automotive clutch springs, select Cr-v steel SAE 6150

Allowable shear stress with a FS = 1.5, $\tau_{max} = \frac{S_{ys}}{1.5} = \frac{770}{1.5} = 513$ MPa

T-VIII-1 Assuming C = 6, Wahl's factor, $K = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.25$

$$\tau_{max} = \frac{8\, F_{max}\, Dm}{\pi\, d^3} K = \frac{8FC}{\pi\, d^2} \times K = \frac{8 \times 687.5 \times 6 \times 1.25}{\pi\, d^2} = 513$$

$\therefore$ $d = 5.06 \approx 5$ mm, $D_m = 5 \times 6 = 30$ mm

To find the number of turns $K_s = 96 = \frac{G \cdot d}{8\, C^3\, n}$, $\quad G = 85 \times 10^3$ MPa

$$n = \frac{85 \times 10^3 \times 5}{8 \times 6^3 \times 96} = 2.56$$

$\therefore$ Total no. of turns with squared and ground ends $= 2.56 + 2 = 4.56$ say 5

$\therefore$ Free length $= 5 \times 5 + \frac{687.5}{96} + 4 \times 1 = 36.16$ mm.

$\frac{\text{Free length}}{Dm} = \frac{36.16}{30} = 1.2$ so o.k.

To check whether nine springs can be accomodated

Mean dia. of the friction disc $= \frac{225 + 150}{2} = 187.5$

$\therefore$ Space available/spring $= \frac{\pi \times 187.5}{9} = 65.41$ mm.

D_o of our spring $= D_m + d = 30 + 5 = 35$ mm so o.k.

Prob. 17.2 : Design a multiple disc clutch to transmit 8 kW at 750 rpm. The mean radius of the discs should be 75 mm. The discs are made of steel and bronze and the clutch operates in oil. What will be the average pressure and the maximum pressure on the discs ?

Ref. **Calculations**

T-XII-4 Assume $K_1 = 1$, $T_d = \frac{8 \times 10^3 \times 60 \times 10^3}{2\pi \times 750}$

Given : $R_m = 75$, $D_m = 150$ mm $= 101.9 \times 10^3$ N-mm

Torque capacity of multiple disc clutch $= \frac{\mu\, Fa\, Dm\, Nf}{2}$

T-XII-6 μ for wet clutch $= .05$, $P_{max} = 0.6$ MPa

$$V_m = \pi \times \frac{150}{1000} \times 750 = 352.2 \text{ m/min.}$$

Ref.	Calculations
T-XII-5	Assume PV = 165 for good heat dissipation $\therefore P_{max} = \frac{165}{353.2} = .467$ MPa $\therefore Fa = \frac{\pi}{4}\left(D_o^2 - D_i^2\right)P$ $= \frac{\pi}{2} D_m (D_o - D_i) P$ Assume $\frac{D_i}{D_o} = .6$ $= \frac{\pi}{2} \times .8 \times .4 D_o^2 \times .467$ $D_m = \frac{D_o + D_i}{2} = .8 D_o$ $= .234\ D_o^2$ $D_o = \frac{150}{.8} = 187.5$ mm $= .234 \times 187.5^2 = 8248.4$ N $D_i = .6 \times 187.5 = 112.5$ mm $\therefore T = \frac{.05 \times 8248.4 \times 150}{2} N_f = 101.9 \times 10^3$ $\therefore N_f = 3.3$ N_f must be even so $N_f = 4$ 3 steel discs & 2 bronze discs. Actual $Fa = \frac{2 \times Td}{\mu D_m N_f} = \frac{2 \times 101.9 \times 10^3}{.05 \times 150 \times 4} = 6793.2$ N $\therefore$ Average pressure $= \frac{4 \times Fa}{\pi\left(D_o^2 - D_i^2\right)} = \frac{4 \times 6793.2}{\pi\left(1875.5^2 - 112.5^2\right)}$ Because we have assumed uniform wear theory $Fa = 2\pi P_{max} r_i (r_o - r_i)$ $= 2\pi \times \frac{112.5}{2}\left(\frac{187.5 - 112.5}{2}\right) P_{max}$ $\therefore P_{max} = \frac{6793.2}{\pi \times 112.5 \times 37.5} = .512$ MPa

Prob. 17.3 : A leather faced cone clutch must transmit 15 kW at a speed of 1000 rpm. The mean diameter of the cone is 200 mm. Design the clutch and calculate the force required to engage the clutch.

Ref.	Calculations
T-XII-4	Assume $K_1 = 1$, $T_d = \frac{15 \times 10^3 \times 60 \times 10^3}{2\pi \times 1000} = 143.3 \times 10^3$ N – mm For leather, semi-cone angle, $\alpha = 12.5°$

T-XII-6	$\mu = 0.3$, P = .07 – .3 MPa ; $V_m = \pi \times \frac{200}{1000} \times 1000 = 628$ m/min. Assuming PV = 110, $P_{max} = \frac{110}{628} = .175$ MPa Assume average P = .15 MPa $Fa = \pi D_m b . P . \sin\alpha$, $T = \frac{\mu Fa D_m}{2 \sin\alpha} = \frac{\mu \pi b P D_m^2}{2}$ $143.3 \times 10^3 = \frac{.3 \pi b \times .15 \times 200^2}{2}$ $b = 50.66 \simeq 51$ mm $\frac{D_m}{b} = \frac{200}{50.66} \approx 4$ so o.k. ; $(D_o + D_i) = 2 D_m = 400$ $(D_o - D_i) = 2 b \sin\alpha = 2 \times 51 \sin 12.5 = 22$ mm $\therefore D_o = 211$ mm, $D_i = 189$ mm $r_o = 105.5$ mm, $r_i = 94.5$ mm $Fa = \pi \times 200 \times 51 \times .15 \sin 12.5 = 1039.8$ N Force for engagement $F'a = Fa (1 + \mu \cot\alpha) = 1039.8 (1 + .3 \cot 12.5) = 5730$ N To find P_{max} $Fa = 2\pi P_{max} r_i (r_o - r_i)$ $P_{max} = \frac{Fa}{2\pi r_i (r_o - r_i)} = \frac{1039.8}{2\pi \times 94.5 \times 11} = .16$ so o.k.

Prob. 17.4 : Determine the main dimensions and the necessary spring force for a cone clutch to transmit 30 kW a 2000 rpm (a) with a leather facing (b) with a asbestos fabric facing and (c) cost iron contact surface. Assume $\alpha = 12.5°$ for all cases.

Ref.	Calculations
T-XII-4	Assume $K_1 = 1$, $T_d = \frac{30 \times 10^3 \times 60 \times 10^3}{2\pi \times 2000} = 143.3 \times 10^3$ N – mm
T-XII-6	(a) For leather $\mu = 0.3$, P = .07 – .3 Assume P = .10 MPa $Fa = \pi D_m P b \sin\alpha$ Assume $\frac{D_m}{b} = 4.5$ or $b = \frac{D_m}{4.5}$ $= \pi \times .10 \times \frac{D_m^2}{4.5} \sin 12.5 = .015 D_m^2$ $T = \frac{\mu Fa D_m}{2 \sin\alpha} = \frac{0.3 \times .015 D_m^3}{2 \times \sin 12.5} = 143.3 \times 10^3$ $\therefore D_m = 239$, $b = \frac{239}{4.5} = 53$ mm

$D_o + D_i = 2 \times 239 = 478$ $\quad D_o = 250.5$ mm

$D_o - D_i = 2 \times 53 \sin 12.5 = 22.94$ $\quad D_i = 227.5$ mm

$Fa = .015\ D_m^2 = .015 \times 239^2 = 856.8$ N

$$P_{max} = \frac{Fa}{2\pi r_i (r_o - r_i)} = \frac{2Fa}{\pi D_i (D_o - D_i)} = \frac{2 \times 856.8}{2 \times 227.5 \times 23} = .1637 \text{ MPa}$$

$V_m = \pi \times \frac{239}{1000} \times 2000 = 1501$ m/min. $\quad \therefore\ p V_m = 1501 \times .1 = 150$ so o.k.

(b) For asbestos fabric $\mu = 0.2$, $P = .7 - 1.5$ Assume P = 1 MPa, $\frac{D_m}{b} = 4.5$

$$Fa = \pi D_m b P \sin \alpha, = \frac{\pi}{4.5} \times 1 \times D_m^2 \sin 12.5 = .15 D_m^2$$

$$T = \frac{\mu\, Fa\, D_m}{2 \sin \alpha} = \frac{0.2 \times .15 D_m^3}{2 \sin 12.5} = 143.3 \times 10^3$$

$D_m = 127$ mm, $b = 28.2$ mm ; $\quad D_o + D_i = 2 \times 127 = 254$

$D_o - D_i = 2 \times 28.2 \times \sin\alpha = 12.2$ $\quad \therefore\ D_o = 133$ mm, $D_i = 120.8$ mm

$$P_{max} = \frac{Fa}{2\pi r_i (r_o - r_i)} = \frac{2Fa}{\pi D_i (D_o - D_i)} \qquad Fa = .15 \times 127^2 = 2419.35 \text{ N}$$

$$= \frac{2419.35 \times 2}{\pi \times 120.8 \times 12.2} = 1.046 \text{ MPa so o.k.}$$

$V_m = \pi \times \frac{127}{1000} \times 2000 = 789$ m/min. ; $\quad P V_m = 789 \times 1 = 789$ very high.

T-XII-6 (c) For C.I. $\mu = .05$ $P = 1 - 1.6$ assume P = 1 MPa ; $\frac{D_m}{b} = 4.5$

$$Fa = \pi D_m b P \sin \alpha = \frac{\pi}{4.5} \times 1 \times \sin 12.5 D_m^2 = .15 D_m^2$$

$$T = \frac{\mu\, Fa\, D_m}{2 \sin \alpha} = \frac{.05 \times .15 \times D_m^3}{2 \times \sin 12.5} = 143.3 \times 10^3$$

$D_m = 201$ mm, $\quad b = 44.66 = 45$ mm, $\quad Fa = .15 \times 201^2 = 6060$ N

$D_o + D_i = 402$, $\quad D_o = 220.5$ mm

$D_o - D_i = 2 \times 45 \times \sin 12.5 = 19.48$ $\quad D_i = 201.0$ mm

$$P_{max} = \frac{2Fa}{\pi D_i (D_o - D_i)} = \frac{2 \times 6060}{\pi \times 201 \times 19.48} = .985 \text{ MPa so o.k.}$$

$V_m = \pi \times \frac{201}{1000} \times 2000 = 1262$ m/min, $P V_m = 1262$ very high.

Prob. 17.5 : A centrifugal clutch is required for a centrifuge drive. The centrifuge is to rotate at 7500 rpm and preliminary calculations have indicated that the rated torque of a 3.75 kW, 1750 rpm electric motor will bring the centrifuge upto running speed in 2 min 45 secs. The proposal is to use six shoes in the centrifugal clutch. The coefficient of friction is 0.35, that the friction material has an allowable pressure of 0.2 MPa, that the shoe width is to be 1/4 the drum diameter, that the seperator shoe is 85% of the drum radius and that the normal and tangential forces (friction) act at the centre of the contact between each shoe and the drum, determine (a) the drum diameter and the width of the shoe (b) the weight of each shoe.

Ref. Calculations

$N = 7500$ rpm, $\quad$ Torque capacity $= \frac{3.75 \times 10^3 \times 60}{2\pi \times 1750} = 20.4$ N – m

$\mu = 0.35$, $\quad P = 0.2$ MPa, $\quad b = 1/4$ D, $\quad r_g = .85$ D/2 No. of shoes = 6

Let D be the drum diameter in m & kg mass of each shoe

$\therefore$ Centrifugal force on each shoe, $F_N = M \cdot r_g\, \omega^2$, $\qquad \omega = \frac{2\pi \times 7500}{60}$

$$= M \times \frac{.85 D}{2} \times 785^2 \qquad = 785 \text{ rad/sec.}$$

$$= 262\ MD \times 10^3 \text{ N}$$

$\therefore$ Frictional force on each shoe $= \mu F_N = 0.35 \times 262\ MD \times 10^3$

$$= 91.7\ MD \times 10^3$$

Torque capacity of each shoe $= 91.7$ M.D. $\times 10^3 \times \frac{D}{2}$

$$= 45.85 \times 10^3 \text{ M.D}^2$$

With six shoes, $6 \times 45.85 \times 10^3\ M D^2 = 20.4$ $\qquad \therefore\ M D^2 = 74.15 \times 10^{-6}$

Angle subtended by each shoe $= \frac{360 \times .85}{6} = 51^\circ$ so short shoe.

$$\therefore\ F_N = \frac{D}{2} \times \sin\left(\frac{51}{2}\right) \times 2 \times b \times P = \frac{D^2}{4} \times 0.2 \times .43 \times 10^6 = 21.5 \times 10^3 D^2$$

Equating the values of F_N: $\quad 262 \times 10^3$ M.D. $= 21.5 \times 10^3 D^2$

$\therefore\ M = .082\ D$

$\therefore\ MD^2 = .082\ D.D^2 = 74.72 \times 10^{-6}$ (Found earlier)

$$\therefore\ D^3 = \frac{74.15 \times 10^{-6}}{.082} = 904.3 \times 10^{-6} \qquad \text{or} \qquad D = .097 \text{ m or } 97 \text{ mm}$$

$b = \frac{97}{4} = 24.2$ mm ; $\quad M = .082 \times .097 = .0079$ kg $= 7.9$ gms.

Prob. 17.6 : A spring controlled centrifugal coupling has a single rotating ring of shoes that are spring loaded so that they do not make contact with the drum until the inner member reaches a predetermined speed. The drum diameter is 400 mm, each of the 8 shoes has a mass of 1 kg, the radius to the centres of gravity of the shoes is 180 mm, and the spring force at the instant of contact is 360 N on each shoe. (a) What is the speed at which the shoes first contact the drum ? (b) What is the power capacity of the coupling at 1200 rpm ?

Ref. **Calculations**

No. of shoes 8, Mass of each shoe = 1kg., Drum dia. = 400 mm,

Radius to the c.g. = 180 mm

Spring force at the point of contact = 360 N

∴ Centrifugal force at speed ω rad/sec.

$F_N = M\,r_g\,\omega^2 = 1 \times .18\,\omega^2 = .18\,\omega^2$

∴ At the point of contact F_N = spring force $= .18\,\omega^2 = 360$

∴ $\omega = 44.72$ rad/sec. $N = 427$ rpm

∴ at 1200 rpm, $\omega = \dfrac{2\pi \times 1200}{60} = 125.6$ rad/sec.

∴ Centrifugal force $= .18 \times 125.6^2 = 2839.6$ N

∴ Net force at the contact $= 2839.6 - 360 = 2479.6$ N Assume $\mu = .30$

∴ Torque capacity of each shoe $= .3 \times 2479.6 \times \dfrac{.4}{2} = 148.8$ N-m.

∴ Power at 1200 rpm with eight shoes ; $P = \dfrac{8 \times 148.8 \times 1200 \times 2\pi}{60} = 149.5$ kW

Prob. 17.7 : The double block brake as shown in the figure below has wooden brake shoes applied to a cast iron drum. The coefficient of friction for these two materials is 0.3. The drum rotates at 1500 rpm. Suggest suitable width for the shoes. Determine the power lost as heat.

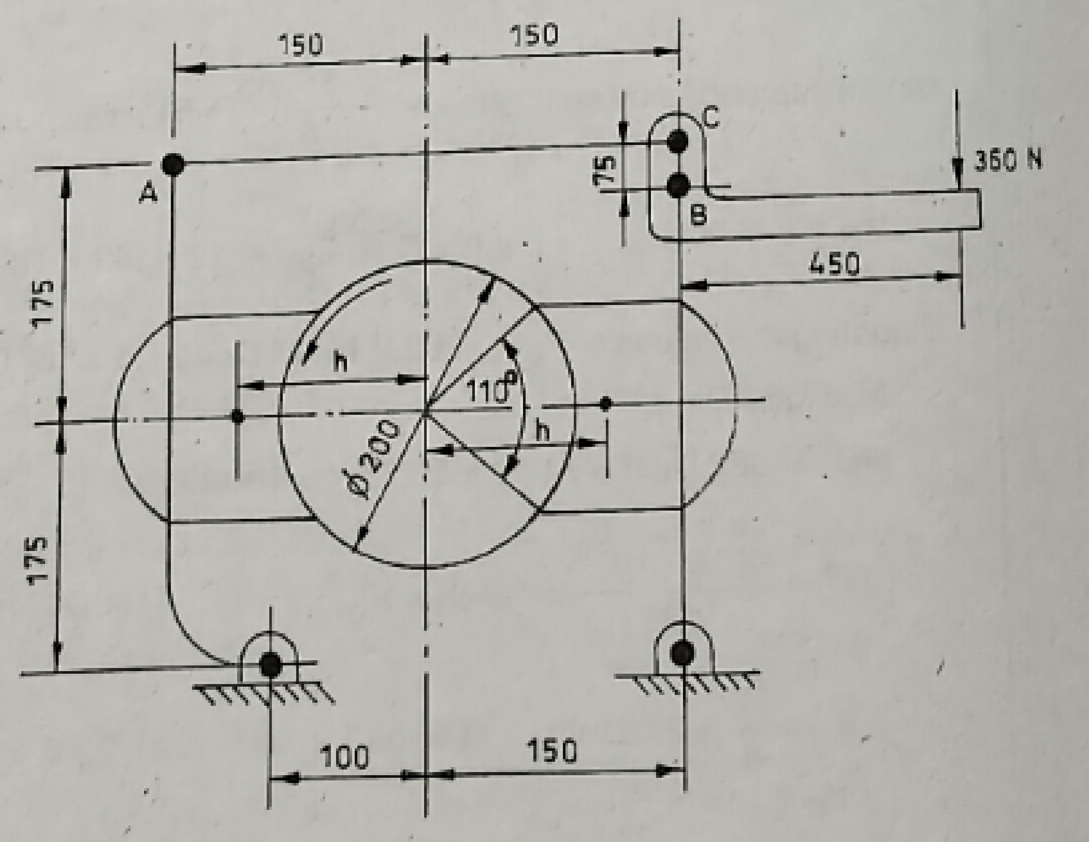
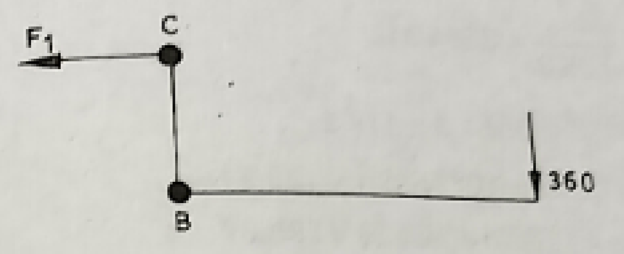


Fig.

Ref. **Calculations**



For the operating lever

taking moments about B :

$F_1 \times 75 = 360 \times 450$

$\therefore F_1 = \dfrac{360 \times 450}{75} = 2160$ N

Fig.

Therefore this is the operating force on both the levers.

The free body diagrams of these levers are shown below. As the shoe angle 2θ is greater than 60°, the frictional force will pass through the pivot located at a distance h from the centre of the drum, given by the relation.

$$h = \frac{D}{2}\left[\frac{4\sin\theta}{2\theta + \sin 2\theta}\right] = \frac{200}{2}\left[\frac{4\sin 55^\circ}{\frac{110}{180}\pi + \sin 110^\circ}\right] = 100\left[\frac{3.28}{1.92 + .94}\right]$$

$$= 114.7 \text{ mm}$$

Taking moments about P_1 :

$R_{N_1} \times 125 + \mu R_{N_1} \times (114.7 - 100) = 2160 \times 300$

$R_{N_1} = \dfrac{2160 \times 300}{125 + 0.3 \times 14.7} = 5007.3$ N

∴ Breaking torque $T_1 = 0.3 \times 5007.3 \times 114.7 = 172.3 \times 10^3$ N-mm $= 172.3$ N-m

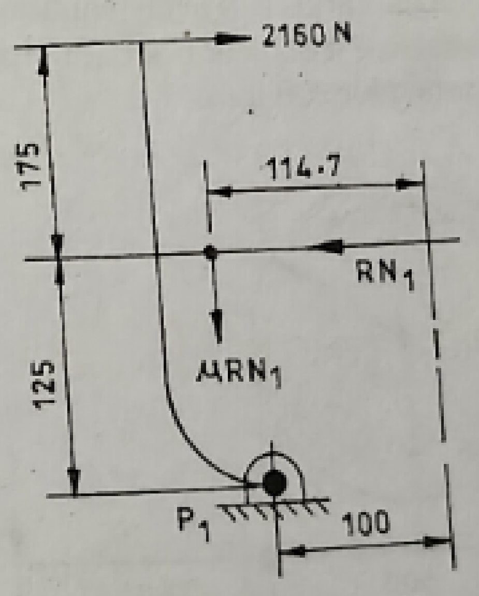


Fig. P_1

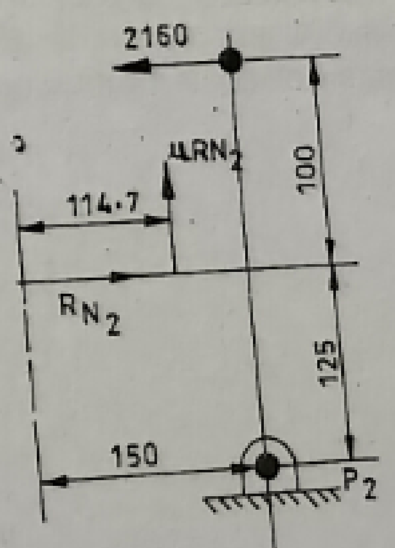


Fig. P_2

Taking moments about P_2 :

$$R_{N_2} \times 125 + \mu R_{N_2} (150 - 114.7) = 2160 \times 225$$

$$R_{N_2} = \frac{2160 + 225}{125 + .3 \times 35.3} = 3584.3 \text{ N}$$

Breaking torque $T_2 = 0.3 \times 3584.3 \times 114.7$

$= 123.3 \times 10^3$ N-mm = 123.3 N-m

∴ Total breaking torque = (172.3 + 123.3) = 295.6 N-m

∴ Heat generated $= 2\pi N \times T = 2\pi \times \frac{1500}{60} \times 295.6$

$= 46.4 \times 10^3$ W = 46.4 kW

To determine the width of the shoe :

T-XII-6 Maximum $R_N = 5007.3$ N, P_{max} for wood & C.I. = .4 – .6 MPa ;

$V = \pi \times 0.2 \times 1500 = 942$ m/min.

T-XII-5 Assuming PV = 110, $P = \frac{110}{942} = .12$ MPa

Let b be width of shoes

∴ $2 \times R \times b \times P \times \sin \theta/2 = R_N$,

$200 \times b \times .12 \times \sin 55 = 5007.3$

∴ b = 245.69 which is very high

So we can use $P_{max} = .6$ MPa which will give

$$b = \frac{245.69 \times .12}{0.6} = 50.93 \text{ so o.k.}$$

This will not satisfy PV limits criteria. Actually wood is never used for such high speeds.

Prob. 17.8 : A band brake uses a V belt as band. The pitch diameter of the V grooved sheave is 500 mm. The groove angle is 45° and the coefficient of friction is 0.25. For the diamensions shown in the figure, determine the maximum power rating for 300 rpm.

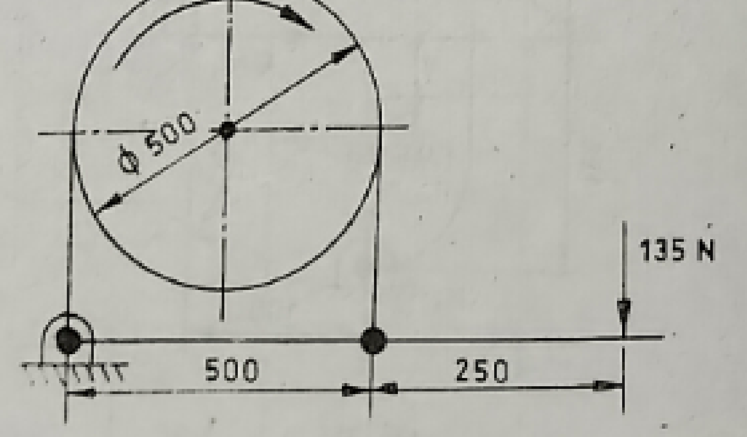


Fig.

Ref. Calculations

From the fig. $\theta = 180° = \pi$

For a V-belt, $\frac{F_1}{F_2} = e^{\frac{\mu\theta}{e^{\sin\alpha/2}}}$ where α is the groove angle.

$= e^{\frac{.25 \times \pi}{e^{\sin 22.5}}} = 7.8$

Taking moments about the fulcrum pin P :

$F_2 \times 500 = 135 \times 750$ ∴ $F_2 = \frac{135 \times 750}{500} = 202.5$ N

$F_1 = 7.8 \times 202.5 = 1579.5$ N

∴ Braking torque $= (F_1 - F_2)\frac{D}{2} = (1579.5 - 202.5) \times \frac{500}{2}$

$= 344.25 \times 10^3$ N-m = 344.25 N-m

∴ Power rating $= 2\pi NT = 2\pi \times \frac{300}{60} \times 344.25 = 10.8 \times 10^3$ W or 10.8 kW.

Prob. 17.9 : The figure shows a band brake used with punch press. When in use, the clutch is released when the crank is 120° past bottom dead centre. The brake is to be engaged at this point, and bring the crank to rest at top dead centre. The crank assembly has a mass moment of inertia of approximately 12 N-m sec^2 and is rotating at the rate of 40 rpm when the brake is engaged. The brake will be used about 3 times per minute, so the maximum pressure on the band should be limited to about 0.18 MPa for long life. Coefficient of friction can be taken as 0.35.

(a) Determine the required band width.

(b) Determine the required force F.

(c) Would any combination of direction of rotation and value of coefficient of friction make the brake self locking ? Explain briefly.

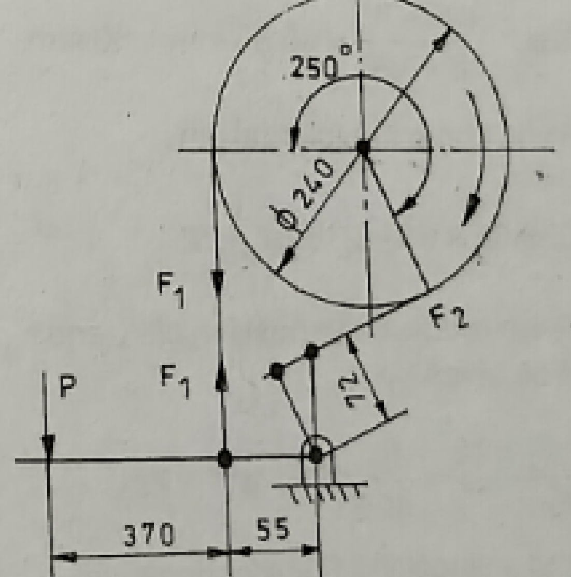


Fig.

Ref.

Calculations

$\theta = \frac{250}{180} \times \pi = 4.5$ rad. $\quad \therefore \frac{F_1}{F_2} = e^{0.35 \times 4.5} = 4.83$

Let the angular velocity when the brake engages be ω_o

$\omega_o = \frac{2\pi \times 40}{60} = 4.19$ rad/sec.

Angle turned for coming to rest $= 180° - 120° = 60°$ or $\frac{\pi}{3}$ rad.

Now $\omega^2 = \omega_o^2 + 2\,\alpha\,\theta$ where ω = final velocity = 0

$$\therefore \alpha = -\frac{\omega_o^2}{2\theta} = \frac{4.19^2 \times 3}{2 \times \pi} = -8.37 \text{ rad/sec}^2$$

The minus sign shows that it is retardation.

$\therefore$ Braking torque $= I\,\alpha = 12 \times 8.37 = 100.48$ N-m

$$\therefore F_1 - F_2 = \frac{100.48}{.12} = 837.33 \text{ N}.$$

Braking torque $= (F_1 - F_2)\frac{D}{2} = 100.48$

$$\frac{F_1}{F_2} = 4.83 \quad \therefore F_2 = 218.62 \text{ N}, \ \& \ F_1 = 1055.95 \text{ N}$$

It can be shown that for a band brake :

$F_1 = P_{max} \frac{D}{2} . b$ Where b is the width of the band

$$\therefore b = \frac{1055.95}{.18 \times 120} = 48.88 \text{ mm} \approx 50 \text{ mm}.$$

Taking moments about the fulcrum pin :

$P \times 425 = 1055.95 \times 55 - 218.62 \times 72 \quad \therefore P = \frac{58077.25 - 15740.64}{425} = 99.6$ N

For anticlockwise rotation, the position of F_1 and F_2 will be interchanged. Therefore the value of P will be given by,

$$F = \frac{F_2 \times 55 - F_1 \times 72}{425} = \frac{F_2}{425}\left(55 - e^{\mu\theta} \times 72\right)$$

If the brake is to self locking P = o or negative

$$e^{\mu\theta} = \frac{55}{72} = .7638$$

$\therefore \mu\,\theta = -.269$ which is not possible.

For clock wise rotation

$$P = \frac{F_1 \times 55 - F_2 \times 72}{425} = \frac{F_2}{425}\left(e^{\mu\theta} \times 55 - 72\right)$$

for $P = 0, \quad e^{\mu\theta} = \frac{72}{55} = 1.31$

$$\therefore \mu\,\theta = .269 \text{ But } \theta = 4.5$$

$$\therefore \mu = \frac{.269}{4.5} = .06$$

$\therefore$ For clockwise rotation if $\mu \leq .06$ the brake will be self-locking.

* * *

Chapter 18

BELT, CHAIN & ROPE DRIVES

18.1 Introduction :

Flexible machine elements, such as belts, ropes or chains are used for the transmission of power over comparatively long distances. When these elements are employed, they usually replace a group of gears, shafts and bearings or similar power transmission devices. They thus greatly simplify a machine and consequently a major cost reducing element. In addition, since these elements are elastic and usually long, they play an important part in absorbing shock loads and in damping out the effects of vibrating forces. Although this advantage is important as concerns the life of the driving machine, the cost reduction element is generally the major factor in the selection of these means of power transmission.

18.2 Belt Drives :

Ordinarily, belts are used to transmit power between two parallel shafts. The shafts must be separated to certain minimum distance, which is dependent upon the type of belt used, in order to work most efficiently. Belt drives have the following characteristics :

i) They are used for relatively long centre distances.
ii) The angular velocity ratio between the shafts is neither constant nor exactly equal to the ratio of pulley diameters because of the slip and creep of belts.
iii) It is possible to obtain the clutch action when flat belts are used, with the help of the fast and loose pulleys.
iv) It is possible to vary the angular velocity ratio, within a range, with the help of V belt drives by employing a spring loaded split pulley.
v) Some provision to adjust the centre distance is always required in a belt drive.
vi) An economical means of changing the velocity ratio in discreet steps can be obtained by employing step pulleys.

The commonly used belts are of the following types :

(a) Flat Belts : The flat belt consists of a strong elastic core surrounded by chrome leather or rubber. The elastic core consists of number of thin plies, made of cotton, rayon or nylon. For special applications *Textile belts or Balata belts* are used. Flat belts are very efficient for high speeds, they are quiet, they can transmit large power over long centre distances, they don't require large pulleys, and they can transmit power around corner or between pulleys at right angles to each other.

(b) V Belts : The V belts are made of cords or strands usually of cotton, rayon or nylon, and impregnated with rubber. Because of the wedging action between the belt and the pulley groove, the power transmitting capacity of a V belt is increased in comparison to a flat belt. These belts are used with smaller pulleys and at shorter centre distances. These drives are less efficient than flat belt drives but a number of them can be used on a single pulley. These belts are endless which eliminates the joints used in flat belts.

(c) Link V Belts : These belts are composed of a large number of rubberized fabric links joined by suitable metal fasteners. These are special purpose belts.

(d) Timing Belts : These are made of rubberized fabric and steel wire having teeth which fit into grooves cut on the periphery of the pulleys. They transmit power at a constant angular velocity ratio. Hence they are used in situations where constant velocity ratio is required through a flexible drive.

18.3 Design Procedure For Drives :

The minimum data required for design of any drive is :

1. Rated Power, type of load and type of service.
2. Speed of the two shafts.

For designing a drive, the following additional data is required which has got to be selected on the basis of the established design practice, if not already specified.

1. Centre distance.
2. Diameter of pulleys or pitch line velocity.
3. Size of the power transmitting element i.e. width and thickness of a flat belt, section and number of V belt, pitch and strands of a roller chain.
4. Other dimensions of pulleys and sprockets.

18.4 Design of Flat Belt Drives :

The steps to be followed in designing a flat belt drive are as follows :

Step 1 : Calculate the design power P_d as per T-XV-1, T-XV-2 and T-XV-3.

Step 2 : Calculate the pitch line velocity V_p if pulley diameters are given, otherwise assume suitable value for V_p as per T-XV-4. The selection of belt velocity depends on shaft sizes and their rotational speeds. Moreover there is a limit to the peripheral speeds of pulleys depending on its material of construction ; some guiding values are given in T-XI-6. Calculate the diameters of pulleys from the assumed value of V_p and suitable value for slip as per T-XV-4.

Step 3 : Calculate $(F_1 - F_2)$ as per T-XV-1.

Step 4 : Calculate the belt tension ratio $\frac{F_1}{F_2}$ as per T-XV-1. & T-XV-4 by assuming suitable centre distance, if not specified. Calculate the maximum belt tension F_1. Also calculate the initial tension F_i.

Step 5 : Select suitable value for the design stress, S_d as per T-XV-5 and T-XV-6.

Step 6 : Calculate the value of the centrifugal stress, S_{cf} as per T-XV-1 and T-XV-5.

Step 7 : Calculate the required belt section, b × t as per T-XV-1.

Step 8 : Assume suitable value for stress due to initial tension S_i as per T-XV-4.

Step 9 : Calculate the required belt section b × t as per the calculated value of F_i.

Step 10 : Select the larger value of belt section b × t calculated in steps 7 and 9.

Step 11 : Assume suitable value of t as per T-XV-4 & T-XV-5.

Step 12 : Calculate the required width of the belt 'b'.

Step 13 : Calculate the length of the flat belt required from the formula given below

$$L = \frac{\pi}{2}(D_1 + D_2) + 2C + \frac{(D_2 - D_1)^2}{4C}$$

This will complete the design of the flat belt. For designing the pulleys refer to article 18.7.

18.5 Design of V Belt Drives :

The steps to be followed in designing a V belt drive are as follows :

Step 1 : Calculate the design power P_d as per T-XV-9. Select the sections of the belts which can be used for the design power P_d. Generally two sections will be suitable.

Step 2 : Assume a suitable value for V_p as per T-XV-10 if not specified.

Step 3 : Calculate the pulley diameters. Select the suitable belt section based on the recommended minimum pulley diameter as per T-XV-8.

Step 4 : Calculate the power capacity per belt as per T-XV-9. Assume suitable value of angle α as per T-XV-11 or assume $\alpha = 40°$ for safe design. Assume suitable value of centre distance C as per T-XV-10 if not specified.

Step 5 : Calculate the number of belts required for the given P_d.

Step 6 : Calculate the width of the pulley rim as per T-XV-11. Generally it should be kept below 300 mm because for arm construction of pulley one set of arm is required below 300 mm width while two sets of arms are required above 300 mm width. The use of two sets of arms makes the manufacture of pulleys more difficult.

Step 7 : Calculate the length of the belt as per the formula given in article 18.4.

18.6 V-Flat Drives :

On V belt drives with a speed ratio of 3 to 1 or more and with short centre distance, the grooving of the larger pulley can be omitted without sacrifice of power or efficiency. A belt has its maximum pulling power when the arc of contact on the larger pulley is 240° to 250°. This angle is obtained when the following equation is satisfied.

$$\frac{D_2 - D_1}{C} = 0.5$$

The use of V flat belts is increasing very rapidly particularly for low and medium torques.

To design a V flat drive calculate the values of the products $\mu\,\theta$ and $\frac{\mu\theta}{\sin\alpha/2}$ for the larger and the smaller pulley. The power capacity will be decided by the smaller value of the product. This point will be clear in the solved problem.

18.7 Design of Belt Pulleys :

Generally the belt pulleys are made of cast iron which is an ideal material for belt pulleys since it can be cast in any designed, shape is readily machined, and gives pulleys which do not change their form. Solid cast iron pulleys can be used with peripheral speeds upto 25 m/sec, while split pulleys with the split through arms can be used upto about 20 m/sec. *Pressed steel* pulleys are lighter and less expensive than cast iron pulleys but they are not balanced and are not used for higher speeds. The rim of pulleys can be lined with wood or paper to increase the coefficient of friction.

The belt pulley construction is of three types depending on their diameters.

1. Solid construction 2. Web construction 3. Arms construction.

Having selected the type of construction, the flat belt pulleys can be completely designed as per T-XV-7. The procedure is illustrated in solved problem. For V belt pulleys the details of grooves are to be designed additionally as per T-XV-11. This will be illustrated in the solved problem.

18.8 Chain Drives :

The types of chains used for power transmission are *Block chains*, *Roller chains* and *Inverted tooth* or *Silent chains*.

1. Block chains are used only for transmitting power at a low rate and at moderate speeds, not exceeding 250 m/min. For steady loads, they can be used upto 8 kW. For fluctuating loads they should not be above 3 kW.

2. Roller chains are the most widely used for power transmission. They can be used without harmful effects upto 450 m/min. They can be used as a single strand chain or as multiple strand chain for larger powers.

3. Silent chains are quiet in operation and they can be used upto 750m/min. Under very favourable conditions and precision manufacturing, they can be used upto 1500 m/min. However they are costly and require good lubrication. Power capacities are not limited.

Out of these three, roller chains are most widely used. Hence they will be discussed in detail. The main advantage of chain drives is their positive drive coupled with flexibility.

18.9 Roller Chain Drives :

The roller chain consists of roller links and pin links which are assembled alternately to each other to make the full length of the chain. The dimensions of roller chain links are standardised in terms of pitch of the chain as shown in T-XV-3 of data book. The load carrying capacity of a chain depends on the pitch of the chain. In order to increase the load capacity of a chain with a certain pitch, several chain strands are assembled side by side by using long through pins. The maximum no. of strands used in practice are limited to 4.

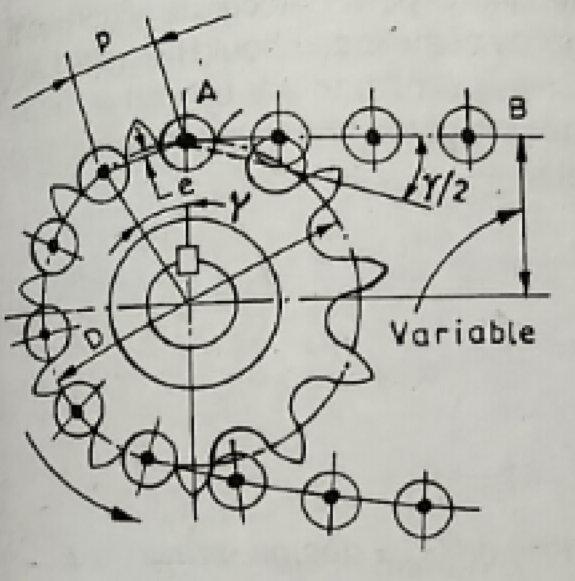


Fig. 18.1

An important factor affecting the operating smoothness of a roller chain drive, particularly at high speeds is *chordal action* as shown in Fig. 18.1.

The angle γ/2 through which the link swings as it enters contact is called angle of articulation. It can be seen that the magnitude of this angle is a function of the number of teeth. Rotation of the link through this angle causes impact between the rollers and sprocket teeth and also wear in chain joint. Since the life of a drive is a function of the wear and the surface fatigue strength of the rollers, it is important to reduce the angle of articulation as much as possible. Hence we try to select as large number of teeth on the smaller sprocket as possible with the constraint of space requirements. The number of sprocket teeth also affect the velocity ratio during the rotation through the pitch angle θ.

18.10 Design of Roller Chain Drives :

Similar to all other drives the generally given data is :

1. Rated power and 2. Speeds of the two machines.

The centre distance may be given or it will be assumed as discussed below. In designing the drive we are required to select the following :

1. Pitch of the chain
2. No. of strands
3. No. of teeth on the two sprockets
4. Length of the chain.

The steps to be followed for the design are as follows :

Step 1 : Calculate the design power P_d as per table T-XIV-1 and T-XIV-2.

Step 2 : Select the proper pitch and no. of strands as per F. 14.1. If the point falls between two pitches, select the higher pitch.

Step 3 : Assume suitable no. of teeth on the smaller sprocket as per T-XIV-3. Calculate the pitch diameter of the smaller sprocket as per T-XIV-1. Calculate the pitch line velocity V_p and check whether it is within the given range as per T-XIV-3. If not, change the number of teeth and repeat the calculations till a suitable pitch line velocity is obtained. Only odd number of teeth are selected for smaller sprocket. If the sprocket is to be keyed to a given shaft check that the maximum permissible bore as per T-XIV-3 is larger than the given shaft diameter.

Step 4 : Calculate the power capacity for single strand chain as per the exact equation given in T-XIV-1.

Step 5 : Calculate the number of strands for the design power P_d. The capacity factor for double, triple and quadruple strands are 1.7, 2.5 and 3.3 respectively.

Step 6 : Calculate the number of teeth on the larger sprocket and its pitch circle diameter.

Step 7 : Calculate the length of chain as per T-XIV-1 and check whether it is as per T-XIV-3.

Step 8 : Calculate other dimensions of the sprockets as per T-XIV-4.

The procedure will be clear from the solved problems.

18.11 Rope Drives :

Rope drives consisting of Manila, Cotton or Dacron rope on multiple grooved pulleys may often be most economical form of drive over long distances & for large amounts of power. Accurate alignments of the pulleys is not necessary because they are grooved. The velocity of the rope should be quite high - 26 m/sec is good speed for greatest economy. The action of drive is similar to a V belt drive. If the groove angle, rope diameter, sp. gravity and allowable tensile stress is given, the power transmitting capacity per rope can be found from the equations used for V belts.

$$\text{Power capacity/Rope} = (F_w - F_c)\,\frac{e^{\frac{\mu\theta}{\sin\alpha/2}} - 1}{e^{\frac{\mu\theta}{\sin\alpha/2}}}\cdot V_P$$

where $F_w = \frac{\pi}{4} d^2 \times S_d$, d = diameter of rope, S_d = design stress

$F_c = \frac{\pi}{4} d^2 \times S_{cf}$ S_{cf} = stress due to centrifugal tension

$= \rho\, V_P^2 \times 10^{-6}$

α = Groove angle ρ = Density of rope, kg/m^3

SOLVED PROBLEMS

Prob. 18.1 : A belt is required to transmit 20 kW from a pulley 1.2 m diameter running at 250 rpm to another pulley which runs at 450 rpm. The distance between the centres of the two pulleys is 2.7 m. The centre line of the drive makes an angle of 50° with the horizontal. Specify a suitable flat leather belt for the drive. Also design the larger pulley and draw a neat sketch for the same.

Ref.	Calculations

The drive is shown in the sketch below.

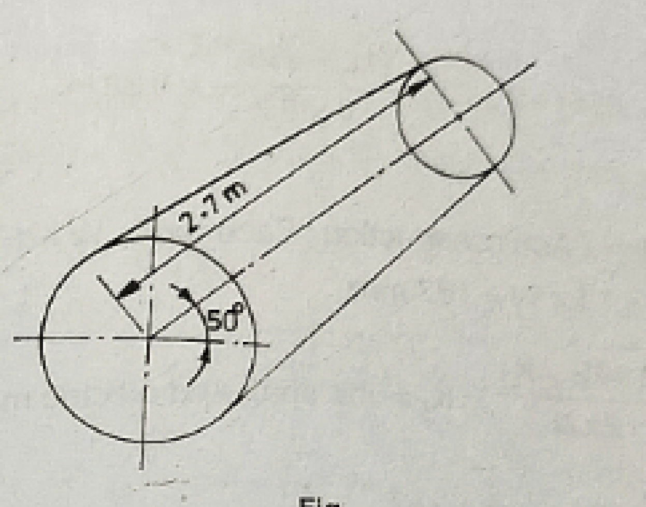


Fig.

T-XV-1 Given $P_R = 20$ kW, $P_d = P_R \times K_1 \times K\theta$

T-XV-2 Assuming it to be line shaft driven by electric motor, $K_1 = 1.10$, $K\theta = 1.1$

T-XV-3 $\therefore P_d = 20 \times 1.1 \times 1.1 = 24.2$ kW

$$V_p = \pi \times 1.2 \times \frac{250}{60} = 15.7 \text{ m/sec.}$$

$$\therefore (F_1 - F_2) = \frac{P_d}{V_p} = \frac{24.2 \times 10^3}{15.7} = 1541\text{ N}$$

To calculate D_2, assume slip = 2%

$$\therefore \pi D_2 N_2 = (1 - S)\,\pi D_1 N_1 \quad ; \quad D_2 = \frac{.98 \times 1.2 \times 250}{450} = .653 \text{ say } 655 \text{ mm.}$$

$$\therefore \theta = \pi - \frac{1.2 - .655}{2.7} = 2.94 \text{ rad}$$

T-XV-4 $\mu = 0.35$ for leather and cast iron

$$\therefore \frac{F_1}{F_2} = e^{0.35 \times 2.18} = 2.144$$

As $(F_1 - F_2) = 1541$ $\therefore F_1 = 2888$ N, $F_2 = 1347$ N

Initial tension $2\sqrt{F_i} = \sqrt{F_1} + \sqrt{F_2} = \sqrt{2888} + \sqrt{1347} = 90.44$

$\therefore F_i = 2045$ N

T-XV-5 Belt section, $b \times t = \frac{F_1}{S_d - S_{cf}}$ S_d for chrome tanned leather wire laced hand $= 3$ MPa

For leather, sp. gr. = .97 $S_{cf} = \rho p^2 \times 10^{-6}$

$\therefore \rho = .97 \times 1000 = 970$ kg/m^3 $= 970 \times 15.7^2 \times 10^{-6} = .239$ MPa

$$\therefore b \times t = \frac{2888}{3 - .239} = 1046 \text{ mm}^2$$

T-XV-4 From stress due to initial tension : Assume $S_i = 1.5$ MPa

$\therefore b \times t = \frac{F_i}{S_i} = \frac{2045}{1.5} = 1363$ mm^2 $\therefore$ Assume $b \times t = 1363$ mm^2 (larger value)

$t_{max} = .02 \times 655 = 13.1$ mm. But the power is quite low, hence select a medium double belt, t = 8.

$$\therefore b = \frac{1363}{8} = 170 \text{ mm.}$$

$$\text{Length of the belt} = \frac{\pi}{2}(1.2 + .655) + 2 \times 2.7 + \frac{(1.2 - 655)^2}{4 \times 2.7} = 8.38 \text{ m}$$

To design the larger pulley

T-XV-7 D = 1.2 m, Type of construction = Arm construction ; Face width, W = 1.1 b = 187 mm.
No. of arms -- 6, No. of sets = 1 for W = 187 mm.

T-XI-1 To design the shaft, $T_d = \frac{60 \times P_R \times K_1}{2\pi N}$ K_1 = line shaft and electric motor = 1.75

$$= \frac{60 \times 20 \times 1.75 \times 10^3}{2\pi \times 250} = 1340 \text{ N-m}$$

Assuming $\tau_{max} = 40$ MPa, low value when the shaft is being designed on the basis of T_d alone.

$$\tau_{max} = \frac{16\, T_d \times 10^3}{\pi D^3} \quad ; \quad \therefore d = \sqrt[3]{\frac{16 \times 10^3 \times 1340}{\pi \times 40}} = 55.4 \text{ mm.}$$

T-XV-7 Assume shaft diameter, $d_s = 60$ mm

$\therefore$ Hub diameter $D_h = 1.5 \times 60 + 25 = 115$ mm

$L_h = 1.5 \times 60 = 90$ mm.

Rim thickness $t = .375\sqrt{1200} + 3 = 16$ mm.

$$\text{Moment on each arm} = \frac{(F_1 - F_2)(D - D_h)}{n}$$

$$= \frac{1541 \times (1.2 - .155)}{6} = 278.66 \text{ N-m}$$

Assume elliptical cross section with major axis = 2 minor axis.

T-XI-6 $Z = \frac{\pi h^3}{64}$, Allowable stress in bending = 15 MPa

$$\sigma_b = \frac{M}{Z} \qquad \therefore 15 = \frac{278.66 \times 10^3 \times 64}{\pi h^3}$$

$$\therefore h = \sqrt[3]{\frac{278.66 \times 10^3 \times 64}{\pi \times 15}} = 72.6 \text{ mm}$$

$$\therefore \text{Hub dia. to accomodate 6 arms} = \frac{72.6 \times 6}{\pi} = 138.7 \text{ mm}$$

$\therefore$ Bending diameter = 140 mm

Generally bending is provided at rim as well as at the hub to get a gradual change in thickness of casting.

To design the key : Key cross section 18 × 11

T-VII-5 Depth of key way in hub = 4.4 mm

Length = 50 – 200, Length of hub = 90 mm. so o.k.

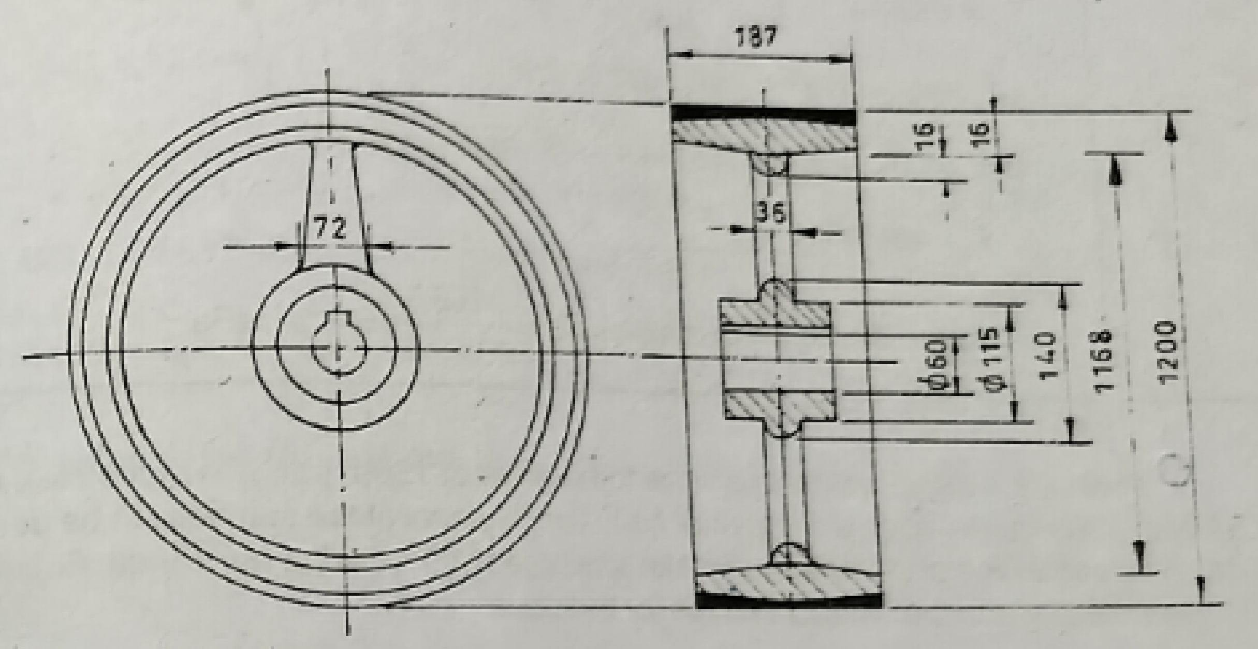


Fig.

Prob. 18.2 : A 1m diameter pulley transmits power to a 500 mm dia. pulley by means of medium double leather belt which is 400 mm wide. The centre distance is 4.2 m. Assume the coefficient of friction as 0.3. What maximum power can be transmitted by this drive if the stress in the belt is not to exceed 2.5 MPa. What is the speed of the belt when this power is transmitted ? What is the stress due to initial tension ?

Ref.	Calculations
T-XV-1	To determine the condition of maximum power transmission.
	Max. tension $F_1 = b \times t\,(S_d - S_{cf})$ $\quad S_{cf} = \rho v^2 \times 10^{-6}$
	$\frac{F_1}{F_2} = e^{\mu\theta}$ $\quad$ Power $= (F_1 - F_2)\,V = F_1 \times \frac{e^{\mu\theta - 1}}{e^{\mu\theta}} \times V$
	Combining all these relations we get,
	$P = K\,(S_d - \rho V^2 \times 10^{-6})\,V$ where K = numerical constant.
	For max. power $\frac{d(P)}{dV} = 0,$ $\quad S_d - 3\,\rho V^2 \times 10^{-6} = 0$ or $S_d = 3\,S_{cf}$
	In our case $S_d = 2.5$ MPa, $\rho = 970$ kg/m³ for leather belt

	$\therefore V=\sqrt{\frac{S_d\times10^6}{3\rho}}$ = 29.3 m/sec. velocity for max. power transmission.
	$S_{cf}=970\times29.3^2\times10^{-6}=.833$ MPa
T-XV	b = 400 mm, t = 8 mm $\therefore F_1=400\times8\times(2.5\times.833)=5334.4$ N
	$\frac{F_1}{F_2}=e^{\mu\theta}$, $\mu=0.3$, $\theta=\pi-\frac{1.0-0.5}{4.2}$ = 3rad $=e^{0.3\times3}=2.46$
	$F_1=5334.4$, $F_2=\frac{5334.4}{2.46}=2168.4$ $\therefore F_1-F_2=3166$ N
	$\therefore$ Power $=3166\times29.3=92.76$ kW
	$2\sqrt{F_i}=\sqrt{F_1}+\sqrt{F_2}=\sqrt{5334.4}+\sqrt{2168.4}=139.56$
	$\therefore F_i=4869.6$ N
	$\therefore$ Stress due to initial tension $S_i=\frac{4869.6}{400\times8}=1.52$ MPa

Prob. 18.3 : A centrifugal pump is to be drive at 1500 rpm by a four stroke diesel engine of 7.5 kW at 1000 rpm through a flat leather belt. The flywheel of the engine is to be used as the driving pulley. The centre line of the belt drive make an angle of 50° with the horizontal. Suggest suitable size of the leather belt and the centre distance for the drive.

Ref.	Calculations
	Given : $P_R=7.5$ kW at 1000 rpm Diesel engine Drive M/C = Centrifugal pump at 1500 rpm $\theta=50°$ Engine flywheel to be used as the driving pulley.
	To select the diameters of pulleys.
T-XI-6	Surface speed of flywheels $V_s<1600$ m/min. for C.I. and $P<75$ kW
T-XV-4	Belt velocity $V_p=1500$ to 1800 m/min.
	Selecting $V_p=1600$ m/min $\pi D\times1000=1600$
	$\therefore D=\frac{1600}{\pi}=.509\approx510$ mm
T-XV-1	$\therefore$ Pulley dia. of centrifugal pump with 2% slip $=\frac{510\times1000}{1500\times.98}\doteq347$ mm.
T-XV-4	Centre distance, minimum $=3.5\times$ larger pulley dia. $=3.5\times510=1785$ mm. max. = 7.5 m. Select C = 2m

	Taking $\mu=0.35$, $\theta=\pi-\frac{510-347}{2000}=3.06$ rad
	$\therefore\frac{F_1}{F_2}=e^{\mu\theta}=e^{.35\times3.06}=2.9$
T-XV-2	$P_d=P_R\times K_1\times K\theta=7.5\times1.35\times1.1=11.14$ kW
T-XV-3	$V_p=\frac{1600}{60}=26.67$ m/sec. $\therefore F_1-F_2=\frac{11.14\times10^3}{26.67}=418$ N
	$\frac{F_1}{F_2}=2.9$ $\therefore F_2=220$ N, $F_1=638$ N
	$2\sqrt{F_i}=\sqrt{F_1}+\sqrt{F_2}=\sqrt{638}+\sqrt{220}=40.09$ $\therefore F_i=402$ N
	$\therefore$ Belt section from consideration of initial tension.
T-XV-4	Assuming $S_i=1.5$ MPa,. $b\times t=\frac{F_i}{S_i}=\frac{402}{1.5}=268\ \text{mm}^2$
	Considering the stresses due to F_1 and F_c, belt section.
T-XV-5	$b\times t=\frac{F_1}{S_d-S_{cf}}$ $S_d=3.0$ MPa
	$=\frac{638}{3-.69}$ $S_{cf}=\rho P^2\times10^{-6}$
	$=276\ \text{mm}^2$ $=.97\times1000\times(26.67)^2\times10^{-6}$
	$=.69$ MPa
	So selecting the higher value of $b\times t=276\ \text{mm}^2$
T-XV-4	Selecting $t=.02\times347=6.94$
	We have a choice between light double or single medium
	Selecting light double t = 6 mm $\therefore b=\frac{276}{6}=46$ mm.
	For medium single t = 4mm, $b=\frac{276}{4}=69$ mm.
	So we can use either 1. Medium single of 70 mm width or 2. Light double of 46 mm width.

Prob. 18.4 : A 37 kW, 960 rpm, A.C. split phase motor is to be drive a pump at a speed of 320 rpm. The pump is for 12 hours per day service. The centre distance is to be approximately 1.25m. Determine the details of a multiple V belt drive i.e. pulley diameters, section and number of V belts. Also design and draw a neat sketch of the pulley on the pump shaft.

Ref.	Calculations

Given : $P_R = 37$ kW, $N_1 = 960$ rpm, $N_2 = 320$ rpm

T-XV-9 Electric motor driving a pump for 12 hr/day, $C = 1.25$ m

T-XV-8 $P_d = P_R \times K_1$, $K_1 = 1.1$

T-XV-2 $= 37 \times 1.1 = 40.7$ kW

For 37 kW we either use C or D section belts. The recommended min pulley dia. for the two sections are, C = 225 mm, D = 325 mm

$$\therefore V_p \text{ for C section belts } = \pi \times \frac{225}{1000} \times 960 = 678.24 \text{ m/min.}$$

$$\therefore V_p \text{ for D section belts } = \pi \times \frac{325}{1000} \times 960 = 980 \text{ m/min.}$$

T-XV-1 Recommended range of $V_p = 300 - 1500$ m/min.

Diameters of pulley on the pump shaft will be,

$$\text{For C section -- } 225 \times \frac{960}{320} = 675 \text{ mm.}$$

$$\text{For D section -- } 325 \times \frac{960}{320} = 975 \text{ mm.}$$

Recommended range of centre distance D_1 or $D_1 + D_2$

For C section -- 675 or 900 mm.

For D section -- 975 or 1300 mm.

Therefore the given centre distance is suitable for D section belts and using D section belts will also require less no. of belts. Hence select D section belts.

T-XV-9 $$\text{Power/belt } = (F_w - F_c) \frac{e^{\frac{\mu\theta}{\sin\alpha/2}} - 1}{e^{\frac{\mu\theta}{\sin\alpha/2}}} \cdot V_p$$

$$F_w = w^2 = 32^2 = 1024, \quad V_p = \frac{980}{60} = 16.33 \text{ m/sec.}$$

$$F_c = K_c \left(\frac{V_p}{5}\right)^2 = 15.75 \left(\frac{16.33}{5}\right)^2 = 168$$

$$\theta = \pi - \frac{975 - 325}{1250} = 2.62 \text{ rad.}, \quad \mu = 0.3, \quad \alpha = 40^\circ$$

$$\therefore \frac{\mu\theta}{\sin\alpha/2} = \frac{.3 \times 2.62}{\sin 20^\circ} = 2.3, \quad e^{2.3} = 9.95$$

$$\therefore \text{Power capacity/belt } = (1024 - 168) \times \frac{9.95 - 1}{9.95} \times 16.33$$

$$= 12574 \text{ w} = 12.574 \text{ kW}$$

$$\therefore \text{No. of belts } = \frac{P_d}{12.574} = \frac{40.7}{12.574} = 3.236 \text{ say 4.}$$

$$\text{Length of each belt } = \frac{\pi}{2}(.975 + .325) + 2 \times 1250 + \frac{(.975 - .325)^2}{4 \times 1.25}$$

$$= 4.6255 \text{ m.}$$

T-XV-11 For designing the pulley on the pump shaft :

For the width of the pulley rim, L

$L = (n - 1)\, e + 2f = 3 \times 37 + 2 \times 25 = 161$ mm.

T-XV-7

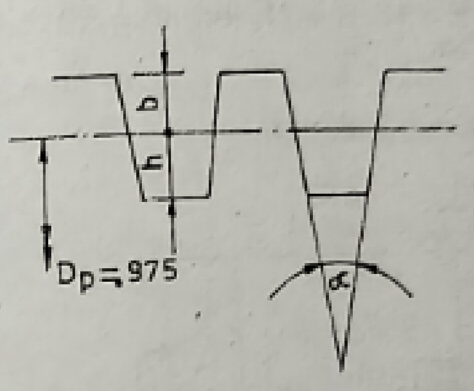


Fig.

For groove details :

α for minimum pitch diameter of 325 mm

$= 36^\circ$, $b = 8.1$, $h = 19.9$

For other pulley details :

Type of construction --

Arm construction No. of arms -- 6

Rim thickness $t = .375 \sqrt{975} + 3$

$= 14.7 = 15$ mm.

T-XI-1 For design of shaft :

T-XI-5 $$\text{Design torque } T_d = \frac{60 \times P \times K_1}{2\pi N} \qquad P = 37 \times 10^3, N = 320 \text{ rpm}$$

$$= \frac{60 \times 37 \times 10^3 \times 1.5}{2\pi \times 320} \; ; K_1 = 1.5 \text{ electric motor and centrifugal pump.}$$

$$= 1657 \text{ N-m}$$

Assume $\tau_{max} = 40$ MPa as we are designing on the basis of only T_d

$$\tau_{max} = \frac{16 \times 10^3}{\pi D^3} \times T_d \qquad \therefore D = \sqrt[3]{\frac{16 \times 1657 \times 10^3}{\pi \times 40}} = 59.75 \text{ say 60 mm.}$$

T-XV-7 Hub diameter $D_h = 1.5 \times 60 + 25 = 115$ mm.

Hub length $L_h = 1.5 \times 60 = 90$ mm.

$$\text{Moment on each arm } = \frac{(F_1 - F_2)(D - D_h)}{n}$$

$$\text{Total } (F_1 - F_2) = \frac{P_d}{V_p} = \frac{40.7 \times 10^3}{16.33} = 2492 \text{ N}$$

$$\therefore M/\text{arm} = \frac{2482 \times (.975 - .115)}{6} = 357.2 \text{ N-m}$$

T-XI-6 Z for elliptic arm section $= \frac{\pi h^3}{64}$

Allowable stress in bending = 15 MPa

$$15 = \frac{357.2 \times 10^3 \times 64}{\pi h^3} \qquad \therefore h = \sqrt[3]{\frac{357.2 \times 10^3 \times 64}{\pi \times 15}} = 79 \text{ mm}$$

Beading dia. to accomodate 6 arms $= \frac{79 \times 6}{\pi} = 150.78$ mm.

T-VII-5 For key design, key cross section b = 18, h = 11, length = 50 to 200

Depth of key in hub = 4.4 mm.

These dimensions are shown in the detail sketch.

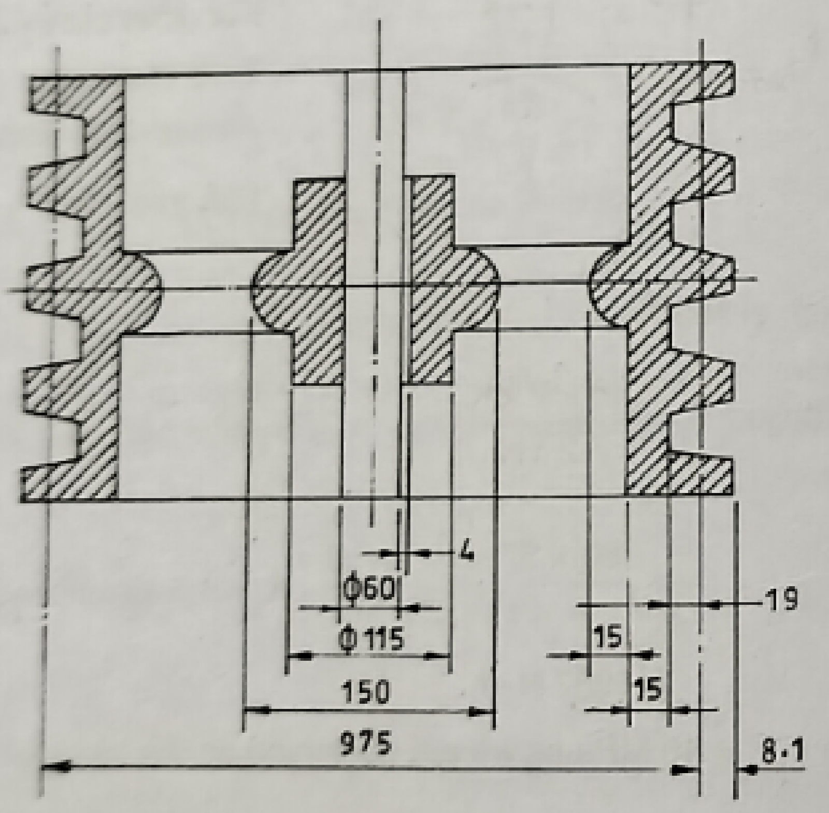


Fig.

Prob. 18.5 : A centrifugal water pump running at 1200 rpm is driven by a 90 kW diesel engine running at 400 rpm. The engine flywheel is to be used as the driving pulley. Design a V flat belt drive with centre as short as possible.

Ref.	Calculations
	Given : Driver is diesel engine of 70 kW at 400 rpm.
	Driven is centrifugal pump at 1200 rpm.
T-XV-8	$P_d = P_R \times K_1 = 90 \times 1.35 = 121.5$ kW

T-XV-2 ∴ We can use either D or E section belts.

For pulley diameters

T-XI-6 $V_s < 1800$ m/min for C.I. flywheel & $P > 75$ kW

T-XV-10 $V_p = 300$ to 1500 m/min for V belts

∴ Selecting $V_p = 1500$ m/min $= 25$ m/sec.

$$\pi D_1 \times 400 = 1500 \qquad \therefore D_1 = \frac{1500}{\pi \times 400} = 1.2 \text{ m}$$

$$\text{with no slip } D_2 = D_1 \frac{N_1}{N_2} = \frac{1.2 \times 400}{1200} = .4 \text{ m.}$$

T-XV-10 $C = D_1 + D_2 = 1.2 + .4 = 1.6$ m

T-XV-1 θ_1 on smaller pulley $= \pi - \frac{1.2 - 4}{1.6} = 2.64$

θ_2 on larger pulley $= \pi + \frac{1.2 - .4}{1.6} = 3.64, \qquad \mu = 0.3, \ \alpha = 40^\circ$

$$\frac{F_1}{F_2} \text{ for V belt drive } = e^{\frac{\mu\theta_1}{\sin\alpha/2}} = e^{\frac{0.3 \times 2.64}{\sin 20}} = 10.1$$

$$\frac{F_1}{F_2} \text{ for flat belt drive } = e^{\mu\theta_2} = e^{0.3 \times 3.64} = 2.98$$

T-XV-8 For V belt drive : As $D_2 = 0.4$ we cannot select E section which requires a min. pulley diameter of 540 mm.

Therefore we will select D section belts.

$$\text{Power capacity/belt} = (F_w - F_c) \frac{e^{\frac{\mu\theta_1}{\sin\alpha/2}} - 1}{e^{\frac{\mu\theta_1}{\sin\alpha/2}}} \cdot V_p$$

$$F_w = w^2 = 32^2 = 1024 \quad ; \quad F_c = K_c \left(\frac{V_p}{5}\right)^2 = 15.75 \left(\frac{25}{5}\right)^2 = 393.75,$$

$$e^{\frac{\mu\theta}{\sin\alpha/2}} = 10.1$$

$$\therefore \text{Power capacity/belt} = (1024 - 393.75) \frac{10.1 - 1}{10.1} \times 25 = 14196 \text{ W} = 14.2 \text{ kW}$$

$$\therefore \text{No. of belts} = \frac{121.5}{14.2} = 8.56 \text{ say } 9$$

For flat belt drive :

$$\text{Power capacity/belt} = (F_w - F_c)\,\frac{e^{\mu\theta_2} - 1}{e^{\mu\theta_2}}\,.\,V_p$$

$$= (1024 - 393.75)\,.\,\frac{2.98 - 1}{2.98} \times 25 = 10469 \text{ w} = 10.469 \text{ kW}$$

$$\therefore \text{ No. of belts} = \frac{121.5}{10.469} = 11.6 \text{ say 12 belts.}$$

Therefore use 12 no. D section belts.

Width of pulley rim = (n – 1) e + 2f = 11 × 37 + 2 × 25 = 457 mm.

Finally use 12 nos. of D section belts with centre distance of 1.6 m.

Prob. 18.6 : A roller chain drive is to be used in a power plant to transmit 15 kW from an electric motor to a reciprocating pump. The pump is to operate continuously 24 hours a day. The speed of the motor is 720 rpm and that of the pump is 250 rpm. Design the drive and give the specifications for the sprockets and the chain.

Ref. | **Calculations**

Given : P_R = 15 kW at 720 rpm, electric motor

Driven m/c -- Reciprocating pump at 250 mpm

T-XIV-1 Design power, $P_d = P_R \times K_1$ K_1 for moderate shock and 24 hr/day = 1.4

T-XIV-2 $= 15 \times 1.4 = 21$ kW

F-14.1 For smaller sprocket running at 720 rpm and P_d = 21 kW or 28.5 HP, the following alternatives are suitable.

Double strand No. 60 chain

Triple strand No. 50 chain

So selecting no. 60 chain with a pitch of 18.75 mm.

T-XIV-3 Selecting no. of teeth on smaller sprocket = 21

(mean value between 17 – 25)

T-XIV-1 Pitch diameter,

$$D_p = \frac{P}{\sin\left(\frac{180}{T}\right)} = \frac{18.75}{\sin\left(\frac{180}{21}\right)} = 125.8 \text{ mm.}$$

$$\therefore \text{ Pitch line velocity, } V_p = \pi \times \frac{125.8}{1000} \times 720 = 284.4 \text{ m/min. so o.k.}$$

$$= 4.74 \text{ msec.}$$

$$\text{Max. permissible bore} = \frac{(T-5)P}{4} \text{ for } P < 25.4 = \frac{(21-5) \times 18.75}{4} = 75 \text{ mm.}$$

T-XII-3 Shaft diameter for 15 kW, 720 rpm electric motor = 55 mm which is less than the max. permissible bore, hence the pitch and the no. of teeth selected seem to be o.k.

T-XIV-1 Power capacity/strand

$$= P^2\left[\frac{V}{104} - \frac{V^{1.41}}{526}\left(26 - 25\cos\frac{80}{T}\right)\right] \times 10^3$$

$$= 18.75^2\left[\frac{4.74}{104} - \frac{4.74^{1.41}}{526}\left(26 - 25\cos\frac{180}{21}\right)\right] \times 10^3$$

$$= 8.382 \times 10^3 \text{ or } 8.4 \text{ kW}$$

$$\text{No. of strands} = \frac{21.0}{8.4} = 2.5$$ which means it should be a triple strand chain.

Because V_p can be increased to 450 m/min and no. of teeth can be increased to 25, we can try to use this combination.

$$t = 25, \qquad D_p = \frac{18.75}{\sin\frac{180}{25}} = 149.6 \text{ mm}$$

$$V_p = \pi \times \frac{149.6}{1000} \times 720 = 338.2 \text{ m/min or } 5.64 \text{ m/sec.}$$

∴ Power capacity for single strand

$$= 18.75^2\left[\frac{5.64}{104} - \frac{5.64^{1.41}}{526}\left(26 - 25\cos\frac{180}{25}\right)\right] \times 10^3$$

$$= 9.77 \text{ kW}$$

$$\text{No. of strands} = \frac{21}{9.77} = 2.15$$

As this factor is greater than 1.7 so triple strand will be required.

Hence we select no. 60 chain with triple strand

No. of teeth on smaller sprocket = 21, D_p = 125.8

$$\text{No. of teeth on larger sprocket} = 21 \times \frac{720}{250} = 60.48$$

Selecting no. of teeth = 60

$$\text{Speed of the pump will be} = \frac{720 \times 21}{60} = 252 \text{ which is o.k.}$$

$$D_p = \frac{18.75}{\sin\left(\frac{180}{60}\right)} = 358.26 \text{ mm.}$$

For assuming the centre distance, we must assume θ_{min} on smaller sprocket = 120° or 2/3 π rad.

$$\theta = \pi - \frac{358.26 - 125.8}{C} \quad \text{or} \quad C = 222 \text{ mm.}$$

Recommended C_{min} = Dia. of larger sprocket + $\frac{1}{2}$ Dia. of smaller sprocket

$$= 358.26 + \frac{1}{2}\,125.8 = 420 \text{ mm.}$$

This is to be adjusted to get the chain length in even no. of pitches.

T-XIV-1

$$L_p = \frac{T_1 + T_2}{2} + \frac{2C}{P} + \frac{P(T_1 - T_2)^2}{40C}$$

$$= \frac{60 + 21}{2} + \frac{2 \times 420}{18.75} + \frac{18.75\,(60 - 21)^2}{40 \times 420} = 87$$

To make it even no. increase C by half pitch i.e.

$$C = \frac{18.75}{2} + 420 = 430 \text{ mm.}$$

$$\therefore L_p = 40.5 + \frac{2 \times 430}{18.75} + \frac{18.75\,(60 - 21)^2}{40 \times 430} = 88$$

To calculate other dimensions of sprocket

T-XIV-4

D_o of smaller sprocket, $D_{o1} = P\left[0.6 + \cot\left(\frac{180}{21}\right)\right]$

$$= 18.75\left[0.6 + \cot\left(\frac{180}{21}\right)\right] = 135.65 \text{ mm.}$$

D_o of larger sprocket $D_{o2} = 18.75\left[0.6 + \cot\left(\frac{180}{60}\right)\right] = 369$ mm

Root diameters $D_r = D_p - .625\,P \quad \therefore D_{r1} = 114.08, \quad D_{r2} = 346.54$

Width of sprocket teeth, $t_o = .56\,P - .15 = .56 \times 18.75 - .15 = 10.35$ mm

Total width of sprocket for triple strand chain

$W = 2\,A + t_o = 2\,(1.1525\,P) + (10.35)$

$= 2\,(1.1525 \times 18.75) + 10.35 = 53.5$ mm.

Prob. 18.7 : A roller chain is to be used in power plant to transmit 180 kW from an engine to a generator. The generator speed is 525 rpm and the engine speed is approximately 75 rpm. Design a roller chain drive and suggest the following dimensions of the sprockets :

1. Pitch diameters 2. Outside diameters 3. Width.

Ref.	Calculations

Given : Engine of 180 kW at 75 rpm driving a generator at 525 rpm in a power plant

T-XIV-1

$P_d = P_R \times K_1 = 180 \times 1.4$ (Moderate shock, 24 hrs/day)

$= 252$ kW or 342 HP

T-XIV-2

F.14.1

Selecting chain no 160 with $P = 50.8$. We should select no. of teeth on smaller sprocket $t = 19$ to get V_p to get V_p within the given range

$$D_{p1} = \frac{50.8}{\sin\frac{180}{19}} = 308.6 \text{ mm.}$$

$$\therefore V_p = \pi \times \frac{308.6}{1000} \times 525 = 508.7 \text{ m/min.} \quad = 8.5 \text{ m/sec.}$$

It is high. But as the power is large we can use it.

Power capacity of single strand chain

$$P = P^2\left[\frac{V}{104} - \frac{V^{1.41}}{526}\left(26 - 25\cos\frac{180}{T}\right)\right] \times 10^3$$

$$= 50.8^2\left[\frac{8.5}{104} - \frac{8.5^{1.41}}{526}\left(26 - 25\cos\frac{180}{19}\right)\right] \times 10^3 = 76.56 \text{ kW}$$

$\therefore$ Quadruple strand chain capacity $= 76.56 \times 3.3 = 252.67$ so o.k.

No. of teeth on larger sprocket $= 19 \times \frac{525}{75} = 133$

$$D_{p2} = \frac{50.8}{\sin\frac{180}{133}} = 2150.8 \text{ mm}$$

For O.D. of sprockets :

$$D_{o1} = P\left(0.6 + \cos\frac{180}{t}\right) = 50.8\left(0.6 + \cos\frac{180}{19}\right) = 334.9 \text{ mm.}$$

$$D_{o2} = 50.8\left(0.6 + \cos\frac{180}{133}\right) = 2180.7 \text{ mm.}$$

Width $W = 3\,A + t_o$

$= 3 \times 58.547 + 28.3 = 203.94$ mm

$A = 1.1525 \times 50.8 = 58.547$

$t_o = .56 \times 50.8 - .15 = 28.3$

* * *

Chapter 19

SPUR & HELICAL GEAR DRIVES

19.1 Introduction :

Gears, defined as toothed members transmitting rotary motion from one shaft to another, are amongst the oldest devices and inventions of man. Among the various means of mechanical power transmission (primarily gears, belts and chains) gears are generally the most rugged and durable. Their transmission efficiency is as high as 98% but they are usually more costly than belts and chains. As would be expected, gear manufacturing costs increase sharply with increased precision as required for the combination of high speeds and heavy loads and for low noise levels. The type of gear to be used depends on the relative orientation of the axes of the two shafts to be connected by the gears. Three types of orientations along with the types of gears used are given below :

1. **Parallel shafts :** For this orientation -- Spur and Helical gears are used.
2. **Non parallel-non intersecting or skew shafts :**

For this orientation -- Spiral and Worm gears are used.

3. **Intersecting shafts :** For this orientation - Bevel gears are used.

The design of gears is based on the strength consideration for intermittent service. However, for continuous duty the gear design is to be checked from wear consideration also. Further for high speed gears, dynamic loads are to be considered. For designing gear drives it is necessary to understand the kinematics of gear teeth and also know the terminology. These are generally given in books on theory of machines. The terms which will be required for the design of the spur gears are given below:

1. Circular Pitch, Pc :

Is the distance measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth. Thus a circular pitch is equal to the sum of the *tooth thickness* and width *of space.*

2. The Module m :

Is the ratio of the pitch diameter to the number of teeth. The module is the index of tooth size in S.I. units.

3. The addendum a : Is the radial distance between the top land and the pitch circle.

4. The debendum b :

Is the radial distance between the bottom land and the pitch circle. The whole depth h of tooth is the sum of addendum and dedendum. Generally the shape of gear teeth for power transmission is of involute form with pressure angles of 14 1/2° and 20°. In 20° teeth, there are two types -- *Full depth* where addendum is equal to module and *Stub teeth* where the addendum is 0.8 times module. The table below shows the standard proportions for these three types of gear teeth along with the minimum number of teeth required to avoid interference.

Table 19.1

m = module, mm

S.No.	Type of gear tooth	Addendum a	Minimum dedendum b	Minimum teeth	Minimum clearance
1	14 1/2° Full Depth	m	1.157 m	32	0.157 m
2	20° Full Depth	m	1.157 m	18	0.157 m
3.	20° Stub	0.8 m	m	14	0.2 m

19.2 Design of Spur Gear Drives :

For designing gears we must find out the following :

1. Type of gear teeth 2. No. of teeth 3. Module 4. Face width 5. Materials of construction 6. Accuracy Grade and 7. Heat treatment.

The data given for any drive is : 1) Rated power, type of load and type of service. 2) Speeds of the two shafts. The most important factor which affects the design is the pitch line velocity of gears V_p which in turn depends on the centre distance and the velocity ratio. In addition to the tangential load F_t acting on the gear tooth to produce the necessary torque, the actual load to which the gear tooth is subjected is increased due to the error in the profile of teeth and inertia effects due to the parts which are rotating with the shafts. This load under dynamic conditions, F_d is equal to (T-XVI-1).

$$F_d = F_t + F_i$$

The gear design will consist of finding the strength of gear tooth under bending, F_B by means of the modified Lewis equation T-XVI-1. Further assuming that the entire load is taken by one pair of tooth we must check that,

$$F_B > F_t$$

Further we must find out the dynamic load F_d acting on the gear tooth and the endurance strength of tooth F_{en} and check that

$$F_{en} > F_d$$

If the drive is being designed for continuous duty, we must calculate the limiting wear load, F_w by Buckingham's equation (T-XVI-1) which will give the required heat treatment for the gears. The procedure for the design of spur gears when the centre distance or number of teeth are given will be illustrated with the help of solved problems.

Step 1 : Select proper tooth profile, if not specified, to meet the constraints of space and noise level. Most commonly used profile is 20° full depth form. If the space availability is low, 20° stub form is used. If noise level is to be kept low & there is no constraint on space, 14 1/2° full depth form is used.

Step 2 : Select the number of teeth for the smaller gear called *pinion*, t_p as per T-XVI-7. Calculate the number of teeth on larger gear called gear, t_g.

Step 3 : By assuming the module of teeth as 'm' mm (unknown) calculate the pitch diameter of the pinion D_p.

$$D_p = t_p \cdot m, \text{ mm.}$$

Step 4 : Calculate the pitch line velocity, V_p in meter/sec.

$$V_p = \pi D_p \times N_p = \pi t_p \cdot \frac{N_p \cdot m}{1000}, \text{ m/sec.}$$

Step 5 : Calculate the design power P_d as per T-XVI-1 & T-XVI-2.

Step 6 : Calculate the tangential tooth load, F_t in terms of m.

$$F_t = \frac{P_d}{V_p} = \frac{P_d \times 1000}{\pi t_p . N_p . m} = \frac{K_1}{m}, N$$

where K_1 = numerical constant.

Step 7 : Assume suitable material for pinion and the allowable basic stress, So as per T-XVI-10. Low strength steel and cast iron is preferred for low speeds and medium power while better grade of steels are used for larger power and higher speeds. For continuous duty which requires heat treatment of gears alloy steels are preferred.

Step 8 : Calculate the modified Lewis form factors for the pinion and gear Y_p and Y_g as per T-XVI-5.

Step 9 : Assume a trial face width, b equal to '10 m' which is a mean value of the range given in T-XVI-7.

Step 10 : Assume a trial value for the velocity factor C_v as given below :

C_v = 0.40 for pinion speeds upto 1000 rpm
= 0.30 for pinion speeds between 1000 – 2000 rpm.
= 0.25 for pinion speeds above 2000 rpm.

Step 11 : Calculate the bending strength of the gear tooth F_B as per modified Lewis equation T-XVI-1.

$$F_B = S_o C_v b Y . m = K_2 m^2$$

where K_2 = numerical constant.

Step 12 : By equating F_B to F_t obtained in step 6, calculate the minimum module required for the drive.

$$K_2 m^2 = \frac{K_1}{m} \text{ or } m = \sqrt[3]{\frac{K_1}{K_2}}$$

Step 13 : Select the nearest standard module, m as per T-XVI-7.

Step 14 : Calculate the actual values of pitch diameters, V_p, F_t and C_v.

Step 15 : Select the material for the gear and its corresponding basic stresses So as per T-XVI-10. The gears of larger size are generally made of cast iron if no heat treatment is required and of cast steel if heat treatment is required. Forged steel gears are used if they are to be mass manufactured.

Step 16 : Calculate the product (S_o Y) for pinion and gear and decide which is weaker.

Step 17 : Calculate the actual width required for the gear by using the actual values of F_t, C_v, m and lower values of (S_o Y). Adjust within the suggested range as per T-XVI-7.

The above steps are sufficient for gears running at low velocities and for intermittent service. For higher velocities and continuous duty the gears are to be checked for dynamic loads and limiting wear strength as discussed below.

1. Calculate the dynamic load F_d by the Buckinghma's equation as per T-XV-1. For calculating the probable error from F. 16.1. Assume the class of gears with the help of F. 16.2 which gives the permissible errors for the given pitch line velocity.
2. Calculate the minimum value of load stress factor K by equating the limiting wear strength, F_w to the dynamic load F_d. F_w is calculated as per T-XVI-1.
3. Suggest suitable hardness for pinion and gear as per T-XVI-6.
4. Calculate the endurance strength of gear teeth F_{en} as per T-XVI-1 and T-XVI-9. Check that $F_{en} > F_d$. If not increase b within the usual range or change the material and the heat treatment.

The above steps will complete the design of the gear teeth. The next part is the design of the gear blank which may be of three types of construction.

1. Solid construction 2. Web construction & 3. Arms construction.

This design can be done as per T-XVI-14 and will be illustrated in the solved problems.

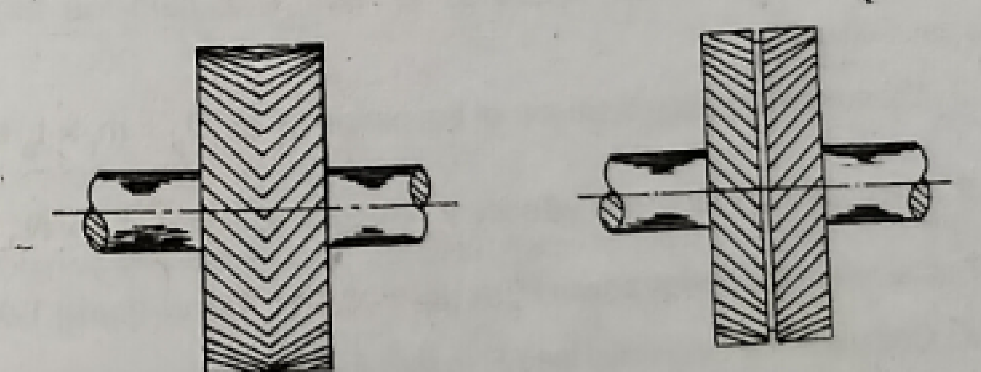
19.3 Design of Gear Trains :

When the velocity ratio is very high, it becomes desirable to reduce the speed in two or more steps. Ordinarily a speed ratio of about 6 for spur gears and a ratio of 12 for helical gears are considered practical limits for a single reduction. The total velocity ratio for a train of gears is equal to the product of the ratios of all pairs. As the speed is reduced, the tooth load goes on increasing which makes it necessary to use larger module teeth for the lower speed gears. Hence larger velocity ratios are used for higher speed pairs than lower speed pairs. Because of the cumulative effect of lost motion, the shock action in consecutive pairs of gears is increased and a gradual decrease of the allowable stress by about 10 to 15 percent in each lower step is advisable.

The procedure for the design of spur gear drives will be illustrated with the help of solved problems.

19.4 Helical Gears :

Helical gears differ from spur gears in that they have teeth that are cut in the form of a helix on their pitch cylinders instead of parallel to the axis of rotation. Helical gears may be used to connect parallel or nonparallel shafts. The gears which are used for non parallel shafts are called *crossed helical* or *spiral gears* which are only used for light loads. A helical gear may be considered to be composed of an infinite number of infinitesimally narrow staggered spur gears. The result is that each tooth is standing across the face so as to form a cylindrical helix. With one end of the tooth advanced over the other, engagement takes place progressively.



(a) Continuous tooth her ringbone gear

(b) Double helical gears

Fig. 19.1

The contact of spur gear teeth is a line extending all the way across the face of the tooth. The initial contact of helical gear teeth is a point which changes into a line as the teeth come into more engagement. In spur gears the line of contact is parallel to the axis of rotation, in helical gears the line is diagonal across the face of the tooth. It is this gradual engagement of the teeth and smooth transfer of load from one tooth to another which gives helical gears the ability to transmit heavy loads at high speeds. The helical gears can be used upto a pitch line velocity of 30 m/sec. However helical gears subject the shaft bearings to both radial and thrust loads. The presence of thrust load reduces the efficiency of helical gears as compared to spur gears. When the thrust loads become high or are objectionable for other reasons, it may be desirable to use double helical gears. A double helical gear is equivalent to two helical gears of opposite hand mounted side by side on the same shaft. They develop opposite thrust reactions and thus cancel out the thrust load. A special form of double helical gear is called *Herringbone gear* as shown in Fig. 19.1.

For connecting parallel shafts, helix angle must be same in both the gears but one has a right hand helix and the other a left hand helix.

19.5 Design of Helical Gears :

The procedure for design of helical gears is exactly same as that of spur gears except for the following differences.

In case of helical gears which are generally cut on hobbing machines, the teeth shape is of true involute form in the plane normal to the helix and not in the transverse plane. Hence we take the normal module, m_n as standard and base the design on normal module. Because of this the Lewis form factor Y is to be found on the basis of the *formative number of teeth*, t_f which are defined as the number of teeth that could be generated on the surface of a cylinder having a radius equal to the radius of curvature at a point at the tip of the minor axis of the ellipse obtained by taking a section of the gear in the normal plane. The formative number of teeth are given by the relation (T-XVI-11)

$$t_f = \frac{t}{\cos^3 \psi}$$,where ψ is the helix angle.

With the difference the design procedure will be as follows :

Step 1 : Select the standard tooth profile in the normal plane as per spur gears.

Step 2 : Select a suitable value of the helix angle between the given range of 15° to 45° (T-XVI-11). Helix angles of 15° and 23° are preferred for single helical gears because they have lower axial thrust while 30° and 45° are preferred for double helical gears.

Step 3 : Assume suitable number of teeth on the pinion t_p by referring to T-XVI-7. These values should be treated as formative number of teeth and the actual number to be used can be calculated as

$$t = t_f \, . \cos^3 \psi$$

Step 4 : Assume the normal module as 'm_n' mm. Calculate the tangential or transverse module 'm_t' as per T-XVI-11.

Step 5 : Calculate the pitch diameter of the pinion D_p as $D_p = m_t \times t_p$ mm.

Step 6 : Calculate the pitch line velocity V_p, m/sec. $V_p = \pi \dfrac{D_p}{1000} \times N_p$

Step 7 : Calculate the design power P_d as per T-XVI-11, T-XVI-2 and T-XVI-12.

Step 8 : Calculate the tangential load F_t in terms of m_n.

Step 9 : Assume suitable material for the pinion and its basic stress as per T-XVI-10.

Step 10 : Assume a trial value of C'_v as given below.

$C'_v = 0.5$ for pinion speed below 1000 rpm.
$= 0.45$ for pinion speed between 1000 – 2000 rpm.
$= 0.40$ for pinion speed above 2000 rpm.

Step 11 : Assume a trial value of the face width b as $b = \dfrac{12 m_t}{\tan \psi}$

Step 12 : Calculate the modified Lewis form factor Y for the pinion as per T-XVI-5 by using $t = t_f$.

Step 13 : Calculate the bending strength F_B from the Lewis equation in terms of m_n as per T-XVI-11.

Step 14 : By equating $F_t = F_B$ we will get the trial value of m_n. Select the nearest standard value of module as per T-XVI-7.

Now with this value of 'm_n' calculate the pitch diameters, face width, no. of teeth. Then check the design for dynamic load and limiting wear load by calculating F_d and F_w as per the modified equations given in T-XVI-11. The gear blank is to be designed on the same lines as spur gears and as per T-XVI-14. The entire procedure will be illustrated in the solved problems.

SOLVED PROBLEMS

Prob. 19.1 : An electric hoist is being designed for a lifting capacity of 30 kN at 80 m/min. The rope drum diameter will be 600 mm and it will be driven by an electric motor running at 1440 rpm through spur gears. Suggest suitable no. of stages for the reduction drive and the velocity ratio in each stage. Design the gears for continuous duty. Design the gear blank for the largest gear and specify the type of construction for all the gears. Suggest suitable capacity of the electric motor.

Ref.	Calculations
	Given : Electric hoist of capacity 30 kN at 80 m/min.
	Drive by electric motor at 1440 rpm
	Rope drum diameter = 600 mm.
	To calculate the no. of stages and velocity ratio for each stage :
	Rope drum speed N_D :
	$\pi \times \dfrac{600}{1000} \times N_D = 80$. $\therefore N_D = 42.46$ rpm.
	$\therefore$ Total velocity ratio $= \dfrac{1440}{42.46} = 33.9$
	Therefore we will need two stage reduction. The max. ratio recommended in one stage with spur gears is 6.
	$\therefore$ First stage reduction, $VR_1 = 6$.
	Second stage reduction, $VR_2 = \dfrac{33.9}{6} = 5.65$,
	The drive will be as shown below :
	Let N_m be motor speed, N_c the counter shaft speed and N_D the drum speed
	$\therefore N_m = 1440$ rpm,

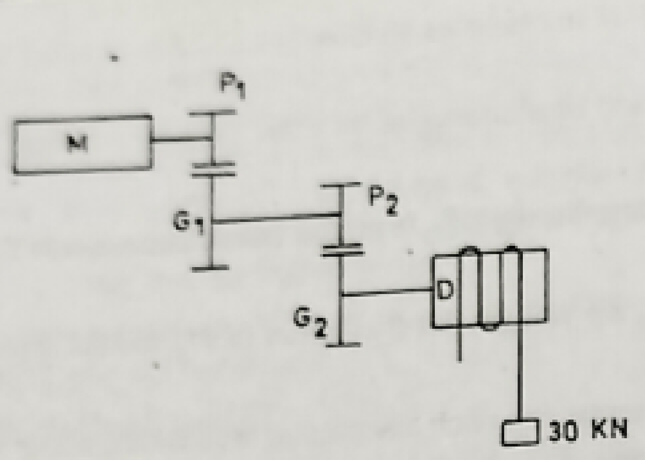


Fig.

$N_c = \frac{1440}{6}$

$= 240$ rpm

$N_c = \frac{1440}{6}$

$= 42.48$ rpm.

To design the gear pair.

P_1 and G_1 :

T-XVI-1 Design power $P_d = P_R \times K_1$, $P_R = \frac{30 \times 80}{60 \times .98 \times .98} = 41.65$ kW

(assuming stage efficiency 98%)

T-XVII-3 It will require a standard motor of 45 kW at 1440 rpm for which the shaft dia. will be 60 mm.

$\therefore P_d = 45 \times 1.5$ (light shock, 24 hrs/day)

$= 67.5$ kW

T-XVI-7 Assume the no. of teeth on $P_1 = 25$ for good balance between strength and wear. Let the module be 'm' mm.

$\therefore$ Pitch diameter $D_p = 25$ m, mm.

Pitch line velocity $V_p = \pi \times \frac{25\,m}{1000} \times \frac{1440}{60} = 1.884$ m, m/sec.

T-XVI-1 $\therefore F_t = \frac{P_d}{V_p} = \frac{67.5 \times 10^3}{1.884} = \frac{35.83}{m} \times 10^3$,N

T-XVI-10 Assuming 1045 steel with heat treatment $S_o = 210$ MPa

T-XVI-1 Bending strength $F_B = S_o\, C_v\, b\, Y\, m$

$S_o = 210$

$C_v = .30$ (trial value as pinion speed 1000 – 2000)

$b = 10$ m (trial value)

$Y = .485 - \frac{2.87}{25}$ (20°FD)

$= .3722$

$\therefore F_B = 210 \times .30 \times 10 \times .3722 \times m^2$

$= 234.48\, m^2$

T-XVI-5 Equating $F_B = F_t$

$234.48\, m^2 = \frac{35.83 \times 10^3}{m}$ $\therefore$ m = 5.33 mm

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T-XVI-7 Select a standard value of m = 6 mm $\therefore D_p = 25 \times 6 = 150$ mm From the consideration of motor shaft dia. = 60 mm. For keyed pinion,

$D_p = 2\,ds + 25 = 2 \times 60 + 25 = 145$ mm.

So m = 6 is o.k. from all considerations.

To calculate actual values of V_p, C_v and F_t :

$V_p = 1.884 \times 6 = 11.3$ m/sec.

T-XVI-3 $C_v = \frac{4.5}{4.5 + V_p} = \frac{4.5}{4.5 + 11.3} = .285$

$F_t = \frac{35.83 \times 10^3}{m} = \frac{35.83 \times 10^3}{6} = 5.97 \times 10^3$,N

T-XVI-10 As the gear is to be heat treated assume the material as cast steel heat treated with $S_o = 196$, $t_g = 25 \times 6 = 150$

T-XVI-5 Y for gear $G_1 = .485 - \frac{2.87}{1.50} = .466$

$\therefore (S_o Y)_p = 210 \times .3722 = 78.162$; $(S_o Y)_g = 196 \times .466 = 91.336$

So pinion is weaker with $(S_o Y)$ value = 78.162

$\therefore$ Actual width requied :

$(S_o Y) \times b \times C_v \times m = F_t$; $78.162 \times b \times .285 \times 6 = 5970$

$b = 44.66$ mm

T-XVI-7 $b_{min} = 8.5\,m = 8.5 \times 6 = 51$ mm. so select b = 51 mm.

To check for F_d and F_w :

F-16.1 : Permissible error at 11.3 m/sec. or 678 m/min = .04 mm.

F-16.2 : $\therefore$ We must use class 2 gears for which the probable error for 6mm module will be – 0.3 mm.

T-XVI-4 Deformation factor C for steel with 20° F D teeth, C = 11800

T-XVI-1 Dynamic load $F_d = F_t + \frac{21 V_p (Ceb + F_t)}{21 V_p + \sqrt{Ceb + F_t}}$

$Ceb = 11800 \times .03 \times 51 = 18054$

$Ceb + Ft = 18054 + 5970 = 24024$

$\sqrt{Ceb + F_t} = \sqrt{24024} = 155$ is printed as: $Ceb + F_t = 24024 = 155$

$21 V_p = 21 \times 11.3 = 23.73$

$\therefore F_d = 5970 + \frac{237.3 \times 24024}{237.3 + 155} = 5970 + 14532 = 20502$, N

T-XVI-1 $F_w = D_p \cdot b \cdot K \cdot Q$

$= 150 \times 51 \times K \times 1.714$

$= 13114.3\, K$

$D_p = 150$ mm, b = 51 mm

$Q = \frac{2 t_g}{t_p + t_g} = \frac{2 \times 150}{25 + 150} = 1.714$

Equating F_w to F_d we get K_{min} ; $K_{min} = \frac{20502}{13114.3} = 1.563$

T-XVI-6 Hardness for pinion = 350 Bhn Hardness for gear = 300 Bhn

To check for endurance strength :

T-XVI-9 $F_{en} = S_{eb} . b . Y . m$ $S_{eb} = 596$ MPa for steel with 350 Bhn

$= 596 \times 51 \times .3722 \times 6$ $b = 51, \quad Y = .3722, m = 6$

$= 67880.3$, N

This is greater than F_d so o.k.

To design the second stage gears P_2 and G_2.

T-XVI-7 Assume the no. of teeth on P_2 = 25, Module = m, mm ; No. of teeth o $G_2 = 25 \times 5.65 = 141$

$\therefore D_p = 25$ m, mm.

$$V_p = \pi \times \frac{25m}{1000} \times \frac{240}{60} = .314 \text{ m, m/sec.}$$

T-XVI-1 $$F_t = \frac{P_d}{V_p} = \frac{67.5 \times 10^3}{.314\,m} = \frac{215 \times 10^3}{m} \text{ N}$$

$F_B = S_o C_v Y m$ $S_o = .9 \times 210$ (Reducing by 10% for every stage) $= 189$

$= 189 \times .4 \times 10 \times .3722\ m^2$ $C_v = 0.4$ (Speed below 1000 rpm)

$= 281.38\ m^2$ $b = 10\ m,\ Y = 0.485 - \frac{2.87}{25} = .3722$

Equating F_B to F_t :

$$281.38\ m^2 = \frac{215 \times 10^3}{m} \qquad \therefore m = 9.14\ m$$

T-XVI-7 Selecting standard module of 10 mm.

T-XVI-3 $\therefore D_p = 25 \times 10 = 250$ mm, $C_v = \frac{3}{3+3.14} = .488$

$V_p = .314 \times 10 = 3.14$ m/sec. $F_t = \frac{215 \times 10^3}{10} = 21.500$ N

T-XVI-10 If we select same material for the gear, i.e. cast steel, heat treated for which $S_o = 196 \times .9 = 175.4$ the pinion will be weaker. To calculate the required face width

$F_B = F_t = (S_o Y) \times b \times C_v \times m$

$$\therefore b = \frac{21500}{189 \times .3722 \times .488 \times 10} = 62.6 \text{ mm.}$$

T-XVI-7 But $b_{min} = 8.5\ m = 85 \times 10 = 85$ mm. So select b = 85 mm.

To check for dynamic loading and limiting wear load.

F-16.1 In this stage with $V_p = 3.14 \times 60 = 188.4$ m/min so the permissible error is quite large and the probable error of class 3 gears is acceptable. But to reduce the dynamic load and the corresponding hardness of gears, we will use class 2 gears in this stage also.

$\therefore$ Probable error = 0.45 mm for the class 2 gears of module 10 mm.

C = 11,800

T-XVI-4 $Ceb = 11800 \times .045 \times 85 = 45135$, N $\sqrt{Ceb + F_t} = \sqrt{66635} = 248$

T-XVI-1 $Ceb + F_t = 45135 + 21500 = 66635$ N $21\,V_p = 21 \times 3.14 = 65.94$

$$F_d = F_t + \frac{21\,V_p\,(Ceb + F_t)}{21\,V_p + \sqrt{Ceb + F_t}}$$

$$= 21500 + \frac{65.94 \times 66635}{65.94 + 258} = 21500 + 13564 = 35064, \text{N}$$

$F_w = D_p . b . K . Q$ $D_p = 250$; $b = 85$; $Q = \frac{2 \times 141}{25 \times 141} = 1.70$

Equating F_w to F_d we get K_{min}

$$\therefore K_{min} = \frac{35064}{250 \times 85 \times 1.70} = .97$$

T-XVI-6 Hardness of pinion = 350 Bhn, Hardness of gear = 250 Bhn K = 1.376

T-XVI-1 $F_{en} = S_{eb} . b . Y . m = 596 \times 85 \times .3722 \times 10 = 188556$ N

T-XVI-9 This is larger than $F_d = 35046$ so o.k.

Therefore the final specifications for the two pair of gears are --

Item	Module mm	No. of teeth	Pitch dia.	Face width	Material	Hardness Bhn	Class	Type of construction
1st stage pinion	6	25	150	51	SAE 1045	350	2	Solid
1st stage gear	6	150	900	51	Cast steel	300	2	Arm
2nd stage pinion	10	25	250	85	SAE 1045	350	2	Web
2nd stage gear	10	141	1410	85	Cast steel	250	2	Arm

Therefore the largest gear is 2nd stage gear with $D_p = 1410$ mm. We design the gear blank for this,

$D_o = D_p + 2m = 1410 + 2 \times 10 = 1430$ mm

$D_r = D_p - 2 \times 1.2\ m = 1410 - 2.4 \times 10 = 1386$ mm.

T-XVI-14 For type of construction : $24m + 80 = 24 \times 10 + 80 = 320$ mm.

D_p is greater than 320 mm we will use arm construction No. of arms = 6.

To design the shaft on the basis of $P_d = 67.5$ kW at 42.48 rpm.

$2\pi NT = P$ or $2\pi \frac{42.48}{60} T_d = 67.5 \times 10^3$ $\therefore T_d = 15.2 \times 10^3$ N-m.

T-XI-1 Finding shaft dia. only on the basis of T_d by assuming $\tau_{max} < 40$ MPa

$$d_s = \sqrt[3]{\frac{16 \times 10^3 \times 15.2 \times 10^3}{\pi \times 40}} = 12.46 \text{ mm say } 125 \text{ mm.}$$

T-XVI-14 Hub dia. $D_h = 1.6\, d_s = 1.6 \times 125 = 200$ mm.

Hub length $L_h = 1.5\, d_s = 1.5 \times 125 = 187.5$ mm.

Rim thickness $= 2 \times m = 2 \times 10 = 20$ mm

Using elliptical cross section for arms

$$\text{Max. B.M. on the arm} = F_t \times \frac{(D_p - D_h)}{2n} = \frac{21500 \times (1410 - 200)}{2 \times 6}$$

$$= 2167.9 \text{ N-m}$$

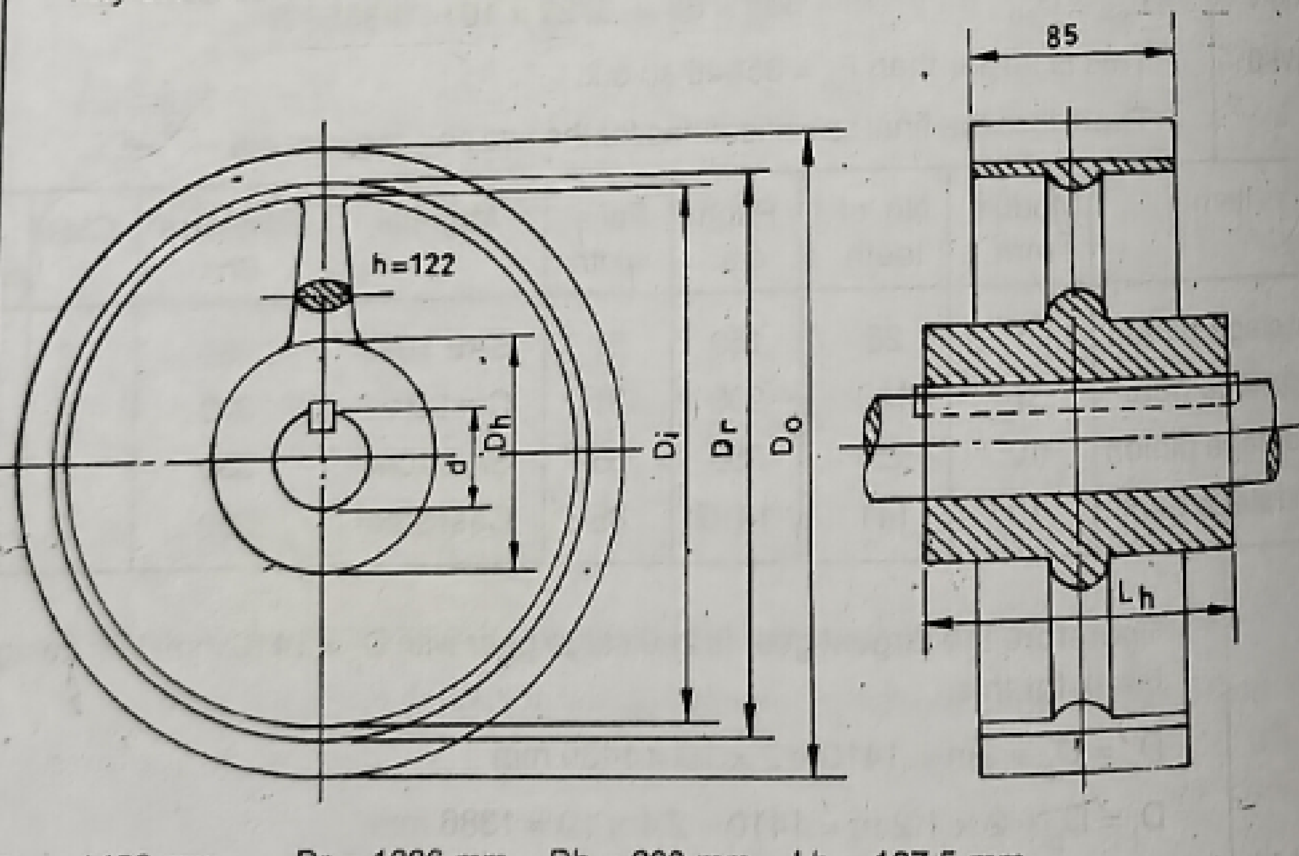
T-XI-6 Allowable stress for cast steel arms = 25 MPa

$$\therefore \text{Required } Z = \frac{2167.9 \times 10^3}{25} = 86716 \text{ mm}^3$$

$$\frac{\pi h^3}{64} = 86716 \quad \therefore h = \sqrt[3]{\frac{86716 \times 64}{\pi}} = 122 \text{ mm}$$

$$\text{Dia. for 6 arms} = \frac{122 \times 6}{\pi} = 233.25 \text{ mm (Bending dia.)}$$

T-VII Key cross section = 32 × 18 Depth of keyway in Hub = 7.4 mm.



Do = 1430 mm, Dr = 1386 mm, Dh = 200 mm, Lh = 187.5 mm

Di = 1386 − 2 × 20 = 1346 mm

Fig.

Prob. 19.2 : A 1.8 m ball mill runs at 25 rpm, the drive being through 14 1/2° involute spur gears with the following specifications.

S.No.	Item	Pinion	Gear
1.	Module	12 mm	12 mm
2.	Face width	125 mm	125 mm
3.	No. of teeth	15	176
4.	Material	SAE 1040 steel untreated 200 Bhn	Cast steel, untreated 150 Bhn
5.	Rated Power	55 kW	55 kW

(a) Check for strength and wear and give decision as regards the suitability of gears for this service.

(b) In practice the pinion wore out. To improve, it is suggested that keeping the gear dimensions same, change the gears to 20° FD type class 2 and make them of SAE 1045 steel heat treated to 300 Bhn. Will this suggestion give satisfactory results ?

Ref. | **Calculations**

$T_g = 176,\ T_p = 15,\ N_g = 25$ rpm $\quad \therefore N_p = \frac{25 \times 176}{15} = 292$ rpm

$D_p = 15 \times 12 = 180$ mm

$$V_p = \pi \frac{180}{1000} \times \frac{292}{60} = 2.75 \text{ m/sec.}$$

T-XVI-3 $C_v = \frac{3}{3 + V_p} = \frac{3}{3 + 2.75} = .52$

T-XVI-10 $S_o = 175$ MPa

T-XVI-5 $Y = .39 - \frac{2.16}{15} = .246$

T-XVI-2 $F_B = S_o\, C_v\, b\, Y\, m = 175 \times .52 \times 125 \times .246 \times 12 = 33579$ N

$\therefore$ Power transmitting capacity $= F_t \times V_p = F_B \times V_p = 33579 \times 2.75$

$= 92342$ W or 92.34 kW

T-XVI-2 $\therefore$ Load factor $= \frac{92.3}{55} = 1.678$ Light shock continuous duty.

To check from wear considerations

T-XVI-4 C = 11500 steel on steel and 13 1/2° FD teeth

F-16.1 e = .097 mm for m = 12 and class 3 gears

$Ceb = 11500 \times .097 \times 125 = 139437.5$

$Ceb = F_t = 139437.5 + 33579 = 173016$

$\sqrt{Ceb + F_t} = \sqrt{173016} = 416$

$21\, V_p = 21 \times 2.75 = 57.75$

T-XVI-1

$$F_d = F_t + \frac{21\,V_p\,(Ceb + F_t)}{21\,V_p + \sqrt{Ceb + F_t}}$$

$$= 33579 + \frac{57.75 \times 173016}{57.75 + 416} = 33579 + 21090 = 54670 \text{ N}$$

T-XVI-6

$\therefore F_w = D_p \,.\, b \,.\, K \,.\, Q$ $\qquad D_p = 180$ mm, $b = 125$ mm $K = .302$

$= 180 \times 125 \times .302 \times 1.843$ $\qquad Q = \frac{2 \times 176}{15 + 176} = 1.843$

$= 12522.7$ N

as $F_w < F_d$ the pinion wore out.

For new conditions : 20° FD teeth with SAE 1045 steel heat treated to 300 Bhn. The dynamic load will change because of change in C and e. New values of F_d and F_w will now be calculated.

T-XVI-4

$C = 11{,}800, \; c = .05$ mm

$Ceb = 11{,}800 \times .05 \times 125 = 73750$

$Ceb + F_t = 73750 + 333579 = 107329$

$\sqrt{Ceb + F_t} = \sqrt{107329} = 327.6$

$21\,V_p = 21 \times 2.75 = 57.75$

T-XVI-1

$$F_d = F_t + \frac{21\,V_p\,(Ceb + F_t)}{21\,V_p\,\sqrt{Ceb + F_t}}$$

$$= 33579 + \frac{57.75 \times 107329}{57.75 + 327.6} = 33579 + 16084 = 49613, \text{ N}$$

T-XVI-6

$F_w = D_p \,.\, b \,.\, K \,.\, Q$ $\qquad K = 1.376$ for 300 BHN steel gear and pinion of 20° FD teeth.

$= 180 \times 125 \times 1.376 \times 1.843 = 57059,$ N

as $F_w > F_d$ so o.k.

Prob. 19.3 : A pair of gears with 20° stub teeth is required to transmit 75 kW at a pinion speed of 5000 rpm. The velocity ratio is 10 and it is meant for continuous duty. Determine the module, face width and pitch diameters of both the pinion and gear. Also suggest suitable heat treatment and the accuracy grade required for these gears. Can a keyed pinion be used ?

Ref. | **Calculations**

T-XVI-1 Assuming steady load with continuous duty $K_1 = 1.25$

T-XVI-2 $P_d = P_R \times K_1 = 75 \times 1.25 = 93.75$ kW

$$N_p = 5000 \text{ rpm}, \; N_g = \frac{5000}{10} = 500 \text{ rpm}$$

T-XVI-7 Assume the no. of teeth on pinion, $t_p = 14$ (Minimum for 20° stub teeth)

Let the module be 'm' mm. $\qquad \therefore D_p = t_p \times m = 14$ m, mm.

$$V_p = \pi \times \frac{D_p}{1000} \times \frac{N_p}{60} = \pi \times \frac{14\text{m}}{1000} \times \frac{5000}{60} = 3.66 \text{ m, m/sec.}$$

$$F_t = \frac{P_d}{V_p} = \frac{93.75 \times 10^3}{3.66 \text{ m}} = \frac{25.6}{m} \text{ kN}$$

T-XVI-10

$F_B = S_o\, C_v\, b\, Y\, m$

$= 245 \times .25 \times 10 \times .344 \times m^2$

$= 210.7\ m^2$

$S_o = 246$ (SAE 1045 heat treated)

$b = 10$ m (trial value)

$C_v = .25$ (trial value for $N_p > 2000$ rpm)

$Y = .551 - \frac{2.90}{14} = .344$

Equating $F_B = F_t$

$$210.7\ m^2 = \frac{25.6 \times 10^3}{m} \qquad \therefore m = \sqrt[3]{\frac{25.6 \times 10^3}{210.7}} = 4.955$$

T-XVI-7

Select standard module of 5 mm.

$\therefore D_p = 14 \times 5 = 70$ mm $\qquad V_p = 3.66 \times 5 = 18.3$ m/sec.

$$F_t = \frac{25.6 \times 10^3}{5} = 5.118 \times 10^3 = 5118 \text{ N}$$

T-XVI-3

$$C_v = \frac{6}{6 + V_p} = \frac{6}{6 + 18.3} = .247$$

Assuming same material for gear i.e. SAE 1045 heat treated steel, Y for gear will be always be more. Hence calculating the required width on the basis of pinion.

T-XVI-1

$F_B = S_o\, C_v\, b\, Y\, m,$

$$b = \frac{5118}{245 \times .247 \times .344 \times 5} = 49.17 \approx 50 \text{ mm}$$

T-XVI-7

$b_{min} = 8.5\ m = 8.5 \times 5 = 42.5$ mm so $b = 50$ mm is o.k.

Check for dynamic load and wear load.

F-16.1 For $V_p = 18.3$ m/sec. $= 18.3 \times 60 = 1098$ m/min.

F-16.2 Permissible error = 0.026 mm.

Probable error with carefully cut gears for m = 5 is e = 0.026 mm.

T-XVI-4

C = 12300, steel on steel 20° stub teeth

$Ceb = 12300 \times .026 \times 50 = 15990$

$Ceb + F_t = 15990 + 5118 = 21108$

$\sqrt{Ceb + F_t} = \sqrt{21108} = 145.28$

$21\,V_p = 21 \times 18.3 = 384.3$

$$F_d = F_t + \frac{21\,V_p\,(Ceb + F_t)}{21\,V_p + \sqrt{Ceb + F_t}}$$

$$= 5118 + \frac{384.3 \times 21108}{384.3 + 145.28} = 5118 + 15317.43 = 20435.43 \text{ N}$$

T-XVI-1	For calculating K_{min}
	$F_w = D_p \,.\, b \,.\, K \,.\, Q \qquad D_p = 70,\ b = 50$
	$Q = \frac{2 \times 140}{14 + 140} = 1.82$
	Equating $F_w = F_d$
	$F_w = 70 \times 50 \times 1.82\ K = 20435$
	$\therefore K_{min} = \frac{20435}{70 \times 50 \times 1.82} = 3.21$
T-XVI-6	Bhn for pinion -- 450, Bhn for Gear = 450
T-XVI-1	$F_{en} = S_{eb} \,.\, Y \,.\, b \,.\, m \quad ; \quad S_{eb} = 700$ for steel Bhn = 450
	$= 700 \times .344 \times 50 \times 5 = 60200$ so o.k.
	To check whether a keyed pinion can be used, we will have to find the shaft dia.
	$\therefore 2\pi \frac{N_p}{60} \times T_d = P_d$
	$T_d = \frac{93.75 \times 10^3 \times 60}{2\pi \times 5000} = 179.14\ N-m$
	Assuming $\tau_{max} = 40$ MPa
T-XI-1	$d_s = \sqrt[3]{\frac{16 \times 179.14 \times 10^3}{\pi \times 40}} = 28.42$ say 30mm
T-XVI-7	For keyed pinion, $D_p = 2d + 25 = 2 \times 30 + 25 = 85$ mm
	But our $D_p = 70$ mm Hence keyed pinion cannot be used.
	For integral pinion $D_p = d + 25 = 30 + 25 = 55$ so o.k.
T-XVI-3	Accuracy grade 4 to 5
	The final results are :
	$m = 5$ mm, $b = 50$ mm, $t_p = 14$, $t_g = 140$
	$D_p = 70$ for pinion and $D_p = 700$ for gear
	Material = SAE 1045 steel heat treated.
	Pinion hardness = 450 Bhn, Gear hardness = 450 Bhn
	Generally plain carbon steel cannot be hardned to this Bhn. So we may use suitable alloy steel.

Prob. 19.4 : A double reduction helical gear reducer is shown in the figure. The input shaft 'a' receives power of 5.5 kW at 960 rpm by an electric motor. Gears 2 & 3 have a 30° helix angle and pressure angle a 20°. Pinion 2 is cut with 14 teeth, left handed, gear 3 has 54 teeth. The second pair of gears in the train 4 and 5 has 23° helix angle and 20° pressure angle. Pinion 4 is left handed and has 16 teeth, gears 5 has 36 teeth. Design all the gears for continuous duty. Calculate the thrust acting on shaft 'b'.

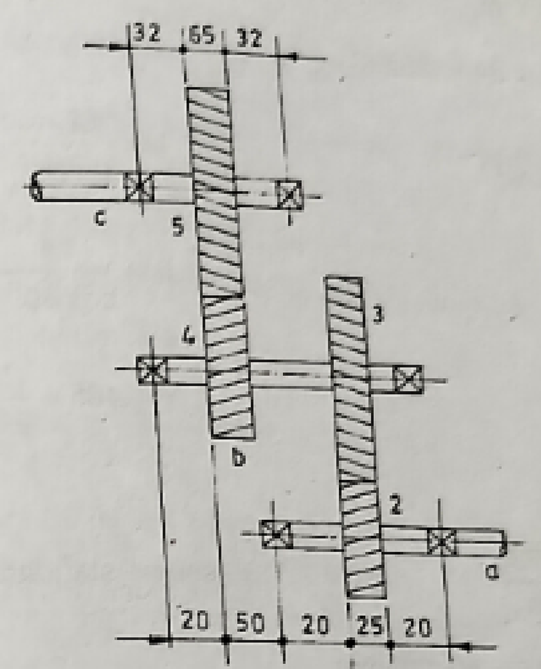


Fig.

Ref.	Calculations
	$P_R = 5.5$ kW at 960 rpm
	$t_2 = 14 \quad t_3 = 54 \quad \psi_1 = 30^\circ \quad \varphi_1 = 20^\circ$
	$t_4 = 16 \quad t_5 = 36 \quad \psi_2 = 23^\circ \quad \varphi_2 = 20^\circ$
	Speeds : $N_a = 960$ rpm
	$N_b = \frac{960 \times 14}{54} = 248.88$ rpm ; $N_c = \frac{248.88 \times 16}{36} = 110.6$ rpm
T-XVI-11	$P_d = P_R \times K_1 \times K_w$ Assume $K_1 = 1.25$ steady load & Cont. duty
T-XVI-2	$= 5.5 \times 1.25 \times 1.15$ $K_w = 1.15$ Continuous lubrication
T-XVI-12	$= 7.9$ kW
	To design Ist stage of gears 2 & 3
	Let the normal module be 'm_n' mm
T-XVI-11	$\therefore D_p = 14\, m_t = \frac{14 \times m_n}{\cos 30^\circ} = 16.17\, m_n$, mm
	$V_p = \pi \times \frac{16.17\, m_n}{1000} \times \frac{960}{60} = .812\ m_n$ m/sec.
	$F_t = \frac{P_d}{V_p} = \frac{7.9 \times 10^3}{.812\, m_n} = \frac{9730}{m_n}$ N
	To find the bending strength

T-XVI-10 $F_B = S_o C'_v\, b\, Y\, m_n$ $\quad S_o = 245$ SAE 1045 steel heat treated

T-XVI-5 $= 245 \times .5 \times 24 \times .352 \times m_n^2$ $\quad C'_v = .50$ Trial value $N_p < 1000$ rpm

$$= 1034.88\ m_n^2 \qquad b = \frac{12\ m_n}{\sin 30^o} = 24\ m_n$$

$$t_f = \frac{14}{\cos 30^o} = 21.55$$

$$Y = .485 - \frac{2.87}{21.55} = .352$$

Equating F_B to F_t

$1034.88\ m_n^2 = \dfrac{9730}{m_n}$ $\quad \therefore m_n = 2.113$ assume standard $m_n = 2.5$ mm.

T-XVI-7 $\therefore D_p = 16.17 \times 2.5 = 40.425$ mm

$V_p = .812 \times 2.5 = 2.03$ m/sec.

T-XVI-13 $C'_v = \dfrac{4.5}{4.5+2} = .69,\ F_t = \dfrac{9730}{2.5} = 3892$ N

Assuming same material for gear, pinion will be weaker. To find actual width

$\therefore F_B = F_t = S_o\, C'_v\, b\, .\, Y\, .\, m_n$

$3892 = 245 \times .69 \times b \times .352 \times 2.5$ $\quad \therefore b = 26.16$ mm

T-XVI-11 $b_{min} = \dfrac{4.5\ m_n}{\sin 30^o} = \dfrac{4.5 \times 2.5}{0.5} = 22.5$ mm so b = 26 mm is o.k.

To check for dynamic load and wea load

F-16.1 Permissible error for $V_p = 2$ m/sec = 120 m/min, e = .12 mm

F-16.2 For class 1 gears probable error for m = 2.5, e = .05 mm

T-XVI-4 C = 11800, b = 26 mm, $F_t = 3892$, $V_p = 2$ m/sec.

T-XVI-11
$$F_d = F_t + \frac{21\,V_p\left(Ceb\cos^2\psi + F_t\right)}{21\,V_p + \sqrt{Ceb\cos^2\psi + F_t}}$$

$$= 3892 + \frac{21 \times 2\left(11800 \times .05 \times 26\cos^2 30^0 + 3892\right)}{21 \times 2 + \sqrt{11800 \times .05 \times 26\cos^2 30^0 + 3892}} = 3892 + 3893.6$$

$$= 7785.6\ \text{N}$$

$$F_w = \frac{K.b.D_p.Q}{\cos^2\psi} \qquad Q = \frac{2t_g}{t_p + t_g} = \frac{2 \times 54}{14 + 54} = 1.588$$

$$K_{min} = \frac{F_d\cos^2\psi}{b.D_p.Q} = \frac{7785.6\cos^2 30^0}{26 \times 40.42 \times 1.588} = 3.5$$

Pinion hardness = 500 Bhn, Gear hardness = 500 Bhn

T-XVI-6 $F_{en} = S_{eb}\, .\, Y\, .\, b\, .\, m_n = 700 \times .352 \times 26 \times 2.5 = 16016$ so o.k.

For designing the second stage gears 4 and 5

$t_4 = 16,\ t_g = 36,\ N_b = N_4 = 248.88$ rpm, $\psi_2 = 23°$

Let the normal module be 'm_n' mm

T-XVI-11 $\therefore m_t = \dfrac{m_n}{\cos\psi_2} = \dfrac{m_n}{\cos 23^o} = 1.09\ m_n$

$\therefore D_p = 16 \times 1.09\ m_n = 17.38\ m_n$ mm

$$V_p = \pi \times \frac{17.38\ \text{mm}}{1000} \times \frac{248.88}{60} = .226\ m_n,\ \text{m/sec}$$

$$F_t = \frac{P_d}{V_p} = \frac{7.9 \times 10^3}{.226\ m_n} = \frac{34895}{m_n},\text{N}$$

Assuming same material for pinion and gear.

$S_o = 245 \times .9$ (because it is second stage)

$= 220.5$ MPa

$C'_v = 0.5$ (trial value speed < 1000 rpm)

$$b = \frac{12\ m_n}{\sin 23^o} = 30.7\ m_n,\ t_f = \frac{t}{\cos^3\psi_2} = \frac{16}{\cos^3 23^o} = 20.5$$

T-XVI-5 $Y = .485 - \dfrac{2.87}{20.5} = .345$

$\therefore F_B = S_o\, C'_v\, b\, Y\, m_n = 220.5 \times 0.5 \times 30.7\ m_n \times .345 \times m_n = 1167.7\ m_n^2$

Equating $F_B = F_t$; $\quad 1167.7\ m_n^2 = \dfrac{34895}{m_n}$ $\quad \therefore m_n = 3.11$

T-XVI-7 Selecting standard $m_n = 4$

$D_p = 17.38 \times 4 = 69.52$ mm. $\qquad V_p = .226 \times 4 = .9$ m/sec.

$F_t = \dfrac{34895}{4} = 8723.75$ N $\qquad C'_v = \dfrac{4.5}{4.5 + .9} = .83$

As material for pinion and gear are same, pinion is weaker. So calculating required b :

$220.5 \times .83 \times b \times .345 \times 4 = 8724$, b = 34.54 mm.

T-XVI-11 $b_{min} = \dfrac{4.5 \times 4}{\sin 23^o} = 46$ mm. So assume b = 46 mm.

	To calculate the dynamic load and wear load :
F-16.1	$e = .05,\quad C = 11{,}800,\quad b = 46,\quad V_p = .9$ m/sec. $\quad F_t = 8724$, N
	$Ceb = 11800 \times .05 \times 46 = 27140$
	$Ceb \cos^2\psi = 27140 \cos^2 23^\circ = 22996$
	$F_d = F_t + \dfrac{21\,V_p\left(Ceb\cos^2\psi + F_t\right)}{21V_p + \sqrt{Ceb\cos^2\psi + F_t}}$
	$= 8724 + \dfrac{21 \times .9\,(22996 + 8724)}{21 \times .9 + \sqrt{22996 + 8724}} = 8724 + 3043 = 11767$, N
	$F_w = \dfrac{D_p . b . k . Q}{\cos^2\psi},\quad Q = \dfrac{2 \times 36}{16 + 36} = 1.38,$
	$K_{min} = \dfrac{11767 \times \cos^2 23}{69.5 \times 46 \times 1.38} = 2.26$
T-XVI-6	Pinion hardness = 450 Bhn, Gear hardness = 350 Bhn
	As the hardness of pinion is quite high its F_{en} will be quite high. So design is o.k.
	To calculate the thrusts on the shaft 'b' which has two gears keyed to it Gear 3 and pinion 4. Gear 3 is right handed and pinion 4 is left handed.
T-XVI-11	$\therefore$ Axial thrust on gear 3, $F_a = F_t \tan\psi_1 = 3892 \tan 30^\circ = 2247$ N
	Axial thrust on pinion 4, $F_a = 8723 \tan 23^\circ = 3702.7$, N
	$\therefore$ Net thrust $= 3702.7 + 2247 = 5949.7$ N

Prob. 19.5 : A helical gear speed reducer is to be used with an 1800 rpm electric motor and a 450 rpm load which involves a moderate amount of shock. The 24 tooth pinion has a normal module of 3 mm, the face width of 40 mm, the normal pressure angle of 20° and helix angle of 23°. (App. hardness 262 Bhn) Both gears are made on SAE 2320 steel with case hardening. Estimate the power that can be transmitted by the reducer for continuous duty. Will the heat treatment be sufficient from the wear point of view ?

Ref.	Calculations
	Given : $N_p = 1800$ rpm, $\quad N_g = 450$ rpm, $\quad t_p = 24, \quad t_g = \dfrac{24 \times 1800}{450} = 96$
	$m_n = 3\text{mm},\ \varphi_n = 20^\circ,\ \psi = 23^\circ,\ b = 40$ mm.
	Both gears of SAE 2320 steel with Bhn = 262
	To find P_R for continuous duty & moderate shock
T-XVI-11	$m_t = \dfrac{m_n}{\cos\psi} = \dfrac{3}{\cos 23^\circ} = 3.26$ mm.
	$D_p = 24 \times 3.26 = 78.22$ mm
	$V_p = \pi \times \dfrac{78.22}{1000} \times \dfrac{1800}{60} = 7.4$ m/sec.
T-XVI-13	$C'_v = \dfrac{6}{6 + V_p} = \dfrac{6}{6 + 7.4} = .447$
	$t_f = \dfrac{t}{\cos^3\psi} = \dfrac{24}{\cos^3 23^\circ} = 30.77$
T-XVI-10	$Y = .485 - \dfrac{2.87}{30.77} = .39$
T-XVI-10	$S_o = 350$
	$\therefore F_B = S_o\, C'_v\, b\, Y\, m_n = 350 \times .447 \times 40 \times .39 \times 3 = 7321.86$ N
	Equating $F_B = F_t$; $P_d = F_t \times V_p = 7321.86 \times 7.4 = 54182 = 54.18$ kW
T-XVI-2	K_1 for medium shock continuous duty = 1.8 and $K_w = 1.15$
T-XVI-12	$\therefore P_R = \dfrac{54.18}{1.8 \times 1.15} = 34.6$ kW
T-XVI-11	
T-XVI-6	$F_w = \dfrac{D_p . b . K . Q}{\cos^2\psi} \qquad K = .920\ ; \qquad Q = \dfrac{2 \times 96}{24 + 96} = 1.6$
	$= \dfrac{78.22 \times 40 \times .92 \times 1.6}{\cos^2 23} = 5435$ N
	As $F_w < F_B$ it will not be suitable from the point of view of wear for continuous duty.

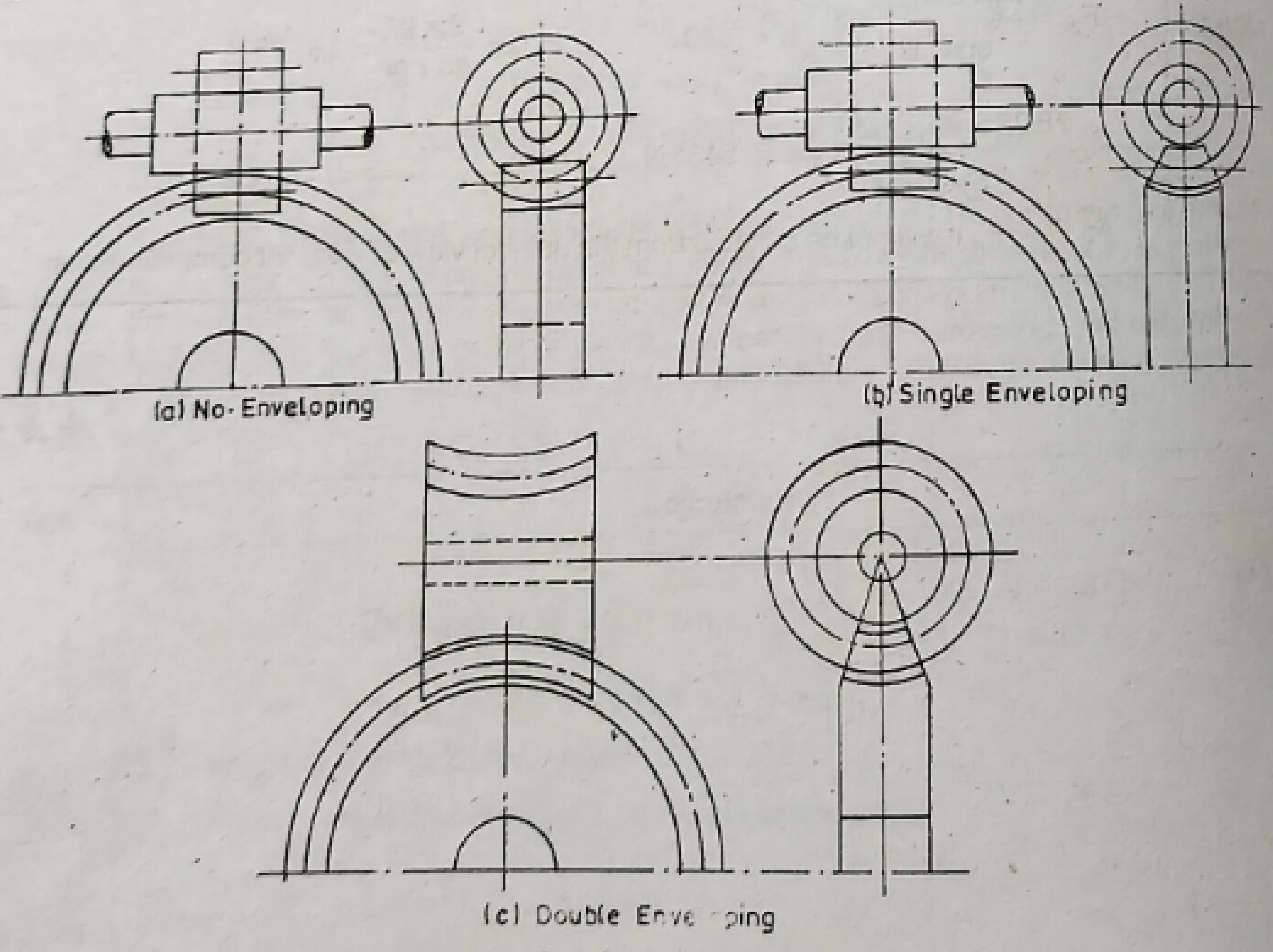
* * *

Chapter 20

WORM & BEVEL GEAR DRIVES

20.1 Worm Gearing :

Worm gearing is widely used to transmit power at large velocity ratios between non-intersecting and non-parallel shafts that are usually but not necessarily at right angles. The worm drive consists of a threaded worm in mesh with a gear, sometimes called worm wheel. Worm gear sets, theoretically have a point contact which limits their power transmission capacity. Therefore to improve their power transmission capacity they are either *single or double enveloping* as shown in Fig. 20.1.A single enveloping gear set is one in which the gear wraps around or partially encloses the worm which results in a line contact between the gears. A gear set in which each element partially encloses the other is double enveloping and this gives area contact between the gears. Thus double enveloping gears have larger power transmission capacity but require extremely precise mounting. Hence only single enveloping gears are used.

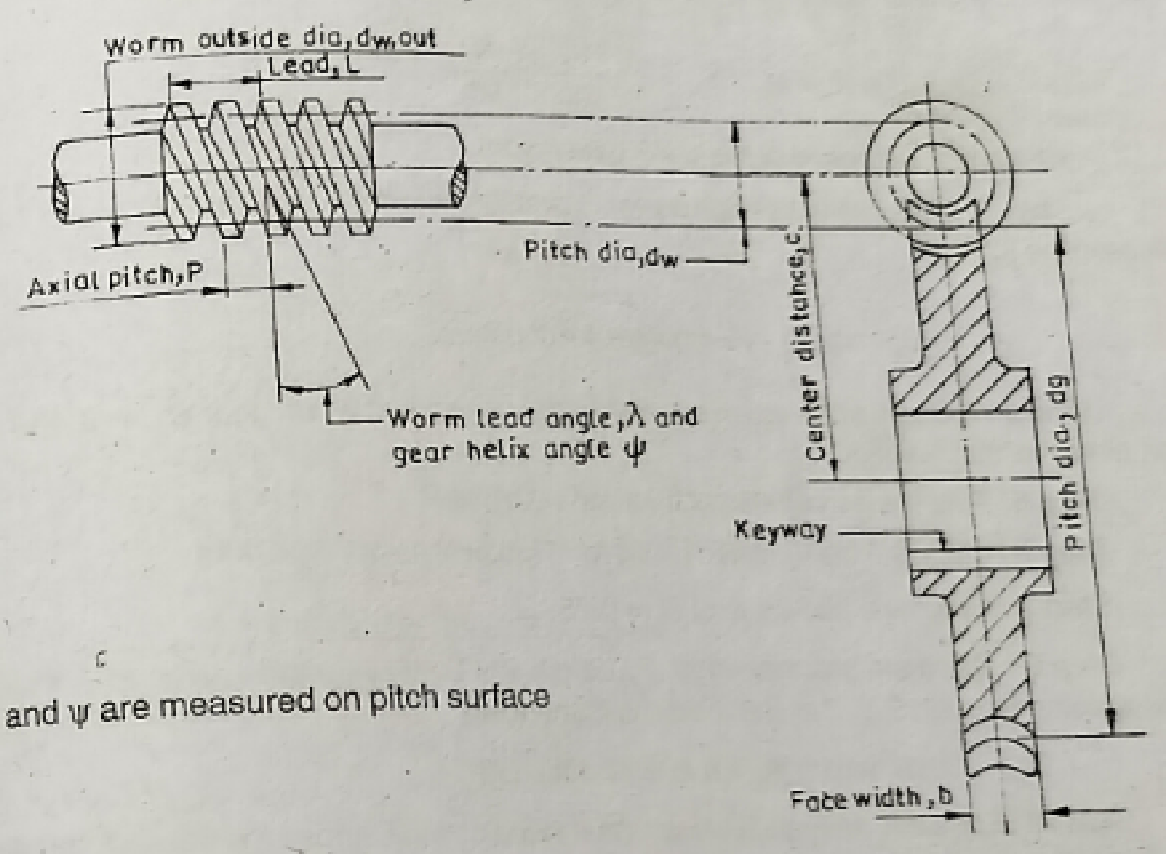

(a) No-Enveloping

(b) Single Enveloping

(c) Double Enveloping

Fig. 20.1

The nomenclature of a worm and worm wheel is also shown in figure 20.2. The worm and wheel of a set have the same hand of helix but the helix angles are usually quite different. The helix angle on the worm is generally quite large, and that on the gear very small. Because of this, it is usual to specify the lead angle λ on the worm and the helix angle ψ_g on the wheel. The lead angle λ and helix angle ψ_g are equal for a 90° shaft angle. In specifying the pitch of worm gear sets, it is customary to state the axial pitch P_a of the worm and the circular pitch P_c of the gear which are equal for a shaft angle of 90°.



Note : λ and ψ are measured on pitch surface

Fig. 20.2

In a worm gear drive, the frictional losses are quite high as compared to other gear drives because of large rubbing velocity. The efficiency of a worm drive depends on the velocity ratio and as a rough estimate the efficiency can be taken as :

$$\eta = 1 - .005\,R$$

where R is the velocity ratio.

It is therefore necessary to calculate the temperature rise of the lubricating oil which depends on the heat generated and the heat dissipated. The heat generated depends on the power and the efficiency of the drive while the heat dissipated depends on the centre distance and the velocity ratio. Based on AGMA recommendations, the limiting input power rating of a plain worm gear unit from the stand point of heat dissipation, for worm speed upto 2000 rpm may be estimated by,

$$P = \frac{3650\,C^{1.7}}{R+5}$$

where P = permissible power input in kW

R = velocity ratio

C = centre distance in meters

20.2 Design of Worm Gear Drives :

The procedure for design of worm gear drives is as follows :

Step 1 : Select the number of teeth on worm, t_w as per T-XVI-16. Calculate the number of teeth on worm gear t_g, which should be greater than 29.

Step 2 : Choose proper lead angle λ and normal pressure angle φ_n, as per T-XVI-16.

Step 3 : Assume the module of worm gear teeth as 'm' mm. Calculate the pitch diameter of the gear and the pitch line velocity of the gear as :

$$D_p = t_g \times m, \text{ mm} ; \quad V_p = \pi \times \frac{D_p}{1000} \times \frac{N}{60} \text{ m/sec.}$$

where N is the speed of the worm gear in rpm.

Step 4 : Calculate the design power P_d and the tangential tooth load F_t as per T-XVI-15. This will be in the form

$$F_t = \frac{K_1}{m} \quad \text{where} \quad K_1 = \text{numerical constant.}$$

Step 5 : Assume materials of construction for worm and worm gear as per T-XVI-18 and their basic stress as per T-XVI-10.

Step 6 : Assume a trial value of b as per T-XVI-19.

Step 7 : Calculate the modified Lewis form factor Y as per T-XVI-15.

Step 8 : Assume a trial value of $C'_v = 0.75$

Step 9 : Calculate beam strength F_B as per the Lewis equation given in T-XVI-15 and the values assumed in step 5, 6, 7 and 8. It will be in the form

$$F_B = K_2 \,.\, m^2 \text{ where } K_2 = \text{numerial oonstant.}$$

Step 10 : Equate F_B to F_t and calculate trial value of 'm'. Choose the nearest standard module as per T-XVI-7.

Step 11 : Calculate pitch diameter of the worm D_w as per T-XVI-19 and see that it is within the given range as per T-XVI-16.

Step 12 : Calculate actual values of D_g, V_p, F_t and C'_v.

Step 13 : Calculate the face width b of the gear. Calculate the beam strength F_B and check that $F_B > F_t$. If not change the module and repeat the procedure.

Step 14 : Calculate the dynamic load on the gear by the relation $F_d = \frac{F_t}{C'_v}$

Step 15 : Calculate the limiting wear load F_w as per T-XVI-15. Check that $F_w > F_d$. If not change the module m.

In case of worm gear drives an additional check for thermal loading and heat dissipation is necessary because these drives are generally meant for continuous duty. For these checks the following steps are required.

Step 1 : Calculate the rubbing velocity V_R and coefficient of friction 'μ' as per T-XVI-16.

Step 2 : Calculate the efficiency of drive as per T-XVI-15.

Step 3 : Calculate heat generated Q_1, heat dissipation capacity Q_2 in terms of ΔT as per T-XVI-15.

Step 4 : Calculate ΔT by equating Q_2 to Q_1. Check that the maximum oil temperature is within the given limits. The procedure will be illustrated with the help of solved problems.

20.3 Bevel Gear Drives :

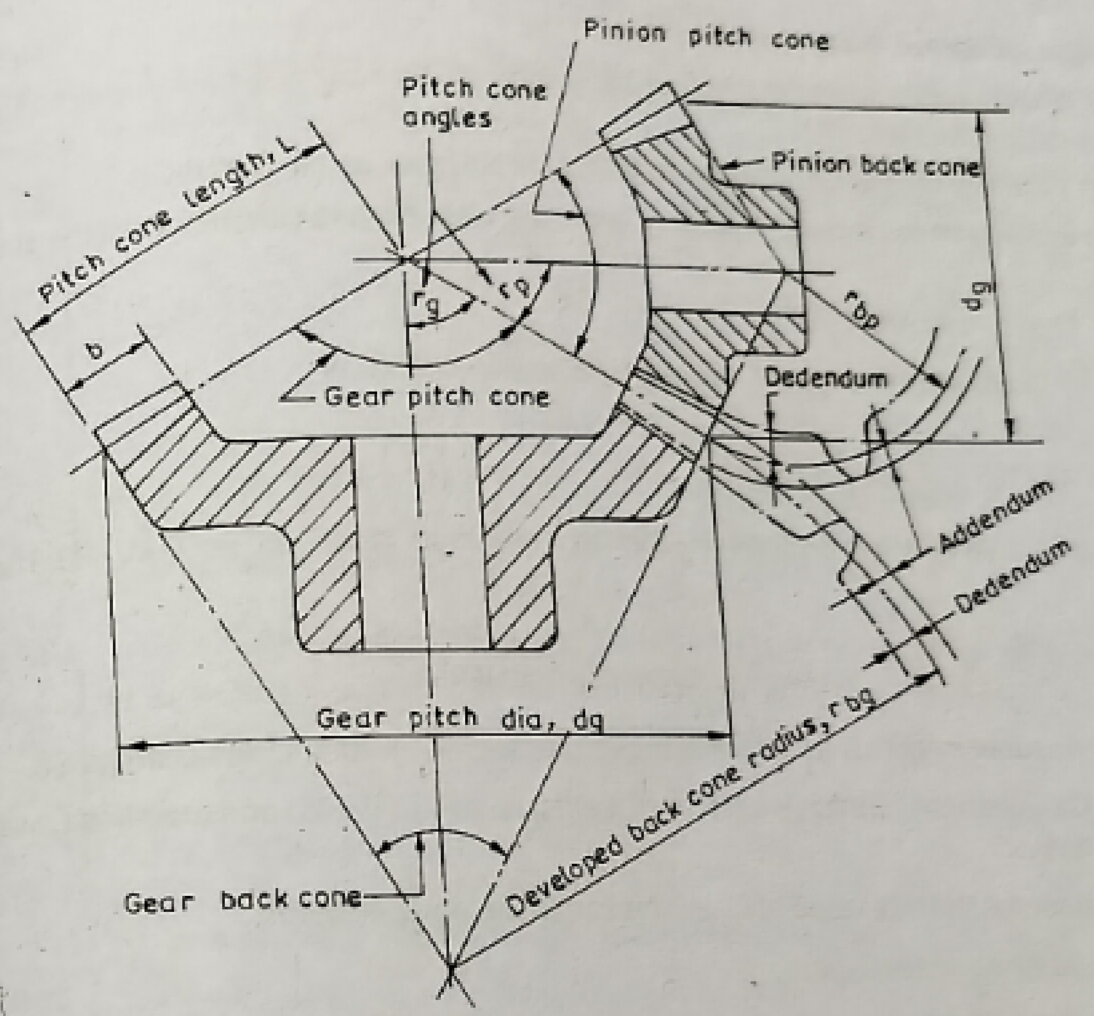


Fig. 20.3

The bevel gears are used to transmit motion between intersecting shafts. Although bevel gears are usually made for a shaft angle of 90°, they may be produced for almost any angle. Generally straight tooth bevel gears are used. These correspond to spur gears. These bevel gears are easy to design and simple to manufacture and give very good results in service if they are mounted accurately and positively. As in the case of spur gears, they become noisy at the higher values of pitch line velocity. In these cases it is often good design practice to go to the sprial bevel gears which is the bevel counterpart of the helical gear. However in this chpater we will only discuss the design of straight tooth bevel gears for 90° shaft angle.

The terminology of bevel gears is illustrated in Fig. 20.3. The pitch or module of bevel gears is measured at the large end of the tooth and the pitch diameters are calculated in the same manner as for spur gears. The pitch angles are defined by the pitch cones meeting at the apex. They are related to the tooth numbers as follows :

$$\tan \gamma_p = \frac{t_p}{t_g}, \quad \tan \gamma_g = \frac{t_g}{t_p}$$

where subscript p and g refer to pinion and gear respectively. In the figure it is shown that the shape of the teeth, when projected on the back cone, is the same as in a spur gear having a radius equal to the back cone distance r_b. This is called Tredgold's approximation. The number of teeth on this imaginary gear called formative number of teeth t_f is related to the actual number of teeth t by the relation $t_f = \frac{t}{\cos\gamma}$

20.4 Design of Bevel Gear Drives :

The procedure for design of these gears is same as for spur and helical gears. The steps required for design are as follows :

Step 1 : Select the number of teeth on the pinion and gear as per T-XVI-21.

Step 2 : Assume the module of teeth m, mm. Calculate the pitch diameters of the pinion and gears as,

$$D_p = t_p \times m \text{ and } D_g = t_g \times m$$

Step 3 : Calculate the pitch line velocity V_p in terms of module.

$$V_p = \pi \times \frac{D_p}{1000} \times \frac{N_p}{60} \text{ m/sec.}$$

Step 4 : Calculate the design power P_d and tangential tooth load F_t as per T-XVI-20. The F_t will be obtained as,

$$F_t = \frac{P_d}{V_p} = \frac{K_1}{m} \quad \text{where } K_1 = \text{numercial constant.}$$

Step 5 : Assume materials for pinion and gear as per T-XVI-10 and basic stress So.

Step 6 : Calculate the formative number of teeth t_f as per T-XVI-20 and modified Lewis form factor Y as per T-XVI-5.

Step 7 : Assume the trial value of C_v on the same line as spur gears.

$$C_v = 0.50 \text{ for } N_p < 1000 \text{ rpm.}$$
$$= 0.40 \text{ for } 2000 < N_p > 1000 \text{ rpm.}$$
$$= 0.30 \text{ for } 3000 < N_p > 2000 \text{ rpm.}$$

Step 8 : Calculate the pitch cone length 'L' in terms of 'm' as per T-XVI-20. Assume the trial face width b = 6 m for L < 30 m and 8m for L > 30 m.

Step 9 : Calculate the beam strength F_B in terms of m with the help of Lewis equation given in T-XVI-20. It will be in the form $F_B = K_2 m^2$ where K_2 = constant.

Step 10 : Equate F_B to F_t and obtain the trial value of 'm'. Select the nearest standard 'm' as per T-XVI-7.

Step 11 : Calculate the values of D_p, D_g, V_p, C_v, F_t and b. Check that b is in the range given in T-XVI-21.

This design is sufficient for intermittent duty. For continuous duty, calculate F_d', F_{en} and F_w using the formulae given in T-XVI-20. Obtain the minimum value of hardness required for the pinion and gear.

The gear blank is to be designed in the same way as for spur gears as per T-XVI-14.

SOLVED PROBLEMS

Prob. 20.1 : An elevator cage is little at the rate of 100 m/min. The elevator drum diameter is 600 mm and the load to be lifted is 20 kN. The worm gear is keyed to the drum shaft. The elevator is driven by an elecric motor running at 960 rpm. Determine (a) the worm and worm gear proportions (b) the temperature rise of the oil and (c) the required power of the motor if the efficiency of the hoist is 94 percent.

Ref.	Calculations
	Given : $N_w = 960$ rpm , Load = 20 kN at 100 m/min. Drum dia = 600 mm, η of hoist = 94%
	Drump speed : $N_g : \pi \times \frac{600}{1000} \times N_g = 100$
	$\therefore N_g = 53$ rpm $\quad \therefore$ V.R., $R = \frac{960}{53} = 18.08$ say 18
	$\therefore$ Approx. efficiency of worm gear drive : $\eta = 1 - .005 \times 18 = .91$
	$\therefore$ Rated power input $= \frac{20 \times 100}{60 \times .91 \times .94} = 38.96$ kW
T-XII-12	Selecting a standard motor of 45 kW for which the shaft diameter will be 75 mm/
T-XVI-6	Selecting $t_w = 2$ for V.R. between 12-36, $t_g = 18 \times 2 = 36 > 29$ so o.k. Lead angle $\lambda = 6°$ per worm tooth $= 6 \times 2 = 12°$
	For compact design, $\lambda = \tan^{-1} \sqrt[3]{\frac{N_g}{N_w}} = \tan^{-1} \sqrt[3]{\frac{1}{18}} = 20.8°$
	$\therefore$ Assume $\lambda = 20°$ and $\varphi_n = 22.5°$ which is normally used.
T-XVI-15	$P_d = P_R \times K_1$ $\quad K_1$ for electric motor and elevator = 1.75 $= 38.96 \times 1.75 = 68.18$ kW Let the module of the gear be 'm' mm.
	$\therefore D_g = 36 \times m$, mm, $\quad V_p = \pi \times \frac{36m}{1000} \times \frac{53}{60} = .1m$, m/sec.
	$F_t = \frac{68.18 \times 10^3}{.1m} = \frac{68.18 \times 10^4}{m}$, N
T-XVI-18 T-XVI-10 T-XVI-19	$F_B = S_o\, C'_v \,.\, b \,.\, Y \,.\, m$ $= 84 \times .75 \times 7.47 \times .435\, m^2$ $= 204.62\, m^2$ Meterial of construction Worm – SAE 3120 steel Gear – Ph bronze SAE 65, $S_o = 84$, $C'_v = .75$ (trial value) $b = 2.38 \times \pi \times m = 7.47$ m $Y = .314 + .0151\,(22.5 - 14.5)$ $= .435$
	Equating F_B to F_t $204.62\, m^2 = \frac{68.18 \times 10^4}{m}$ $\therefore m = 14.94$ mm

T-XVI-7 Selecting a standard module of 16 mm $\therefore D_g = 36 \times 16 = 576$ mm.

$V_p = 0.1\ m = 1.6$ m/sec., $C_v' = \dfrac{6}{6+1.6} = .79$

T-XVI-19 $b = 2.38 \times \pi \times 16 + 6.25 = 125.8$ mm

$$F_t = \frac{68.18 \times 10^4}{16} = 42.6 \times 10^3, N$$

$F_B = 84 \times .79 \times 125.8 \times .435 \times 16 = 58.1 \times 10^3$ N

$\therefore F_B > F_t$ so design is o.k.

To check for wear

$$F_d = \frac{F_t}{C_v} = \frac{42.6 \times 10^3}{.79} = 53.92 \times 10^3 \text{ N}$$

T-XVI-15 $F_w = D_g \cdot b \cdot K_2$ $K_2 = .70$ MPa

T-XVI-17 $= 576 \times 125.8 \times .70 = 50.72 \times 10^3$

As $F_w < F_d$ we may get less wear life. For continuous duty we will redesign as per AGMA standard.

From AGMA formula for thermal rating

$$P = \frac{3650\ C^{1.7}}{R+5}$$

P = Rated Power, kW = 38.96

C = Centre distance, m

R = Velocity ratio.

$$38.96 = \frac{3650\ C^{1.7}}{18+5} \qquad \therefore C = .438\ m = 438 \text{ mm}$$

T-XVI-19 $C = \frac{1}{2}[36m + 2.4 \times \pi \times m + 27.5] = 438$

$$\therefore m = \frac{438 \times 2 - 27.5}{43.536} = 19.49$$

T-XVI-7 Select standard module of m = 20 mm

$\therefore D_g = 36 \times 20 = 720$ mm, $V_p = 0.1 \times 20 = 2$ m/sec.

$$C_v' = \frac{6}{6+2} = .75$$

$$F_t = \frac{68.18 \times 10^4}{20} = 34.1 \times 10^3, N \quad ; \quad F_d = \frac{34.1 \times 10^3}{.75} = 45.47 \times 10^3, N$$

T-XVI-19 $b = 2.38 \times \pi \times 20 + 6.25 = 155.714$ mm.

T-XVI-15 $\therefore$ New $F_w = D_g\, b\, K_2 = 720 \times 155.7 \times .7 = 78.47 \times 10^3$, N

as $F_w > F_d$ so o.k.

We could have got the same result if we had increased m to the next standard module of 20 mm.

T-XVI-19 $D_w = 2.4 \times \pi \times 20 + 27.5 = 178.22$ m.

$$C = \frac{720 + 178.22}{2} = 449.11 \text{mm}$$

T-XVI-16 $\dfrac{C^{.875}}{D_w} = \dfrac{449.11^{.875}}{178.22} = 1.174$ so o.k.

To calculate temp rise ΔT :

$$V_R = \text{Rubbing velocity} = \frac{\pi D_w N_w}{\cos\lambda \times 10^3} = \frac{\pi \times 178.2 \times 960}{\cos 20^\circ \times 10^3}$$

$$= 571.64 \text{ m/min.}$$

$$\mu = .025 + \frac{V_R}{20{,}000} = .025 + \frac{571.64}{20{,}000} = 0.54$$

T-XVI-15 $\tan\theta = \tan\varphi_n$ $\cos\lambda = \tan 22.5^\circ$ $\cos 20^\circ = .4142 \times .94 = .389$

$\theta = \tan^{-1} .389 = 21.26^\circ$

$$\eta = \frac{\cos\theta - \mu\tan\lambda}{\cos\theta + \mu\cot\lambda} = \frac{\cos 21.26^\circ - .054\tan 20^\circ}{\cos 21.26^\circ + .054\cot 20^\circ}$$

$$= \frac{.932 - .02}{.932 + 1.48} = .844$$

$Q_1 = (1 - \eta)\, P_R = (1 - .844)\, 38.96 \times 10^3 = 6.08 \times 10^3$, W

$Q_2 = KA\,\Delta T$, $A = \frac{\pi}{4} D_g^2 + D_w L_w$, $L_w = (4.5 + 0.2 \times 2)\,\pi \times 20$

$= 285.1$ mm

Putting $Q_2 = Q_1$ $= \frac{\pi}{4} 720^2 + 178.22 \times 285.1$

$= 45.78 \times 10^4 \text{ mm}^2$

$$\Delta T = \frac{6.08 \times 10^3}{45.78 \times 10^4 \times .4 \times 10^{-3}} = 33.2^\circ C$$

It is within limits of about 40°C so o.k.

The proportions of worm and gear are,

$D_w = 178.22$, mm, $L_w = 285.1$ mm

$D_g = 720$ mm, $b = 155.7$ mm

$m = 20$ mm $t_w = 2$, $t_g = 36$

Prob. 20.2 : A 2 teeth right hand worm transmits 0.75 kW at 1200 rpm to a 30 teeth worm gear. The worm has an axial of 13 mm, a pitch diameter of 50 mm and a face width of 63 mm. The face width of the gear is 25 mm and the pressure angle is 14 1/2°. Find the centre distance, the lead and the lead angle. Find the forces exerted by the bearings against the worm gear shaft and the output torque. If the materials of construction are SAE 3120 steel for the worm and the Phospher Bronze SAE 60 for the gear, is it suitable for continuous duty ?

Ref.	Calculations
	Given : $t_w = 2$, $t_g = 30$, $N_w = 1200$ rpm,
	$N_g = \frac{1200}{15} = 80$ rpm, $P_a = 13$ mm,
	$D_w = 50$ mm, $L_w = 63$ mm, $b = 25$ mm
	$\phi_n = 14\ 1/2°$; $P_R = 0.75$ kW
	Material of construction Worm – SAE 3120 steel
	Gear – Ph – Bronze SAE 65
	To find D_g, C, L and λ :
	$P_a = P_c = 13$ mm, $D_g = \frac{13 \times 30}{\pi} = 124.2$ mm
	$C = \frac{D_w + D_g}{2} = \frac{50 + 124.2}{2} = 87.1$ mm
T-XVI-15	$L = t_w \times P_a = 2 \times 13 = 26$ mm
	$\lambda = \tan^{-1} \frac{L}{\pi D_w} = \tan^{-1} \frac{26}{\pi \times 50} = 9.4°$
	To determine whether it is suitable for continuous duty.
	V_p of gear $= \pi \times \frac{D_g}{1000} \times \frac{N_g}{60}$
	$= \pi \times \frac{124.2}{1000} \times \frac{180}{60} = .52$ m/sec.
	$\therefore F_t = \frac{.75 \times 10^3}{.52} = 1.442 \times 10^3 = 1442$, N
T-XVI-15	$C'_v = \frac{6}{6 + .52} = .92$, $Y = .314 + .0151\ (14.5 - 14.5) = .314$
T-XVI-10	S_o for gear = 84, $m = \frac{P_c}{\pi} = \frac{13}{\pi} = 4.14$ m
	$\therefore F_B = 84 \times .92 \times 25 \times .314 \times 4.14 = 2511.5$ N
	$F_B > F_t$ so o.k. ; $F_d = \frac{F_t}{C'_v} = \frac{1442}{.92} = 1567.4$

1.8 : A tank open the top and closed at the bottom has a diameter of 500 mm, a wall thickness of 10 mm, and is 4.5 meters high. The tank is filled to the top with liquid mercury which has a specific gravity of 13.6. The tank is simultaneously subjected to equal and opposite torques of 9000 N-m at the top and bottom. What is the magnitude of the principal stresses at the point A which is 2 m below the top and at the outer surface of the tank ?

1.9 : A rigidly built in steel cantilever has a cross section 12 mm deep, 9mm wide, and is loaded at 300 mm from the support. Determine the maximum bending stress in the beam, assuming (a) that a load of 36 N is applied gradually and (b) that a weight of 36 N is dropped on the beam through a distance of 4.5 mm.

1.10 : A bar of 12 mm diameter gets stretched by 2 mm under a steady load of 5 kN. What stress would be produced in the bar by a weight of 800 N which falls through 60 mm before commencing the stretching of rod. Take E = 200 GPa.

1.11 : In a free-rolling conveyor, a crate loaded with non fragile material, 270 N tare, reaches station A at a velocity of 3 m/sec and is normally unloaded at station B. In case it gets away from station B, a pair of helical buffer springs at C are expected to arrest the motion of the crate. Assume that the springs are set up with an initial compression of 50 mm and that the compression of the spring due to impact is to be 150 mm. Determine the required stiffness of springs and the maximum load on each spring.

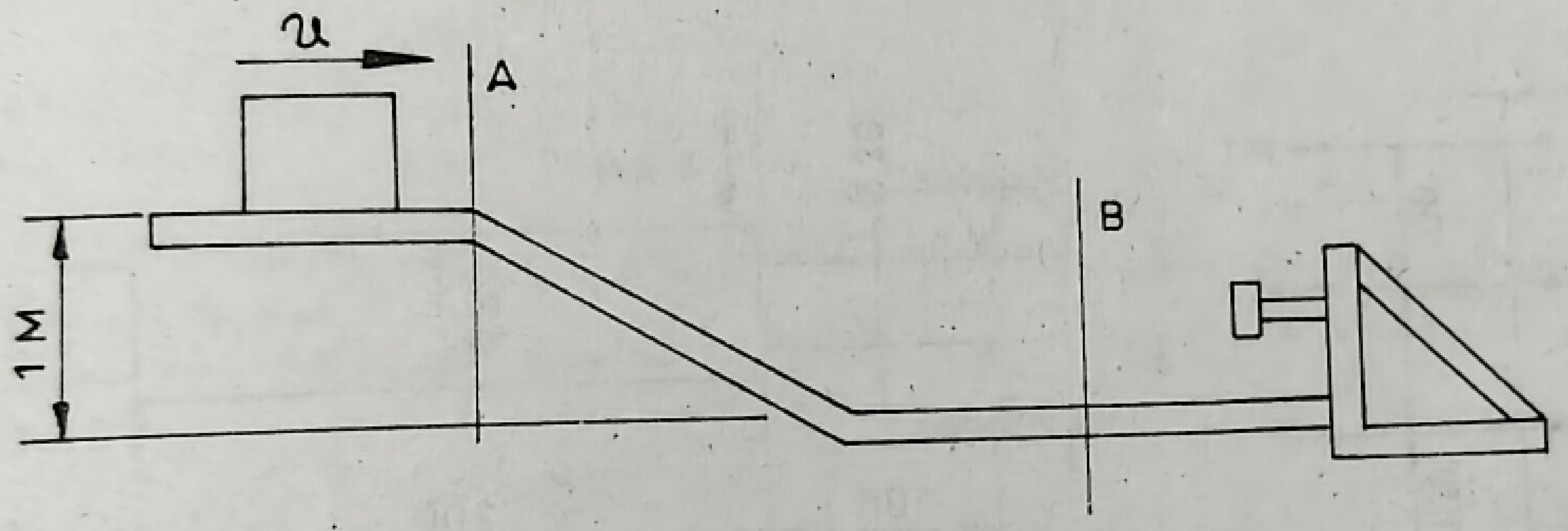


Fig. 1.11

1.12 : A steel bar of 40 mm diameter and 2 m long absorbs a shock energy of 80 N-m. Find the maximum instantenous stress and the elongation produced. Take E = 200 GPa.

1.13 : A thin steel tyre is shrunk on to a bullock-cart wheel of 1 m diameter. Find the internal diameter of the tyre if shrinking on, the hoop stress in the tyre is 100 MPa. Find also the least temp to which the tyre must be heated above the ambient temperature before it could be slipped on.

1.14 : A cast steel bracket shown in the figure is subjected to the horizontal load F = 4.5 kN. The width of the bracket is 100 mm. Determine the maximum stress in the bracket. Neglect the direct shear stress. Find how much this stress will be changed with r = 10 mm. Use Fig. (1.14) of data book for finding K_t.

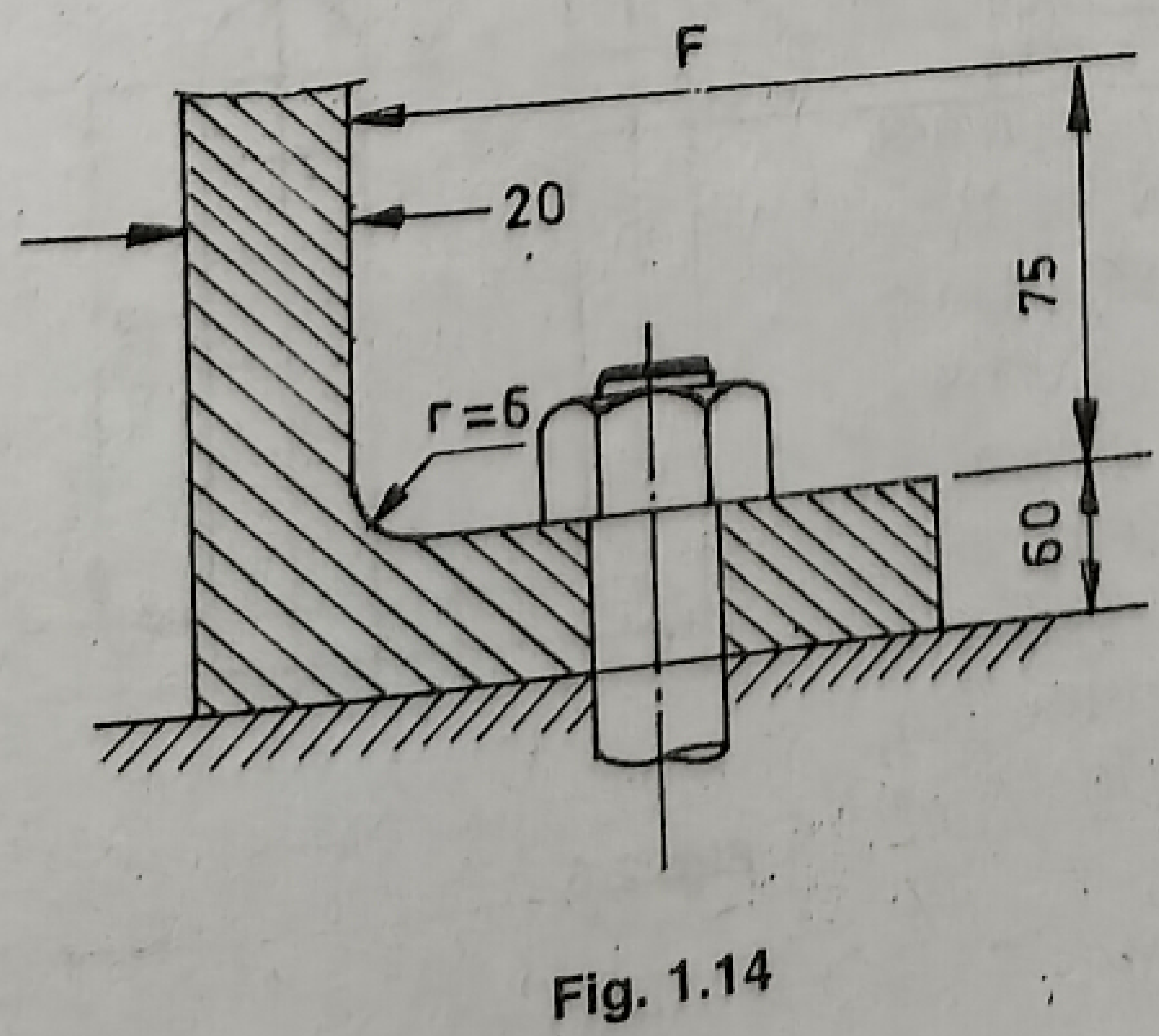


Fig. 1.14

Chapter 2
STRAIN AND DEFLECTION ANALYSIS

2.1 : A steel rod of 25 mm diameter is fitted inside a brass tube of 25 mm internal diameter nd 40 m external diameter. The projecting ends of the steel rod are provided with nuts and washers. The nuts are tightened up so as to produce pull of 6 kN the rod. The assembly is then placed in a lathe and the tube is turned down to a 5 mm thickness. Calculate the stresses in the two materials. Take $E_S = 200$ GPa, $E_B = 100$ GPa.

2.2 : A steel square bar with 30 mm sides and 2 m long is elongated by a load of 600 kN. If the Poisson's ratio is 0.3, find the increase in length and the change in volume.

2.3 : The Fig. (2.3) shows one end of a spring attached to a lever. What is the spring constant of the system.

(a) With respect to a horizontal force applied at A ?

(b) With respect to a horizontal force applied at B ?

(c) With respect to a horizontal force applied at C ?

2.4 : The Fig. (2.4) shows a simply supported steel staff subjected to two loads. Determine the deflection at the points where the loads are applied. Take E = 200 GPa.

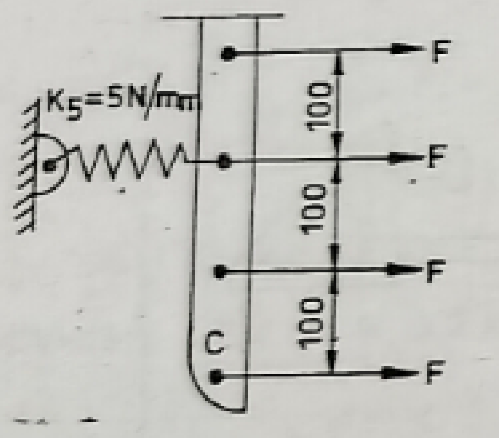


Fig. 2.3

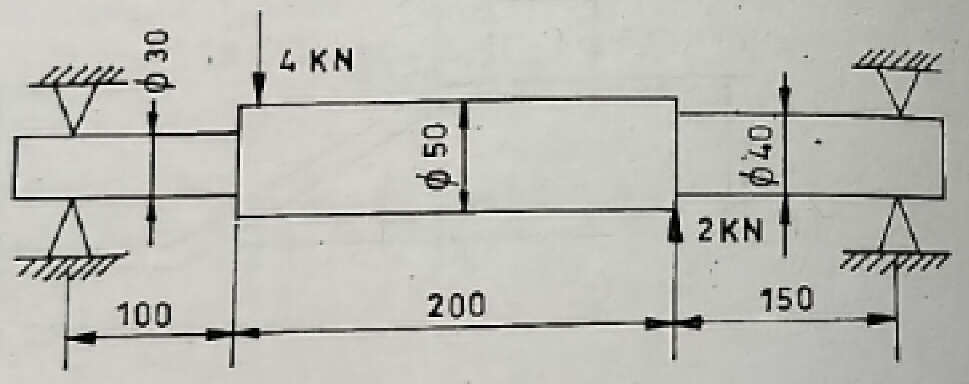


Fig. 2.4

2.5 : Calculate the torsional spring constant (N-m per degree of angular deflection) of a solid round steel shaft 300 mm long if its diameter is 30 mm for half its length and 20 mm for the other half.

2.6 : A bracket is shown in the figure. What are the angular and linear displacements of point A?

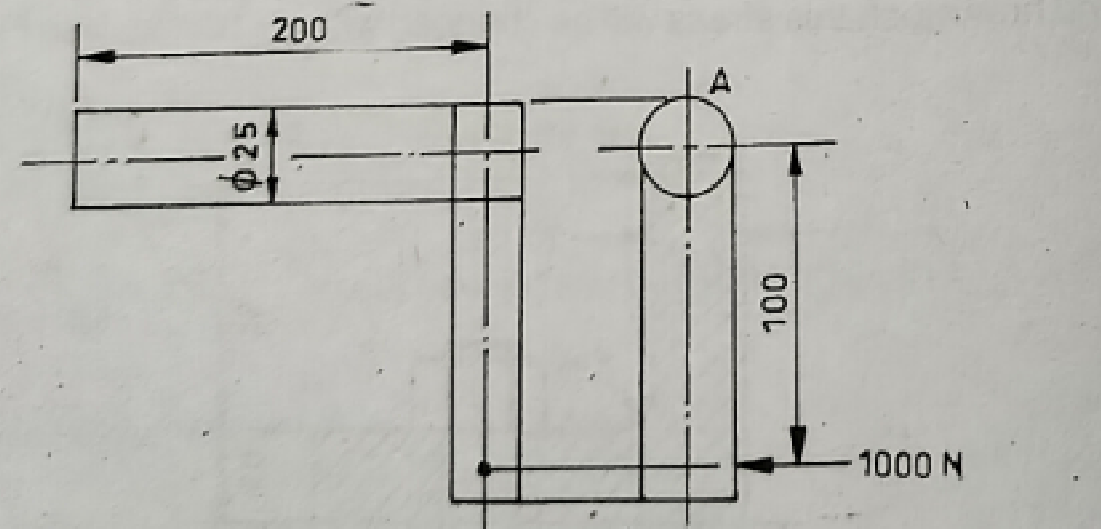


Fig. 2.6

2.7 : The piston rod of a hydraulic cylinder having a bore of 75 mm is made of medium carbon steel with $S_{yt} = 500$ MPa. It has a diameter of 40 mm and the unsupported length when fully extended can be taken as 1m. The maximum pressure of oil is 25 MPa. Because of the stiffness of cylinder and seal, the end conditions can be assumed as one end fixed and the other end guided. Calculate the factor of safety.

2.8 : Find the distance *l* in the Fig. (2.8) that will keep the rigid 9 kN weight level. The lower ends of the two suspension rods were at the same elevation before the two weights were applied. The rods are of the same material, and the cross sectional area of rod 2 is 25 percent smaller than that of rod 1.

2.9 : A cantilever 1 shown in the Fig. (2.9) is made of a 25 × 40 mm flat steel bar. When acted upon by a force F of 27 kN, the deflection at the end was excessive. Find the amount by which the deflection will be reduced if the end of cantilever 1 is tied to the end of a parallel cantilever 2 made of a 25 mm round rod of brass. Take $E_S = 200$ GPa and $E_B = 100$ GPa.

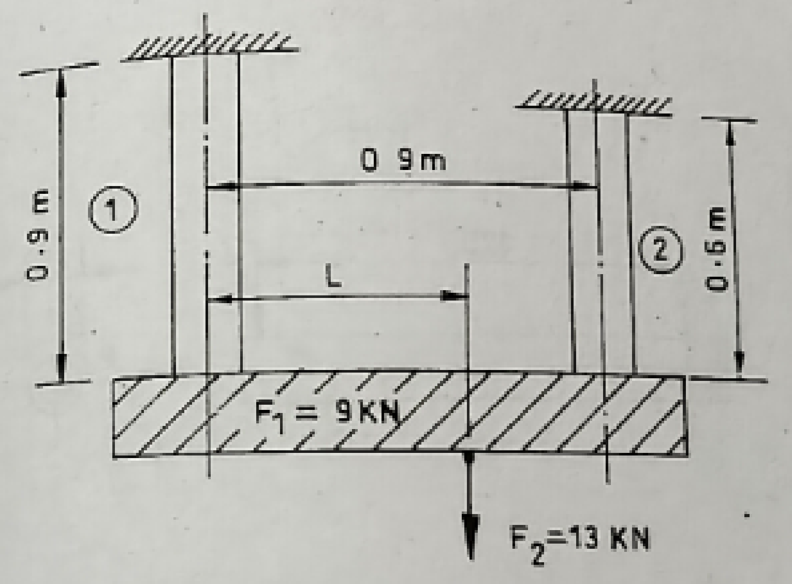


Fig. 2.8

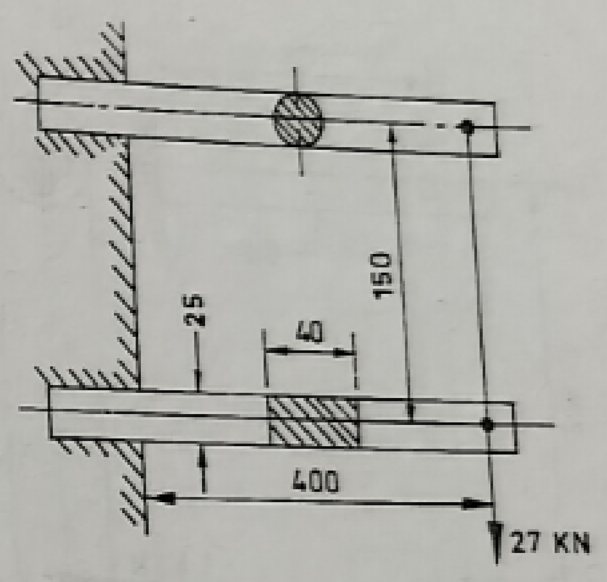


Fig. 2.9

2.10 : The piston rod of steam engine may be considered to be a column. Determine the recommended diameter of the rod, assuming the following ;

Diameter of cylinder .... 450 mm
Maximum net steam pressure on piston .... 1 MPa
Distance from piston to cross head .... 1.5 m
Material .... SAE 1030 steel
Factor of safety 4
End-fixity coefficient n .... 3.

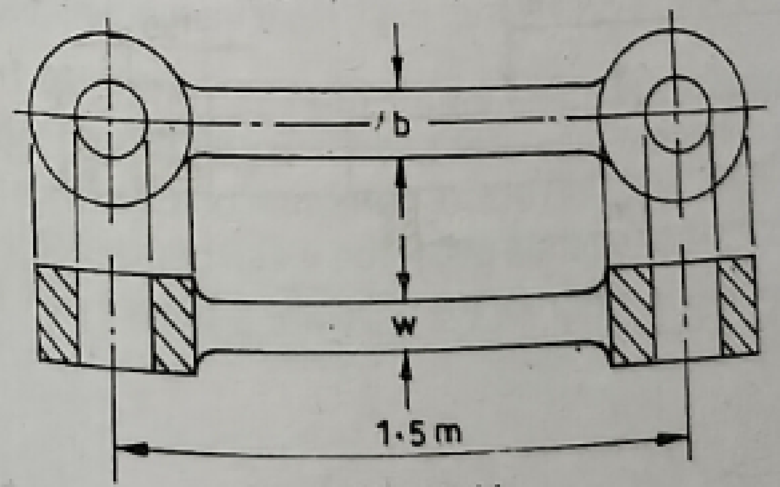


Fig. 2.11

2.11 : A steel strut, as shown in the sketch, is rectangular in cross section and is to be part of the operating mechanism of a bridge. The compressive load to be carried is 180 kN. Determine the following -- (a) the relation between b and w required for equal strengths of the strut in failure by buckling in either plane; (b) the required dimensions b and w, assuming a factor of safety of 4 and SAE 1020 steel.

Chapter 4
DESIGN FOR STATIC AND FATIGUE STRENGTH

4.1 : A lever subjected to a static force of 1.8 kN keyed to a 25 mm round bar as shown in the Fig. (4.1)

(a) The round is made of steel with S_{yt} = 102° MPa. Based on static loading, find the factor of safety using the distortion energy theory.

(b) Find the factor of safety using the maximum shear stress theory.

4.2 : The Fig. (4.2 shows a cantilevered tube to be made of aluminimum alloy SAE N. 260. It is loaded as shown in the figure with F = 0.8 kN, P = 7.2 kN and torque T = 38 N-m. Suggest suitable thickness of the tube if the outside diameter of tube is 25 mm. Use distortion energy theory with a factor of safety of 2.

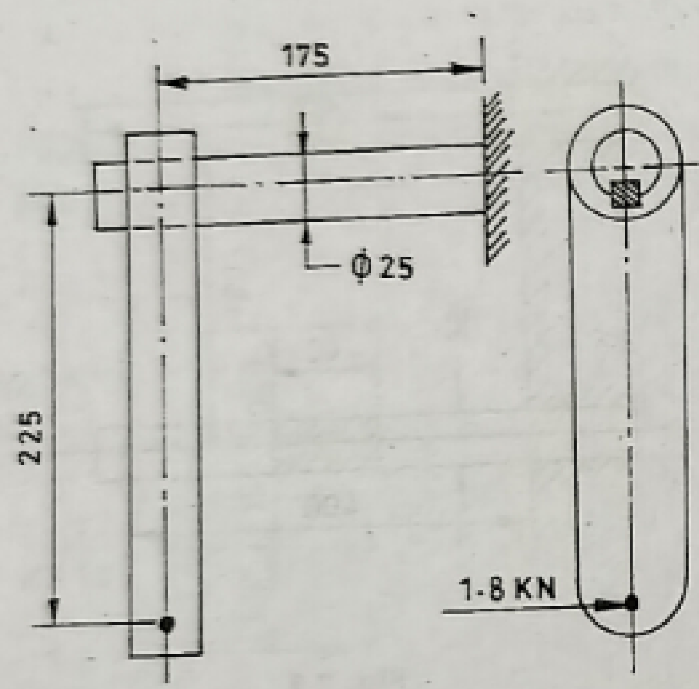


Fig. 4.1

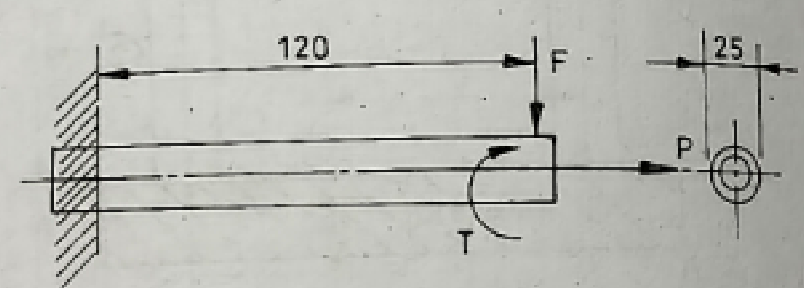


Fig. 4.2

4.3 : Each of the three steel rods made of SAE 1020 steel has the same cross sectional area. Their lengths are such that before the load F of 100 kN is applied, there is no stress in them. Sugges suitable diameter for the rods for a factor of safety of 2. (Fig. 4.3)

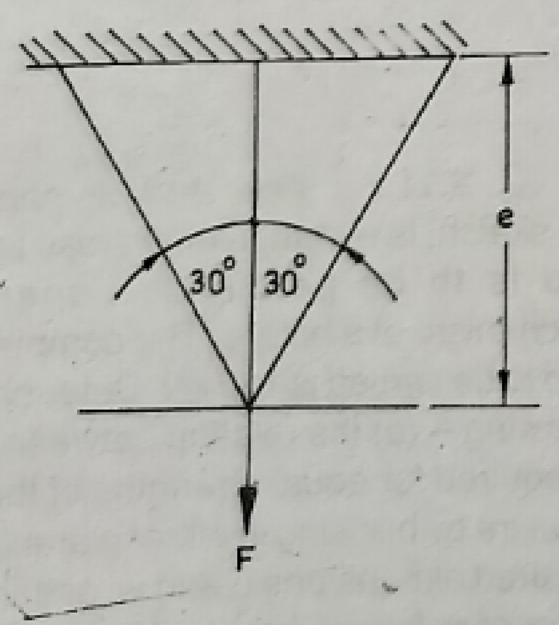


Fig. 4.3

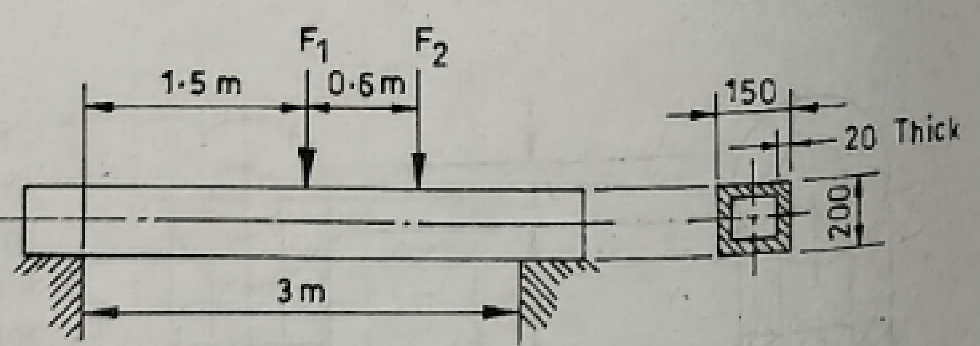


Fig. 4.4

4.4 : The Fig. (4.4) shows the loading and cross section of a simply supported cast-iron beam. Sketch the B.M. diagram and determine the maximum deflection and the factor of safety in bending if it is made of I.S. grade 35 cast iron. What is the maximum shear stress in the beam ?

4.5 : The load on the C clamp shown in the Fig. (4.5) is F = 15 kN. The clamp is made of cast steel IS-CS-125. Determine the dimensions 'h' for a factor of safety of 2.

4.6 : A portion of a machine member is shown in the Fig. (4.6). It is loaded by completely reversed axial forces F which are uniformly distributed across the width. The material is SAE 1020 steel. Determine the maximum force F that can be applied with a factor of safety of 1.5.

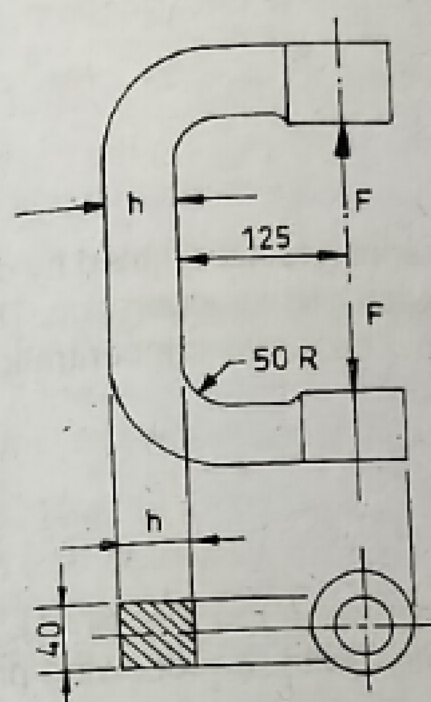


Fig. 4.5

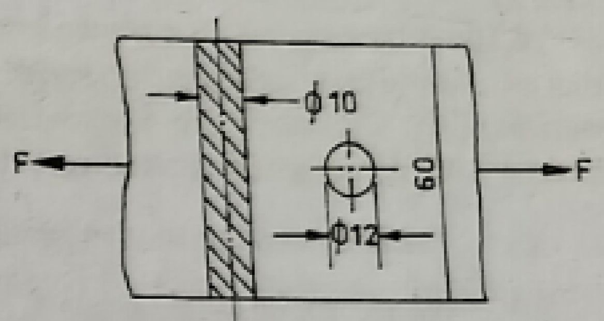


Fig. 4.6

4.7 : A 30 mm diameter machined cantilever 300 mm long is loaded on the end with a force that varies from 300 N down to 500 N up. There is a 6 mm fillet where the member is connected to the support. The notch sensitivity factor may be taken as 0.92. If the material has the following properties, determine the factor of safety based on the Soderberg criteria.

$$S_{ut} = 550 \text{ MPa}, S_{yt} = 400 \text{ MPa}, S_{eb} = 250 \text{ MPa}.$$

4.8 : A steel member of diameter d is subjected to a torsional stress that varies from 0 to 35 MPa and to an axial stress that varies from – 15 MPa to + 30 MPa. The maximum stresses due to torsion and axial load occur at the same time. Determine the factor of safety based upon yield in shear. Assume the following -- S_{eb} = 210 MPa, S_{yt} = 4800 MPa, K_f = 1. The surface has a mirror polish and the diameter is less than 12.5 mm.

4.9 : The Fig. (4.9) shows a stationary torsion-bar spring, loaded by the forces F which remain constant. The spring is subjected to a torque which varies from 0 to 10 N-m. The material is SAE 1050 steel hardened and tempered to 300 Bhn. The ends of the spring are ground upto shoulders. The 2.0 m body of the spring has a hot rolled surface finish. The stress concentration factors at the shoulders are K_t = 1.7 for bending and K_t = 1.4 for torsion. Determine the diameter 'd'.

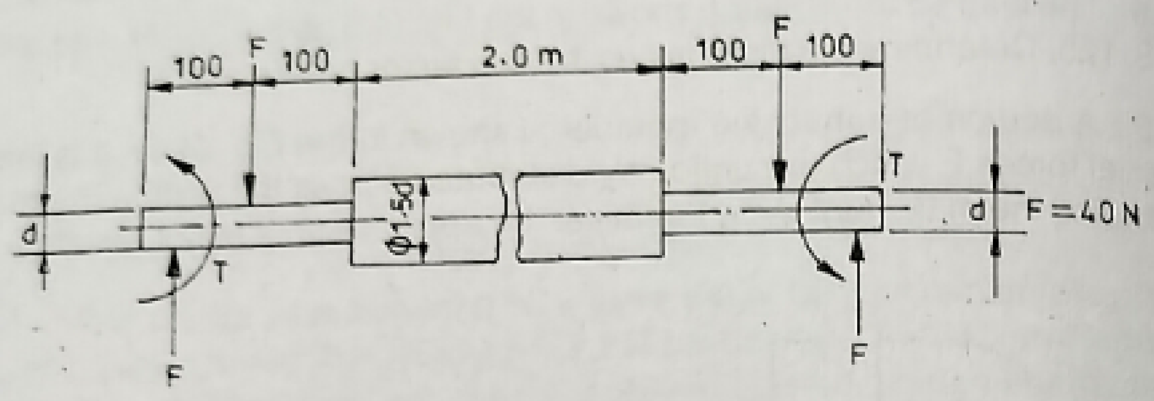


Fig. 4.9

4.10 : A latching spring is shown in the Fig. (4.10). The spring is assembled by deflecting it 1.5 mm initially and then deflecting it an additional 4.5 mm during each latching operation. The material is machined SAE 1095 steel hardened and tempered to 400 Bhn. The stress concentration factor at the band is 1.70 corrected for notch sensitivity.

(a) Find the maximum and minimum latching force F.

(b) Will the spring fail in fatigue ?

4.11 : The 40 mm diameter bar is bent into the shape shown in the Fig. (4.11). The force applied to the bar varies from F to 2 F. The bar is made from steel with the following properties : $S_{yt} = 420$ MPa ; $S_{eb} = 310$ MPa ; what is the max. load that be applied for a safety factor = 2 based upon the static yield point. Assume surface finish factor = 0.8

(a) Determine the load based on stresses at section AA

(b) Determine the load based on stresses at section BB.

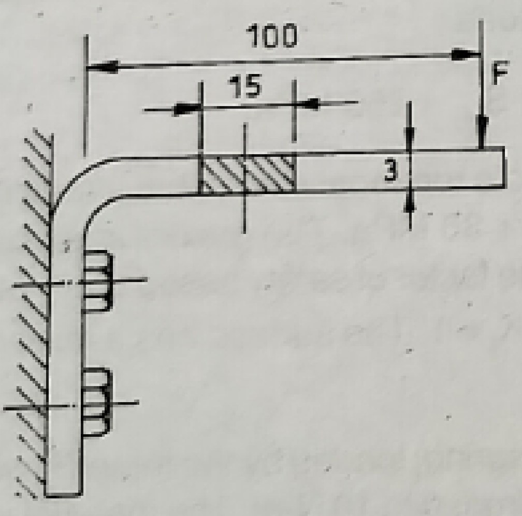


Fig. 4.10

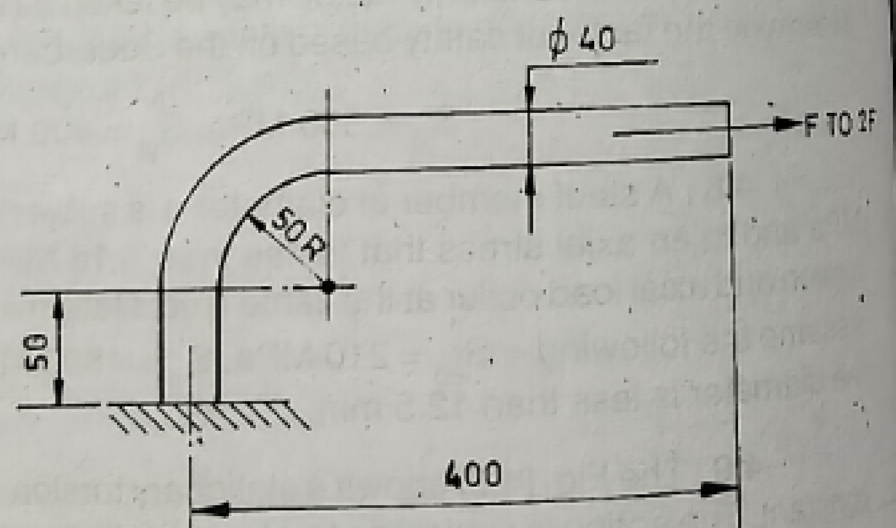


Fig. 4.11

Chapter 5

SCREW FASTENERS & POWER TRANSMISSION SCREWS

5.1 : Two parts of a machine are held together by bolts that are initially tightened to provide in initial clamping force of 10 kN. The elasticities are such that $K_c = 2 K_b$

(a) What external separating force would cause the clamping force to be reduced to 2 kN ? (Assume that the bolts remain within elastic range).

(b) If this separating force is repeatedly applied and removed, what will be the values of mean and alternating force acting on the bolts ?

(c) What should be the diameter of bolts made of SAE 2330 steel ? Assume a factor of safety of 2.

5.2 : An industrial motor weighing 22 kN is to be provided with a steel eye bolt for use when it is lifted.

(a) What size of steel bolt will you recommended if it is made of SAE 2320 steel ?

(b) If the housing into which the bolt is threaded has only half the yield strength of the bolt steel, what minimum number of threads should be engaged ?

5.3 : The Fig. (5.3) shows a bracket which is to support a force F as shown. The bracket is to be bolted to a smooth vertical face by means of four M 10 × 1.5 steel bolts two on centre line AA and the other two on centre line BB. It is assumed that the bolts on AA carry the entire moment load and those on BB carry the entire shear load, determine :

(a) the external shear load carried by the bolts at BB.

(b) the external tensile load carried by the bolts at AA.

(c) what is the minimum factor of safety if the bolts are made of SAE 1120 steel.

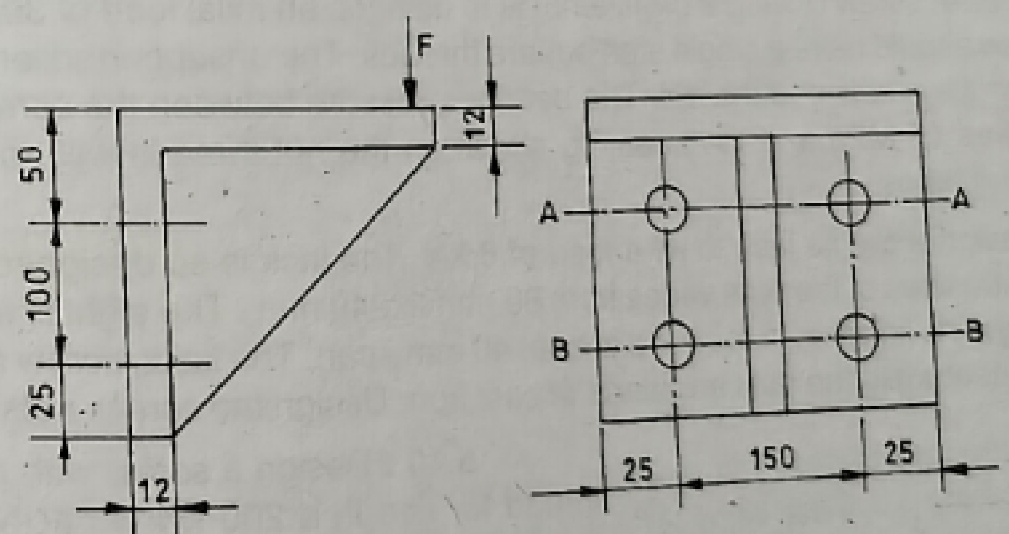


Fig. 5.3

5.4 : Determine the total shear load on each of the three bolts for the connection shown in the Fig. (5.4) and calculate the maximum shear stress and the bearing stress. Calculate the moment of inertia of the 8mm plate on a section through the three bolt holes and find the maximum bending stress in the plate.

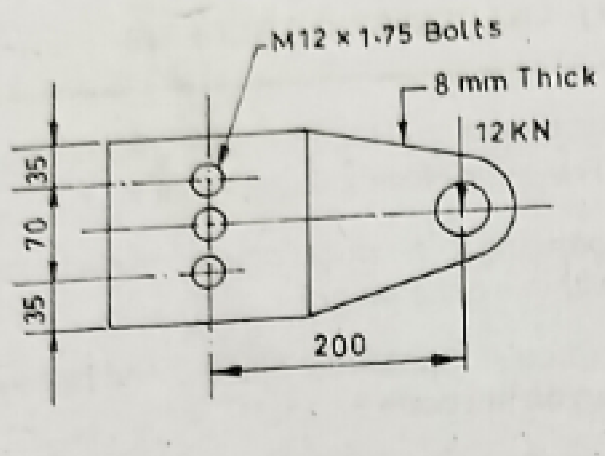


Fig. 5.4

5.5 : A critical application requires the smallest possible bolt for resisting a dynamic separating force varying from 0 to 100 kN. It is estimated that by using an extra high strength bolt steel with proof strength = 1200 MPa, and using special equipment to control initial tightening to the full $A_r \times S_p$, a stiffness ratio of $K_c/K_b = 6$ can be realized. With a safety factor of 1.3 with respect to eventual fatigue failure, what smallest size of fine threaded bolt can be used ? With this bolt tightened as specified, what clamping force will remain when the 100 kN load is applied ? What is the torque required to tighten the bolts ?

5.6 : One M 20 steel through bolt with $A_r = 225\ \text{mm}^2$ is used to tighten up two steel and plates against a steel cylinder with an O.D. of 100 mm and I.D. of 75 mm. The end plates can be considered as extensions of the cylinder when calculating the joint stiffness. An oil pressure of 10 MPa is applied inside the cylinder. Show that an initial tension of 50 kN will prevent leakage from the cylinder joint faces and then find the total load in the bolt.

5.7 : A screw jack uses a single square thread screw to raise a load of 60 kN. The screw has a major diameter of 40 mm and a pitch of 7 mm. The thrust collar mean diameter is 80 mm. The coefficient of friction for the screw and collar can be assumed as 0.12. Estimate the starting torque for raising and lowering the load. Estimate the efficiency of the jack. What power will be required to drive the screw at a constant speed of one revolution per second ?

5.8 : A steel screw driving a bronze nut is to develop an axial load of 300 kN in a extrusion press. The screw should have a single start square threads. The unsupported length of the screw is 500 mm. Design the screw and the nut. The bearing pressure between the screw and nut threads should not exceed 15 MPa and the shearing stress on the nut threads will not exceed 28 MPa. Assume other properties.

5.9 : Design a toggle jack to lift a load of 8 kN. The jack is so designed that the distance between the centre lines of the nuts varies from 60 mm to 240 mm . The eight links are symmetrical and 150 mm long. The link pins in the base are set 40 mm apart. The links, screw and pins are made from SAE 1020 steel while the nuts are made of cast iron. Design the screw, nuts and the links.

5.10 : Design a screw with a modified square thread for use in a 200 kN capacity bell bottom jack using a ball thrust bearing at the collar. It has a maximum lift of 150 mm. The modified form of the thread is shown below. (Fig. 5.10)

What must be the length of the steel operating bar if the maximum operating force is not to exceed 150 N under rated load ? What will be the force required to lower the jack under the rated load with this bar ? Suggest suitable diameter for this bar if it is made of SAE 1030 steel.

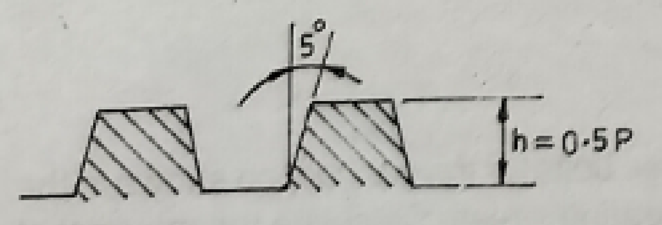


Fig. 5.10

Chapter 6

RIVETTED AND WELDED JOINTS

6.1 : Design a triple riveted butt joint for the longitudinal seam of a steam boiler drum 1 m in diameter and sustaining a steam pressure of 2.2 MPa.

6.2 : Design a single riveted lap joint for the girth seam of a pressure tank 900 mm in diameter and subjected to an internal pressure of 1 MPa. Find efficiency of the joint.

6.3 : The longitudinal seam of a cylindrical boiler drum is of the double riveted butt joint type. The shell is 12 mm thick. The rivet diameter is 22 mm ; the pitch is 125 mm ; the short pitch is 62.5 mm ; the transverse pitch is 70 mm and all margins are 36 mm. The thickness of the wider, inner cover plate is 10 mm and that of the narrow cover plate is 12 mm. Determine the manner of possible failure and the efficiency of the joint.

6.4 : An air tank 1 m in diameter has a double riveted lap joint longitudinal seam. The thickness of the shell is 15 mm, the rivet hole diameter is 27 mm, pitch 82 mm ; transverse pitch 52 mm and margin 35 mm. Determine the safe air pressure and efficiency of the joint so that the working stresses in tension, shear and crushing shall net exceed the ASME boiler code values of 80, 63 and 135 MPa respectively.

6.5 : Determine the safe load F which can be carried by the crane bracket shown in the Fig. (6.5), using the following data. The steel plate a is 8 mm thick, the size of angle b is 80 × 60 × 6, all rivets are 16 mm diameter. Assume a safety factor of 2.5.

6.6 : The connection of a 12 mm thick plate to a girder is shown in the Fig. (6.6). Find the size of the rivets and the distance so that the rivets 1 and 3 are equally stressed.

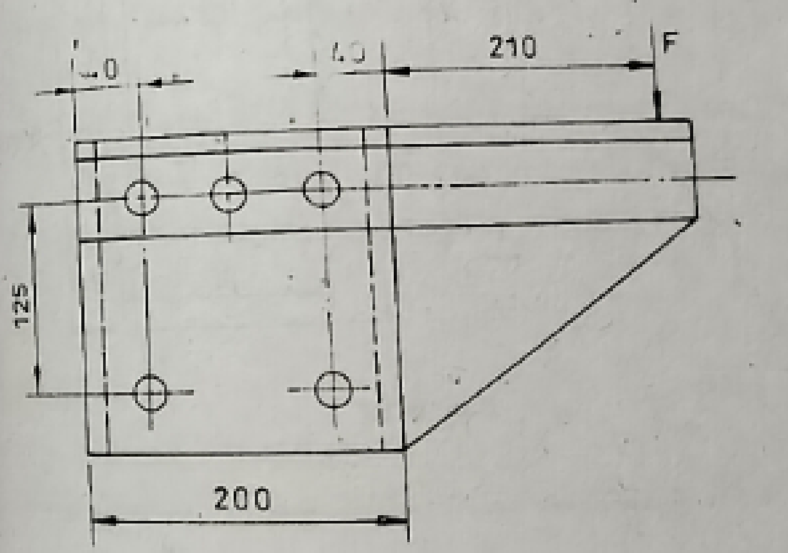


Fig. 6.5

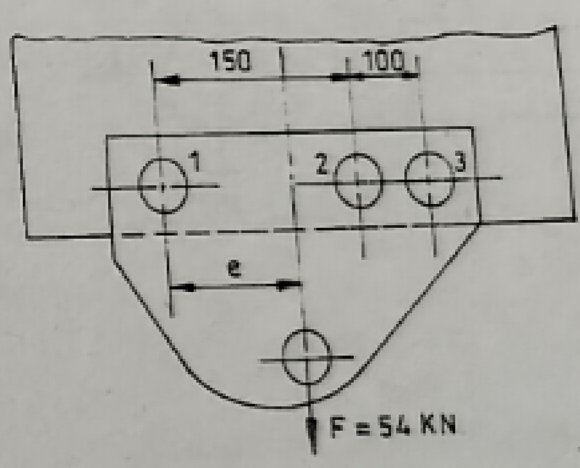


Fig. 6.6

6.7 : Determine the necessary minimum distance 'c' between the welds shown in the Fig. (6.7) for a static load F of 36 kN if $l = 100$ mm and $w = 150$ mm. Use 8 mm shielded arc fillet welds and assume that the direct shear stress is distributed uniformly over the throat area.

6.8 : Find the permissible static load F for the welded ear shown in the Fig. (6.8) if $l = 250$ mm and $e = 50$ mm. use 6 mm shielded arc fillet welds.

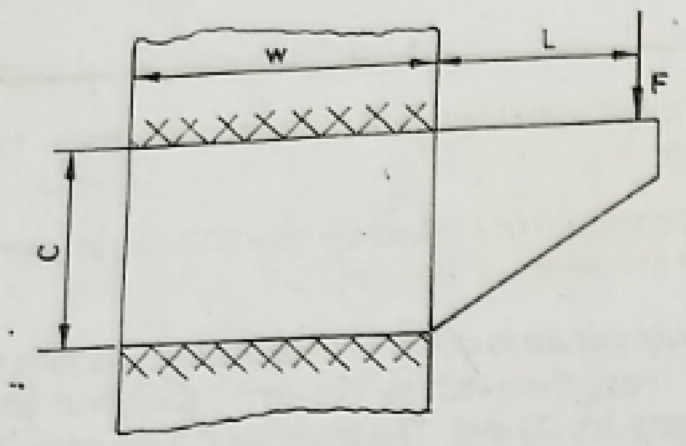


Fig. 6.7

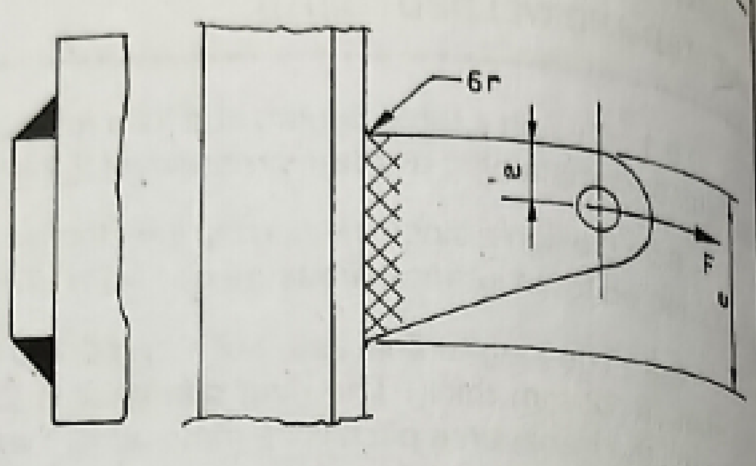


Fig. 6.8

6.9 : Find the load F that will produce a maximum shear stress of 140 MPa in the welds shown in the Fig. (6.9).

6.10 : A force F = 7.5 kN acts on the bracket shown in the Fig. (6.10). Find the maximum combined stress in the weld metal.

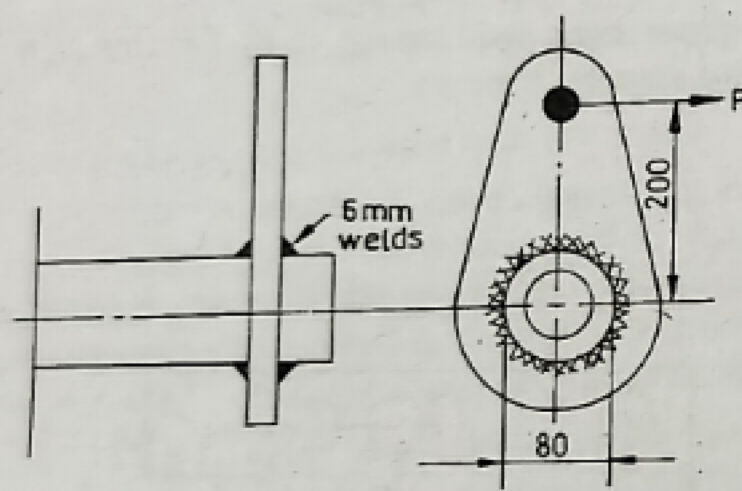


Fig. 6.9

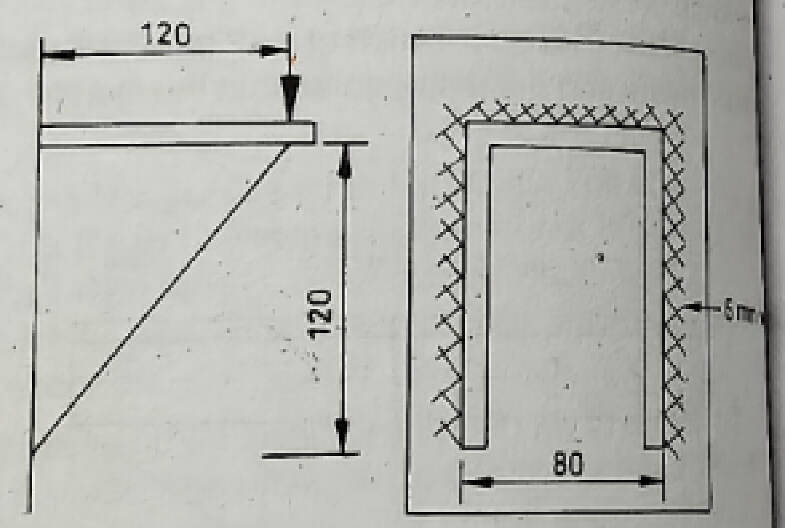


Fig. 6.10

Chapter 7

PIN JOINTS AND COTTER JOINTS

7.1 : Design and draw a neat sketch of a knuckle joint to connect two rods made of SAE 1020 steel which are subjected to a tensile load of 30 kN. The parts of the knuckle joint are also made of the same steel.

7.2 : Two rod ends of a pump are joined by means of a cotter joint. Design the joint for an axial load of 50 kN which alternately changes from tension to compression. All the parts are made of SAE 1020 steel.

Chapter 8

FITS, TOLERANCES, PRESS & SHRINK JOINTS

8.1 : A medium force fit on a 60 mm diameter shaft requires a H7-p6 fit. Give the proper hole and shaft dimensions in accordance with the basic hole standard. What are the maximum and minimum interferences ?

8.2 : A centering fit for a flange coupling is to locate the two halves of the coupling. If an H7-h6 fit is used to provide for a close fit, the allowance is proper but the tolerances are small. Determine the necessary dimensions of the centering fit for a nominal dimension of 100 mm, if a compromise is made by using the allowance of an H7-h6 fit with the tolerances of an H8-f7 fit.

8.3 : A 50 mm O.D. × 28 mm I.D. steel hub is to be assembled on a 28 mm diameter steel shaft with a heavy press fit not using a selective assembly. Design the fit.

(a) What will be the maximum radial contact stress ?

(b) What will be the actual maximum & minimum tangential stresses at the contact surface ?

(c) What is the maximum torque that may be transmitted with this assembly if the hub length is 75 mm ? Assume $\mu = 0.15$.

8.4 : It is desired to assemble two steel cylinders having nominal diameters of 25 mm I.D. × 50 mm O.D. and 50 mm I.D. × 75 mm O.D. with the tangential stress at the inner surface of the outer member limited to 83 MPa. Determine the required interference. Also determine the tangential stresses at the inner and outer surface of the two members.

8.5 : A 450 mm O.D. × 250 mm I.D. steel hub is to have a shrink fit on a 250 mm diameter steel shaft. The tangential stress at the contact surface is to be 100 MPa. The length of the hub is 300 mm.

(a) What is the radial stress at the contact surface ?

(b) How much torque may be transmitted assuming a coefficient of friction of 0.18 ?

To what minimum temperature should the hub be heated in order to permit assembly without interferences ?

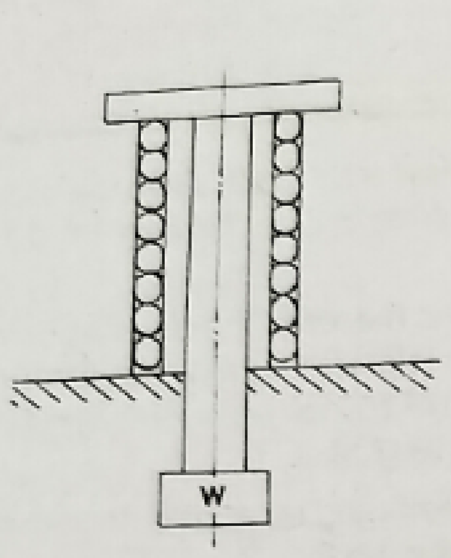
Chapter 9

MECHANICAL SPRINGS

9.1 : A machine uses a pair of concentric helical compression springs to support an essentially static load of 2.25 kN. Both springs are made of steel and have the same length when loaded and unloaded. The outer spring has $D_o = 45$ mm d = 8 mm, N = 5 ; the inner spring $D_o = 25$ mm, d = 5 mm and N = 10. Calculate the deflection and the maximum stress in each spring.

9.2 : A force of 4.45 kN is required to engage a disc clutch. This force is to be provided by nine identical springs equal spaced around the pressure plate of the clutch. Because of space limitations, the outside diameter of the coils can be no more than 40 mm, and the length of the springs when the clutch is engaged cannot exceed 52 mm. The pressure plate must move 3 mm to disengage the friction surfaces which increases the force on each spring by 10 percent. Select suitable material and design the spring.

9.3 : A weight W = 10 kN supported by a spring (or springs) as shown in Fig. (9.3). A certain natural frequency of vibration of the weight is desired, and the spring rate necessary for the prescribed frequency is found to be 250 N/mm.



A junk yard has a group of identical steel springs available with the following specifications.

Mean diameter = 75 mm ; spring index = 6, No. of active coils = 8 ; G = 80 GPa.

Assuming that each spring may be modified by cutting of an many turns as necessary and refinishing the end, how many springs will be required to give the desired spring rate and how many active coils must be removed from each spring ? What is the maximum stress in each spring ?

Fig. 9.3

9.4 : A loaded narrow-gauge car weighing 18 kN and moving at a velocity of 72 m/min is brought to rest by a bumper consisting of two helical steel springs. In bringing the car to rest, the springs are to be compressed by 200 mm. Assuming the spring index of 6 and the allowable stress of 350 MPa, determine the following : (a) maximum load on each spring (b) diameter of wire (c) mean diameter of coils and (d) number of effective coils.

9.5 : A compression spring will be used in a service where the maximum value of the varying load is three times the minimum value. The spring specified below is available in stock and a check of capacity under these load conditions must be made to determine whether it is suitable. The minimum required factor of safety is 1.25. The spring specifications are :

Mean coil diameter = 50 mm; Wire diameter = 10 mm.

No. of active coils = 10 ; Free length of spring = 175 mm.

$F_{(max)}$ = 1.8 kN, Material of the spring = Cr – V steel SAE 6150

Apply the full value of the Wahl factor tot he variable stress; but only the portion of the Wahl factor that corrects for transverse shear is to be applied to the mean stress.

9.6 : a 100 MM O.D. steel coil spring having 10 active coils of 15 mm diameter wire is in contact with a 700 mm long steel cantilever having fire graduated leaves 100 mm wide and 6 mm thick as shown in the Fig. (9.6).

(a) What force if gradually applied to the top of the coil spring will cause the cantilever spring to defelct 30 mm ?

(b) What will be the maximum shear stress in the coil ?

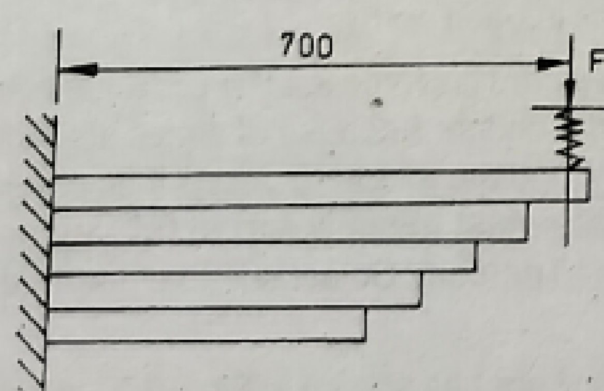


Fig. 9.6

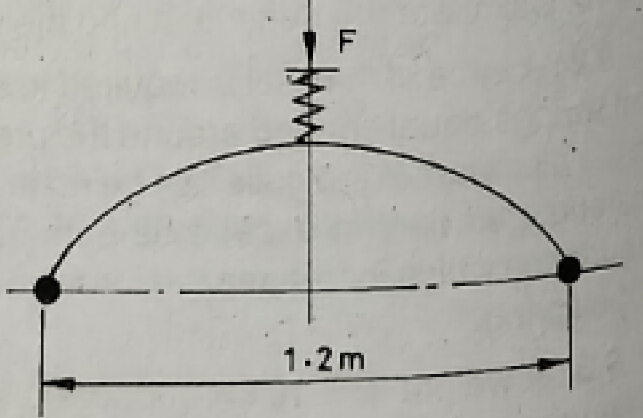


Fig. 9.7

9.7 : A semi-elliptic leaf spring has an effective length of 1.2 m. It is composed of 10 graduated leaves and one extra full length leaf, each 6 mm thick and 50 mm wide. The spring seat, midway between the shackles, carries a helical spring upon which is imposed an impact load equivalent to 1000 N-m of energy. The coil spring is composed of 6 effective turns of 25 mm wire with a mean coil diameter of 100 mm. Calculate the maximum stress induced in each spring. (Fig. 9.7)

9.8 : What thickness of lef is required for a semi-elliptical leaf spring for the following service ?

Load on the spring = 2 kN ; Total no. of leaves = 9

Extra full length leaves = 1 ; Width of leaves = 50 mm

Length of spring = 750 mm.

Material for the spring : SAE 1095 steel.

9.9 : A semi-elliptic leaf spring is to be designed so that it will just have infinite fatigue life at an overload (applied to the centre of the spring) that varies between 0.5 kN to 1.5 kN. The total length of the spring is to be 1m. The steel to be used has properties of S_{ut} = 1200 MPa, S_{yt} = 1030 MPa and S_{eb} = 500 MPa. The endurance limit is corrected for size and surface finish. Stress concentration of the centre hole, K_f = 1.3.

(a) Estimate the appropriate thickness, t and width, b of the leaves for b = 50 t.

(b) Based on a trapezoidal approximation, what will be the defelction at the centre of the spring when a 1.5 kN load is applied ?

9.10 : A semi-elliptic leaf spring for use in a light trailer is to be made of steel, SAE 9250. The spring is 1 m long and has 5 graduated leaves of 5 mm thickness and 100 mm width. The fatigue stress conentration factor, K_f = 1.4. When the trailer is fully loaded, the static load applied to the centre of the spring is 3.5 kN.

(a) Estimate the alternating load (due to driving over a rough road) that, when superimposed onto the full loaded spring, would be just on the verge of causing eventual fatigue failure.

(b) What will be the maximum deflection of the spring when loaded as determined in (a) ?

Chapter 10

CYLINDERS, HEAD AND COVER PLATES

10.1 : A cast iron drum with integral heads must be designed for an internal pressure of 5.10 MPa. The inside diameter must be 250 mm. Assume a better grade of cast iron. After determining the thickness of the cylinder walls, find the radius r of the inner curvature necessary to make the thickness of the head equal to that of the walls.

10.2 : A cast steel cylinder 250 mm in diameter has closed and is subjected to an internal pressure of 14 MPa. Determine the minimum permissible thickness of the cylinder walls using soft steel in the casting.

10.3 : A flat cast steel head 20 mm thick is fastened to the flange of a 200 mm cast steel cylinder by means of through bolts. Assuming for cast iron an allowable stress of 65 MPa, determine (a) the pressure that can be carried in the cylinder (b) the thickness of the cylinder walls and of the cylinder flange (c0 the number and size of bolts to fasten the head and (d) the outside diameter of the head.

10.4 : Determine the thickness of a flat cover made of high grade case iron for a cylinder having an inside diameter of 300 mm and carrying a pressure of 2 MPa. Find the number and size of stud bolts to hold the cover i) with a metal to metal joint and ii) with a gasket joint.

10.5 : The drum of a steam generating unit in 5.4 m long and 1.2 m in diameter. The steam pressure is 3.5 MPa. Assuming an allowable tensile stress of 70 MPa & that the efficiencies of the circumferential and longitudinal joints are 65 & 90 percent respectively, recommended the thickness of the shell.

10.6 : The operating cylinder for an air operated arbour press in shown in the Fig. (10.6). The cylinder must exert a maximum force F = 18 kN. The air pressure in the cylinder is 1 MPa. (a) Determine the diameter of cylinder bore required, assuming that overall friction due to stuffing box and piton packing is equivalent to 10 percent of F. (b) Determine the thickness of cylinder, assuming that it is seamless steel tubing. The allowable tensile stress is 20 MPa. (c) Determine the thickness of cast rion cylinder heads assuming a high grade cast iron. (d) Determine the diameter and numbr of bolts required for the cylinder.

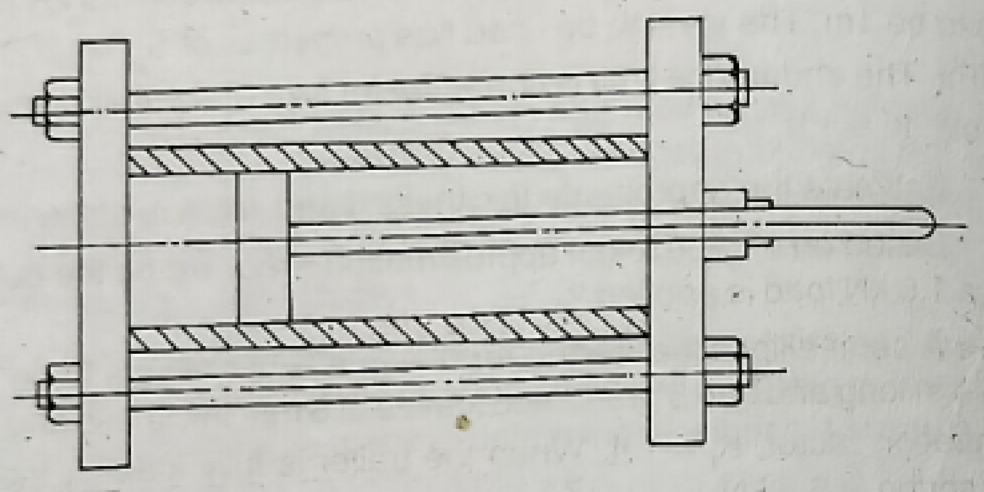


Fig. 10.6

10.7 : The dimensions of the gasketed pressure vessel joint shown in the Fig. (10.7) are. A = 120mm, B = 200 mm, C = 24 mm, D = 4 mm and E = 250 mm. A leak proof seal can be obtainede if the average gasket pressure is at least 15 MPa. Determine the proper material, diameter and number of bolts required for the joints.

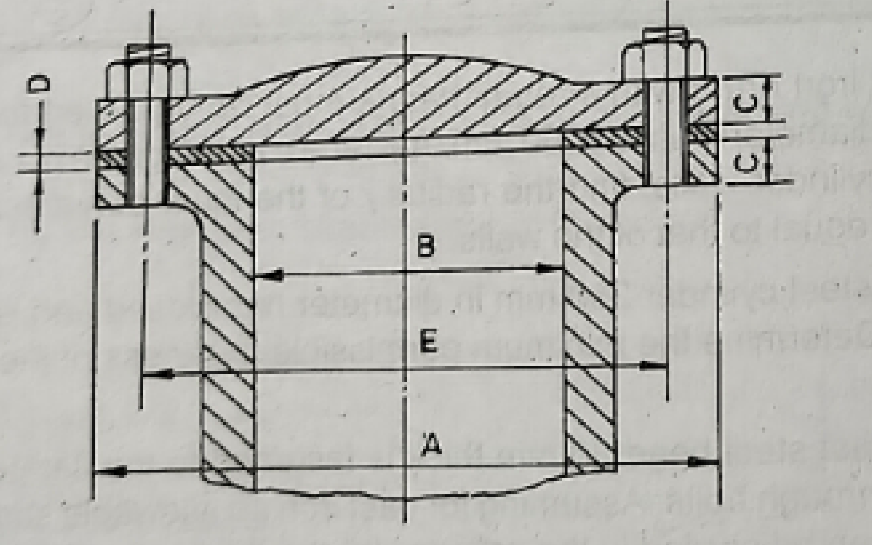


Fig. 10.7

10.8 : The cast steel head of a cast iron cylinder used to storage gas at 6 MPa is connected to the cylinder by means of 10 bolts and a confined gasket. The bore of the cylinder is 100 mm. Determine (a) the thickness of cylinder walls and flange (b) thickness of head (c) pitch circle diameter of bolts, (d0 gasket material, (e) Number and diameter of bolts.

Chapter 11

WIRE ROPES AND ACCESSORIES

11.1 : Determine the size of 6 × 19 plow steel rope for the elevator in a building 180 m high for a total load of 30 kN. The desired speed is 5 m/sec and the full speed must be reached in 12 m. Use four ropes and use large sheaves, selecting twice the minimum diameter recommended. Check the rope size and sheave diameter for fatigue life.

11.2 : The pulley system as shown in the Fig. (11.2) is used to raise a load of 27 kN with a maximum acc^n of 2.4 m/sec^2. Assume a 6 × 37 special flexible rope, average sheave diameter and a loss of 5 percent at each pulley. Determine the diameter of the wire rope for a safety of 5. Determine the effort E. Check the rope size by the bearing pressure ratio.

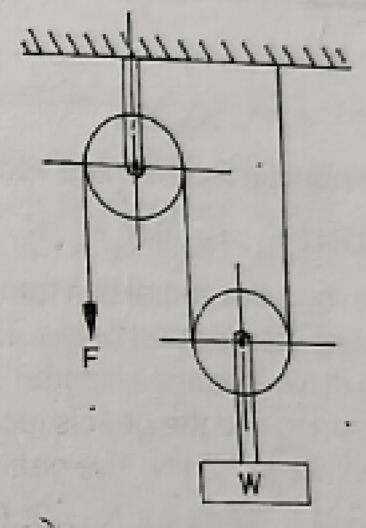


Fig. 11.2

11.3 : A 20 kN electric hoist is being designed for sale with a guaranteed life of five years service on the basis of its being used three times each hour, 16 hours per day and five days a week. As there is uncertainty in the design data it is proposed that the rope be designed to carry 150 percent to rated load for a life 25 percent greater guaranteed. Select proper rope and sheave diameter to make the hoist as light and as compact as possible.

11.4 : The four wire ropes that lift the dipper of a large power shovel must carry a maximum load of 1800 kN when over-burden in a strip mine. The shovel has a cycle time of 45 sec. Considering this service to be equivalent of cranes sepcify the diameter of a 6 × 19 regular lay plow-steel wire rope. How many hours of continuous use will be expected before rope failure if the sheaves have the recommended minimum diameters ? What will be the percent increase in expected life for the rope if average recommended sheave diameters are used instead of the minimum ?

Chapter 12

SHAFTS, KEYS AND SPLINES

12.1 : The Fig. 12.1 shows a counter shaft which receives power through the helical gear 3 and delivers it to another gear through the bevel gear 4. The forces acting on the gears and the pitch diameters are shown in the figure. The gears are keyed to the shaft. The shaft is made of SAE 1030 steel. Suggest suitable diameter for the shaft. Show the steps on the shaft which will be necessary for the location of different parts. The shaft design in such that only the bearing at O can take the axial load.

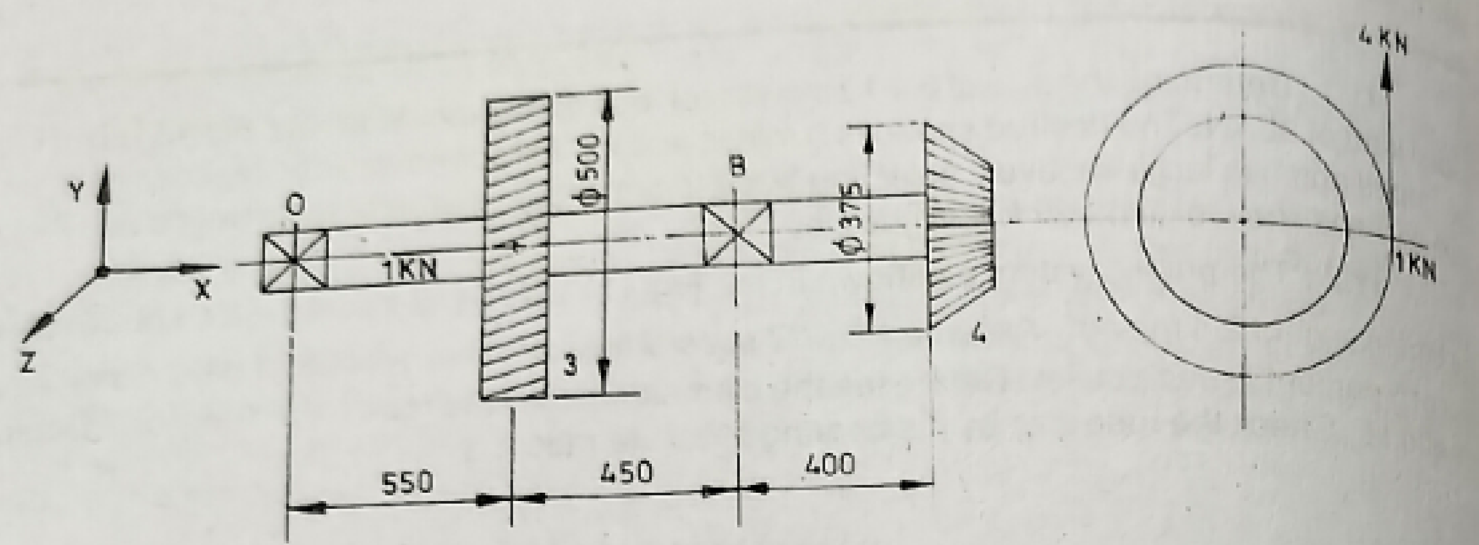


Fig. 12.1

The force on the gear 4 in kN can be represented vectorially as :

$F_4 = -0.242\ F_4 . i - 0.242\ F_4 . j + .940\ F_4 , K$

12.2 : A 600 mm pulley driven by a horizontal belt transmits power thorugh a solid steel shaft to a 250 mm pinion which drives a mating gear located below. The pulley weights 1 kN to provide some flywheel effet. The shaft is supported in two bearings located 750 mm apart. The pulley is located at 250 mm to the right of the left hand bearing and the gear is located at 200 mm to the right of the right hand bearing. The belt tensions are 5 kN and 1.5 kN. The gear has 20° involute teeth. Determine the necessary shaft diameter as per A.S.M.E. code for $K_b = 2.0$ and $K_t = 1.5$. The shaft is made of SAE 1030 steel.

12.3 : The coefficient of friction between the cam and follower disc shown in the Fig. (12.3) is 0.3. The torque required to turn the cam under these conditions is supplied at the right end of the cam shaft. If the camshaft is made of steel for which $S_{ut} = 550$ and $S_{yt} = 390$ MPa. What is the required minimum diameter of the cam shaft ? The shock and fatigue are $K_b = 2.0$ and $K_t = 1.5$.

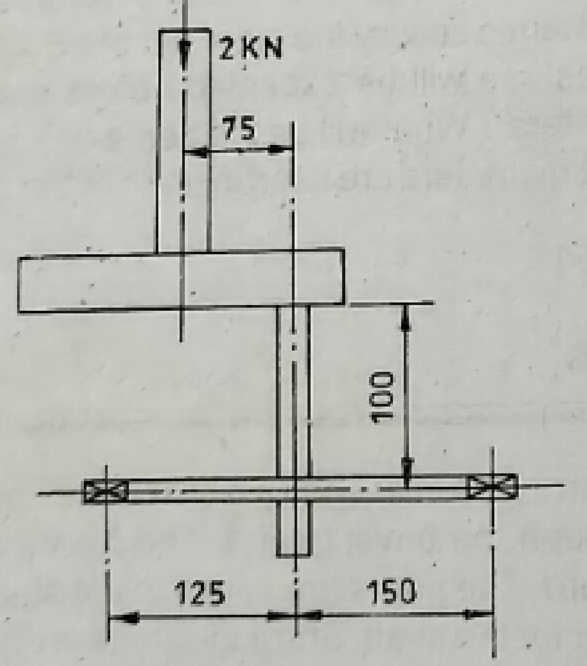


Fig. 12.3

12.4 : A steel shaft 2 m long has applied to it a 1200 N-m torque by a gear located at the centre of the shaft. A pulley at the left end of the shaft applied 600 N-m of torque to the shaft while a gear located 250 mm to the left of the right end of the shaft applies 600 N-m of torque. Calculate the angular deflection of the shaft if the shaft is 60 mm in diameter for a length of 1.2 mm from the left end of the shaft and 40 mm in diameter for the remaining length. Neglect the effect of the keyways.

12.5 : Suggest suitable dimensions for the rectangular key and the minimum length of shaft extension for a kW, 720 rpm three phase induction motor. Assume the key is made of SAE 1030 steel.

12.6 : Determine all the dimensions of splires for a 55 mm shaft that will permit sliding when under load to transmit 75 kW at 800 rpm. Compare the lenght of splines with the length of a rectangular key which will be required to transmit the same torque.

Chapter 13

BEARINGS WITH SLIDING CONTACT

13.1 : A sleeve bearing has a diameter of 75 mm, a journal speed of 10 rev/s and a length 75 mm. The oil supply is SAE 30 at an inlet temperature of 70°C. The bearing carries a radial load of 2.7 kN and has a radial clearance of 35 μm. Calculate the heat loss, the side flow, the total flow, the minimum film thickness and the temperature rise.

13.2 : A 12 kW, 300 rpm centrifugal pump has two identical journal bearings. The journal diameter is 75 mm and the load on the journal 1.2 kN. The ambient temp can be assumed as 30°C. Design the bearing. Select proper lubricating oil and calculate the heat loss.

13.3 : A 1000 kW generator is to be drive at 3600 rpm by a steam turbine. The generator bearings are required to carry only the 16 kN of the rotor. The journal diameters and lengths are 100 mm and 125 mm resp. Assuming that the diametral clearance will be 0.15 mm, the oil will be SAE 10 and the temperature of the oil leaving the bearing will be maintained at 60°C by circulating the oil through a heat exchanger. Determine (a) the operating minimum film thickness, (b) the power loss in the bearings (c) the oil flow required if all the heat is removed by the oil with a temperature rise of 10°C, (d) the speed at which the minimum film thickness will be .0025 mm and (e) the minimum length of bearing that would give a satisfacory oil film minimum thickness.

13.4 : A journal bearing has been operatin satisfactorily for many years under the following conditions.

Load -- 16 kN ; Speed -- 2000 rpm ; Journal Dia. -- 75 mm ; Journal length -- 75 mm ; Diametral clearance -- 0.15 mm ; Oil -- SAE 10 at 70°C (temperature maintained by circulating the lubricating oil through a cooler)

It is proposed to use the machine under identical conditions as before expect that the journal speed is to be 1000 rpm. (a) Do you recommend using the bearing at the new operating speed ? Justify your conclusion (b) If your answer to (a) is yes, determine the power loss in the bearing (c) if your answer to a) is no, suggest a change that may be made easily and prove that your suggestion will work.

13.5 : A 2.8 kN load is supported by a journal bearing 80 mm long on a 60 mm shaft. The bearing has clearance of 0.05 mm and is lubricated with an oil of viscosity 20 mPa-s at its operating temperature. Determine the maximum speed of rotation if the bearing is capable of dissipating 80 watts by means of heat transfer.

13.6 : A full journal bearing is 100 mm diameter by 150 mm long and has a radial load of 30 kN. The shaft speed is 600 rpm and an H7-f7 fit is to be used. The surrounding air temperature is 30°C and the oil used has a viscosity of 90 mPa-s at its operating temperature. Determine the probable temperature of the bearing surface if there is no artificial cooling. Assume a well ventilated bearing.

13.7 : A furnace is being designed for annealing castings on a production line basis. Small four-wheeled trucks loaded with castings are to be run through the furnace at a speed of 1 m per minute. The truck axle bearings are somewhat shielded from the heat when in the furnace but they may still reach a temp. of 175°C. Assuming that the total weight oi the truck and castings will be 17 kN, the wheels will be 300 mm diameter and the journal diameters will be 50,mm, (a) determine the coefficient of friction by suggesting suitable length of journals. (b) what bearing material and lubricant should be under for this bearing ?

13.8 : A multicollar bearing has six collars which has an inside diameter of 65 mm and an outside diameter of 90 mm. Determine the thrust load which the bearing can carry and the power lost through friction if the shaft speed is (a) 1200 rpm, (b) 200 rpm and (c) 20 rpm. The bearing is water cooled. Determine the amount of water to be circulated for a temperature rise of 10°C.

13.9 : The oil engine of a motorboat develops 750 kW at 200 rpm. If 75 percent of the power is transmitted to the propeller shaft, suggest the main dimensions of a multicollar thrust bearing, (a) find the loss of power caused due to friction (b) select the oil to be used for lubrication and determine the amount of oil that must be circulated through the bearing for a temperature rise of 25°C.

13.10 : A slider bearing with a rectangular pivoted shoe has the following specifications :

Length of the shoe -- 75 mm ; Width of the shoe -- 115 mm.

Velocity of the moving member -- 2 m/s

Expected mean oil film temp -- 80°C

Minimum oil film thickness -- 0.025 mm.

Viscosity oil film operating temp -- 28 mPa-s.

Assume that the inclination of the bearing surfaces corresponds to maximum load carrying capacity of the bearing. Determine (a) load carrying capacity of the bearing and (b) the power loss in the bearing.

Chapter 14

BEARINGS WITH ROLLING CONTACT

14.1 : The shaft in the Fig. (14.1) has parallel belt pulls with the tension on the loose side of pulley 4 being 20 percent of the tension on the tight side. The shaft rotates at 900 rpm and the radial ball bearings to be selected for locations at O and B are to be have a reliability of 80 percent corresponding to a life of 30,000 hrs. Suggest the suitable shaft size and specify the size of 02 series bearings needed. Only one size of bearing is to be used.

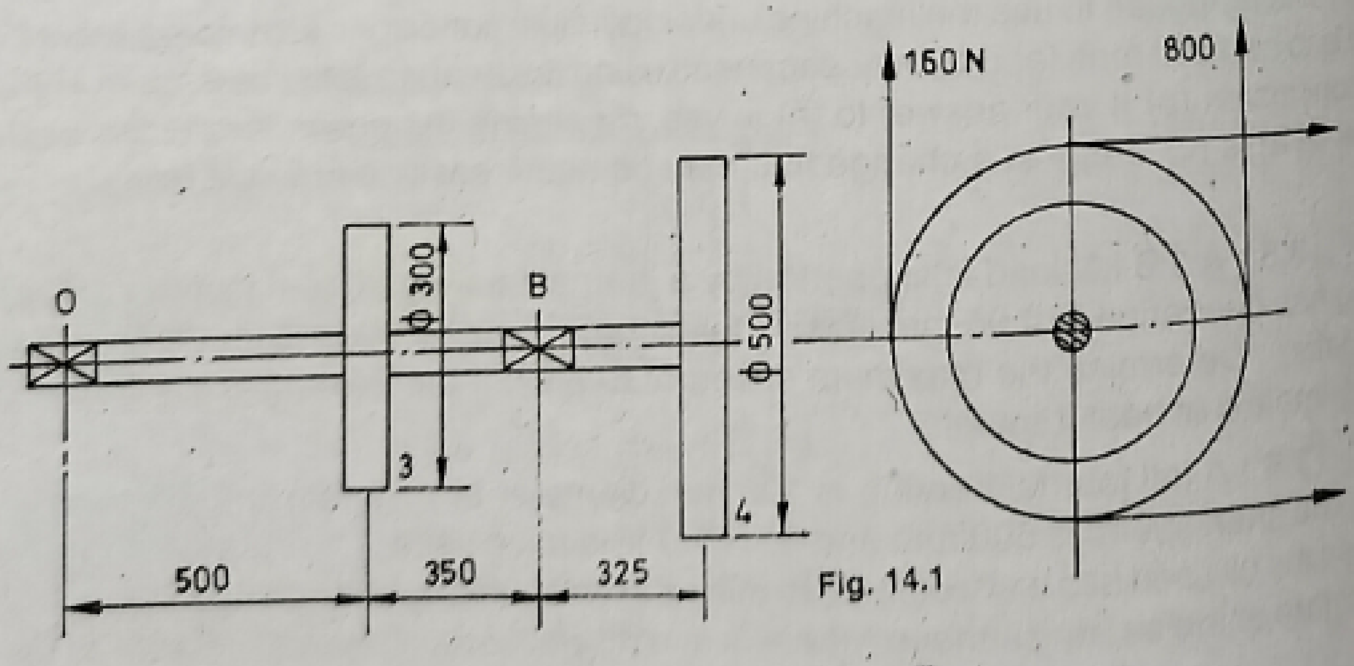


Fig. 14.1

14.2 : A roller bering is rated by one manufacture to have an average life of 1000 hrs when carrying a load of 40 kN at 1800 rpm. (a) What is the 80 percent life expectancy for these bearing under the same load and speed ? (b) What will be the rated load capacity of this bearing for an average life of 50,000 hrs at 1500 rpm ?

14.3 : A bearing supporting a 1000 rpm shaft carries a 3 kN radial load and kN thrust load. The shaft is part of a machine for which loading can be considered as moderate shock. The required life is 5000 hr with 95 percent reliability select a 03 series ball bearing for this application.

14.4 : A no. 312 contact ball bearing is loaded uniformly (without shock) as follows :

55 percent time -- 7 kN at 1800 rpm ; 25 percent time -- 14 kN at 1200 rpm ; 20 percent time --18 kN at 800 rpm.

Estimate the bearing life for 90 percent reliability.

14.5 : A no 204 radial contact ball bearing is used in a gearing application considered to be average with respect to shock loading. The shaft rotates at 3000 rpm, 8 hr/day. The bearing is subjected to a radial load of 1 kN and thrust load of 400 N. Estimte the average life of the bearing in days. What will be the life if the thrust load is changed to 200 N ?

Chapter 15

ROTORS AND FLYWHEELES

15.1 : Design the flywheel required for a punch press for the following conditions : The press requires 12 kN, the complete cylce consists of 7 revolutions of the flywheel, only two and a half of which take place during the working portion of the cycle ; there are 30 complete cycles per minute and the desirable mean diameter si 1.25 m. Assume other suitable data.

15.2 : A multicylinder engine is to run at a constant speed of 600 rpm. In the T - θ diagram the areas above and below the mean torque line in mm^2 are as follows ;

$+ 160, - 172, + 168, - 191, + 197, - 162$

The speed is to be kept within ±2 percent of the mean speed of the engine. Design a suitable cast-iron flywheel for the engine.

15.3 : A shaft fitted with a flywheel rotates at 250 rpm and drives a machine, the resisting torque of which varies in a cyclic manner over a period of three revolutions. The torque increases from 600 N-m to 2400 N-m gradually during half revolution and remains constant for the following one revolution. If then decreases uniformly to 600 N-m during the next one revolution and remains const. for the remaining half revolution, the cycle being then repeated. The coefficient of fluctuation of speed is to be limited to 5 percent. Design a suitable flywheel with a mean radius of 0.5 m.

15.4 : Suggest suitable weight and cross section for the rim of a cast iron flywheel to be used on a two cylinder four stroke I.C. engine which develops 50 kW at 1000 rpm. The cranks have a phase difference of 180°. The total fluctuation of speed should be limited to 5 percent of the mean speed. The work done during the power stroke may be assumed to be 1.3 times the work done during the whole cycle in a cylinder. The flywheel diameter should not be greater than 1.4 m.

15.5 : A one piece cast iron flywheel has an outside diameter of 1.35 m and a rectangular rim section with 175 mm width and 100 mm thickness. Determine (a) safe speed of the flywheel (b) the theoretical bursting speed, and (c) the safe speed if it was mode of cast steel.

Chapter 16

COUPLINGS AND POSITVE CLUTCHES

16.1 : In a flanged shaft coupling having a 40 mm bore, it is decided that the torsional stress int he shaft will not exceed 25 MPa. The outside diameter of the coupling is 190 mm. There are three 16 mm bolts on a bolt circle 65 mm in radius. The radial flange is 20 mm. Determine the following (a) the power that may be transmitted at 600 rpm ; (b) the shearing stress in the bolts and (c) the bearing pressure on the bolts.

16.2 : Design a bushed pin type of flexible coupling for connecting a motor shaft to a centrifugal pump shaft for the following service conditions :

Electric Motor --- 30 kW at 960 rpm.

Dia. of pump shaft --- 60 mm.

The allowable bearing pressure on the rubber bush = .45 MPa.

The pins are made of SAE 1030 steel.

16.3 : Design and draw neat sketh of a square jaw coupling for connecting 100 mm shafts when they are to be engaged during running. Using machine jaws.

Chapter 17

FRICTION CLUTCHES AND BRAKES

17.1 : An automotive type single disc clutch has inside and outside diameters of 150 and 225 mm. respectively. Clamping force is provided by mine springs, each compressed 6.5 mm to give a foce of 625 N when the clutch is new. The moded friction material provides a conservatively estimated coefficient of friction of 0.35 when in contact with the flywheel and pressure plate. Maximum engine torque is 280 N-m.

(a) What is the safety with respect to slippage of a brand new clutch ?

(b) What is the safety factor after "initial wear" has occured ?

(c) How much wear of the friction material can take place before the clutch will slip ?

(d) Design the springs.

17.2 : A multiple disc clutch is to operate in oil, and be able to transmit a design overload torque of 650 N-m. The discs are alternately high carbon and moded asbestos, with inside and outside diameters of 90 and 150 mm respectively. Design the clutch with P_{max} = 1000 kPa. How many discs will be required ? What should be the axial clamping force ?

17.3 : When in use, the motor and flywheel of a punch press run continously. A compressed air actuated, multiple disc clutch connects the flywheel to the counter shaft each time the press is to punch or form metal. The clutch torque required is 525 N-m. Alternatte sintered metal and cast iron discs are to be used. Assume the coefficent of friction as 0.2. Space requirements limit the disc outside diameter to 220 mm. Assume a PV value of 110 if P is in MPa and V in m/min. In addition a safety factor of 1.15 is to be used with respect to torque capacity.

(a) Determine the minimum satisfactory total number of discs.

(b) Using this number of discs, determine the minimum area of compressed air actuating surface that must be provided if air will always be available at a pressure of 0.40 MPa.

17.4 : A cone clutch is mounted on a shaft which transmits power at 225 rpm. The small diameter of the cone is 230 mm, the cone face is 50 mm, and the cone face makes an angle of 15° with the horizontal. Determine the axial force necessary to engage the clutch to transmit 4.5 kW if the coefficient of friction of the contact surfaces is 0.25. What is the maximum pressure on the contact surfaces assuming uniform wear ?

17.5 : An engine developing 30 kW a 1000 rpm is fitted with a cone clutch built into the flywheel. The cone has a face angle of 12.5° and a mean diameter of 350 mm. The coefficient of friction is 0.20 and the normal pressure on the clutch face is not to exceed 80 kN/m^2. Determine the required face width and the force necessary to engage the clutch, assuming uniform wear.

17.6 : A small manufacturing plant has a 1200 rpm 200 kW generator that may be drive by either a steam turbine or diesel engine. Both power units are to be connected to the generator by means of spring-controlled centrifugal clutches to permit either unit to be at rest or warning up at an idling speed, while the other is driving. Assuming that the coefficient of friction is 0.35, that the allowable pressure for the friction material is 0.2 MPa, that the shoe width is 30 percent of the drum diameter, that the separator ribs between the shoes will take up 15 percent of drum area, that the radius to the centre of gravity of the shoes is 85 percent of drum radius, and that the normal and tangential forces act at the centre of the contact between each shoe and the drum, determine (a) the required wieght of each of the eight shoes and (b) the spring force required on each shoe at the instant the shoe contact the drum if the clutches are to meet the following specifications : (1) the shoes are to contact the drum at 500 rpm and (2) the capacity at 1200 rpm is to be 140 percent of the generator rating.

17.7 : A double block brake is actuated as shown in the Fig. (17.7). The drum rotates at 300 rpm when the applied force is 2.5 kN and the coefficient of friction is 0.3. Determine the braking torque and the amount of heat being generated per unit time.

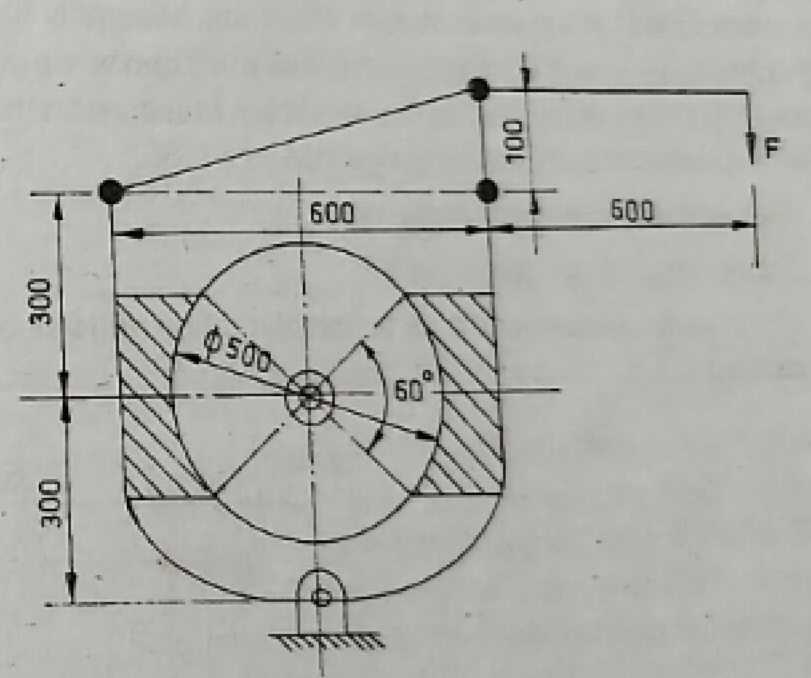


Fig. 17.7

17.8 : A spring set hydraulically released brake is to be designed to have torque capacity of 54 N-m under almost continuous duty when the brake drum is rotating at 400 rpm in either direction. The brak lining is to be an asbestos composition, the coefficient of friction will be 0.36, the width of the brake shoes is to be one third the diameter of the drum and the remaining proportions are to be as shown in the Fig. (17.8). Determine (a) the required brake drum diameter and width of the brake shoes, (b) the spring force required to set the brake and (c) the hydraulic pressure required to release the brake if the bore of the cylinder is 50 mm.

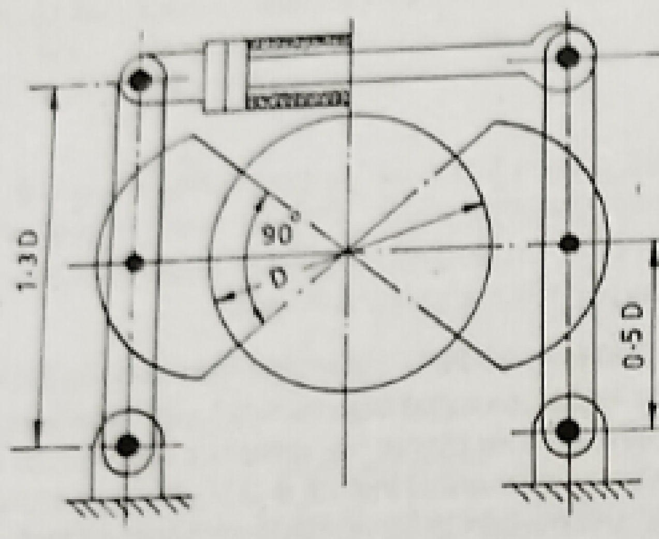


Fig. 17.8

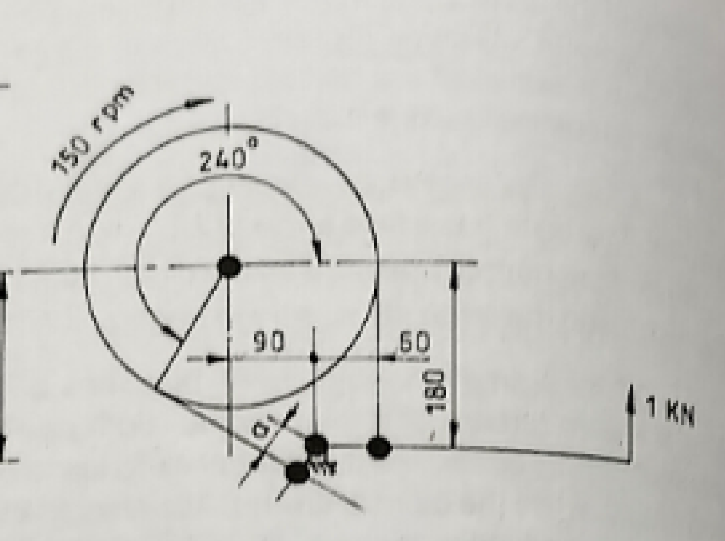


Fig. 17.9

17.9 : A band brake is designed as shown in the Fig. (17.9). The ratio between the band tensions is 2.00. The belts are fastened normal to the lever.

(a) What must be the distance 'a' in order that the bands be fastened normal to the operating lever ?

(b) How much power can be brake absorb ?

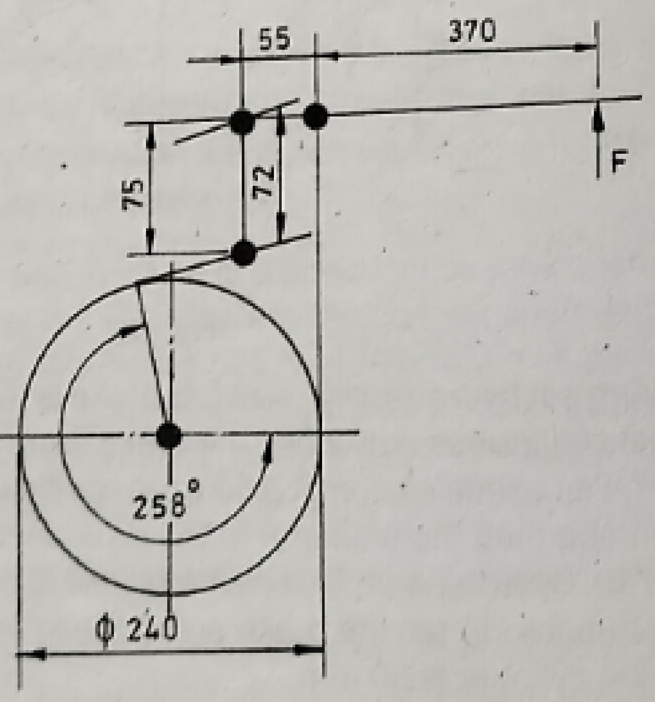
17.10 : The Fig. (17.10) shows a band brake used with a punch press. When in use, the clutch is released when the crank is 120° past bottom dead centre. The brake is to be engaged at this point, and bring the crank to rest at top dead centre. The crank assembly has a mass moment of Inertia of approximation 12 $N-m-s^2$ and is rotating at the rate of 40 rpm when the brake is engaged. The brake will be used about 3 times per minute, so the maximum pressure on the band lining should be limited to about 0.2 MPa. Coefficient of friction can be taken as 0.35.

(a) Determine the required band width

(b) Determine the operating force F

(c) Would any combination of direction of rotation and value of coefficient of friction make the brake self-locking ?

Fig. 17.10



Chapter 18

BELT, CHAIN AND ROPE DRIVES

18.1 : A machine requiring 37 kW at about 320 rpm is to be driven by an 1440 rpm electric motor by use of a flat leather belt. It is assumed that the centre distance will be 1.5 m and that both pulleys will be cast-iron, what are your recommendations for (a) pulley diameters, (b) combination of width and thickness of belting (c) length of belt and (d) minimum initial tension.

18.2 : A 7.5 kW, 1440 rpm electric motor is to drive the input shaft of a hammer mill at 600 rpm. Space is at a premium and one proposal is to use a short centre flat-belt drive with a centre distance equal to the diameter of the larger pulley. Design the flat belt drive and the larger pulley.

18.3 : A line shaft must be connected to be driving motor by a medium double leather belt. The C.I. motor pulley is 450 mm in diameter and runs at 900 rpm. The actual speed of the line shaft should be redued in the ratio of 4.75 : 1. If all the machines driven from the lineshaft are operating simultaneously, the total maximum load is 85 kW. However, the actual load is never higher than 72 percent of the max. load. Determine the necessary width of the belt, and the diameter and width of the line shaft pulley. The centre distance is 5.4 m and makes a 50° angle with the horizontal line. Design and draw a neat sketch of the line shaft pulley.

18. 4 : Select a V belt drive for connecting a 1.5 kW, 960 rpm motor to a 300 rpm blower which is part of an air conditioning unit. The centre distance may be taken as approximately 1.5 times the diameter of the larger pulley. What will be the width of the pulleys and length of the belts ?

18.5 : An electric motor drives a compressor thorugh V belts. The following data are known :

Sr.No.	Item	Motor pulley	Compressor pulley
1.	Pitch diameter	225 mm	1200 mm
2.	Angle of wrap	2 rad.	4.28 rad.
3.	Groove angle	36°	Flat
4.	Power transmitted 17.5 kW	17.5 kW	
5.	Speed	1800 rpm	?

Select proper V bet section and suggest suitable number of belts for the drive. What will be the width of the pulleys ? What will be the outside diameters of the two pulleys ?

18.6 : A leather belt is to transmit 25 kW from a 250 mm fibre covered pulley running at 1200 rpm to a 600 mm steel pulley. The coefficient of friction between the steel pulley and belt is 0.30 and that between the driver and belt is 0.40. The centre distance between the shaft is 1.8 m. Assume that the maximum allowable working stress in the belt is 2 MPa and that the belt mass density - 970 kg/m^3. Which of the following belt will be more suitable --9 mm thick or 12.5 mm thick ? Calculate the width of the belt.

18.7 : A V belt transmits 12 kW at 960 rpm of the smaller sheave. The pitch diameters are 173 mm and 346 mm. The centre distance is 750 mm. Determine the proper section and number of V belts required for the drive.

18.8 : A double strand no. 60 roller chain is used to transmit 13 tooth driving sprocket rotating at 300 rpm and a 52 tooth driven sprocket.

(a) What is the rated powre of this drive ?

(b) Determine the approx. centre distance if the chain length is 82 pitches.

18.9 : A 960 rpm motor is to be used to drive an oil field pumping unit that requires 400 N-m torque at approximately 300 rpm. Assuming that a roller chain drive is to be used and that there is an efficiency of 95 percent and service factor of 1.5, determine the following : size of motor, number of teeth on the sprockets, pitch and number of strands of the chain and pitch diameters, outside diameters and width of sprockets.

18.10 : A roller chain drive, as specified below, is to be replaced by a V belt drive. Assuming that 150 mm sheaves are to be used, determine the number of B section V belts required for the replacement drive.

Pitch	18.75 mm
No. of strands	2
No. of teeth on sprockets	21
Speed	960 rpm
Velocity ratio	1 : 1

Chapter 19

SPUR AND HELICAL GEAR DRIVES

19.1 : A reciprocating compressor running at 600 rpm is driven by a 15 kW, 1440 rpm electric motor through a pair of spur gears for continuous duty. Design the gears and give all the specifications including heat treatement. Design the blank for the compressor gear and draw a neat sketch of the same.

19.2 : The Fig. (19.2) below shows a ddrive for a rotary kiln driven by a 7.5 kW, 960 rpm electric motor through a counter shaft. The kiln is required to be rotated at 60 rpm. The gears may be designed for a starting overload of 30 percent. The kiln has to operate 24 hours per day. Design all the geas and the counter shaft along with the ball bearings for the shaft. Give a dimensioned sketch of the counter shaft.

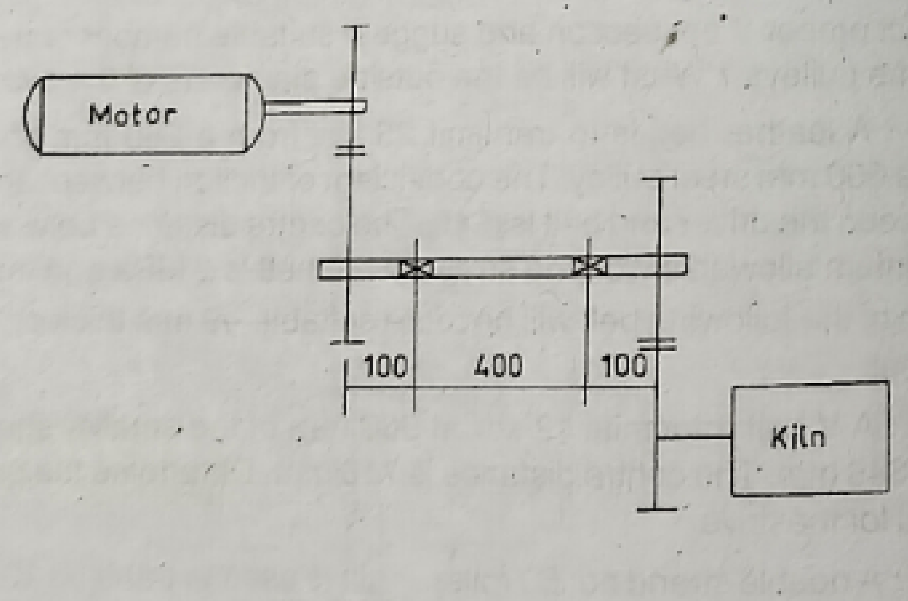


Fig. 19.2

19.3 : A pair of gears with 20° stub teeth must transmit 75 kW at a speed 5000 rpm of the pinion. The velocity ratio is 8. Design the gears for continuous duty and specify the accuracy grade required for these gears. Specify all the details including materials and heat treatment.

19.4 : A pair of mating gears have 3 mm module, 20° full depth teeth. Both gear and pinion are made of steel SAE 1045 heat treated to 350 Bhn. Both have a face width of 30 mm. The teeth are cut with a top quality hobbing operation. The pinion has 18 teeth and drives a 36 tooth gear. The pinion rotates at 1800 rpm. The pinion drives the gear fixed to the blower shaft for continuous operation. Estimate the poewr that the gears can transmit.

19.5 : A 11 kW electric runs at 960 rpm and drives a ventilating fan through a gear reduction having a velocity ratio of 4 : 1. The gears are to have 20° stub teeth. The pinion is to be made of micarta and the gear is to be made of cast iron. Design the gears and give all the specifications.

19.6 : The gate of sluice valve weighing 60 kN is raised by means of a C.I. rack and pinion. Design the train of gears, including the rack so that the gate may be raised by two men working on 400 mm crank handles and exerting a pressure of 150 N each. Give also the linear speed of the rack motion, assuming that the hand crank makes 25 rpm.

19.7 : A pair of helical gears are to transmit 15 kW. The teeth are 20° stub and have a helix angle of 45°. The pinion operates at 10,000 rpm and the velocity ratio is 4:1. The gears are made of cast steel with $S_o = 100$ MPa. The pinion is heat treated to 300 Bhn and gear to 200 Bhn. Suggest suitable module, number of teeth and face width for the gears.

19.8 : The helical gear speed reducer shown in the Fig. (19.8) is rated to transmit 20 kW at an input speed of 2900 rpm. The helix angle is 30° and the 22 teeth pinion and 54 teeth gear were hobbed with 3 mm, 20° full depth hob. Determine (a) the pitch diameter of gears. (b) the outside diameter of gears (c) the face width of the gears (d) the axial or the thrust force on the output shaft and (e) the resultant forces acting on bearings. A and B. Assum suitable materials.

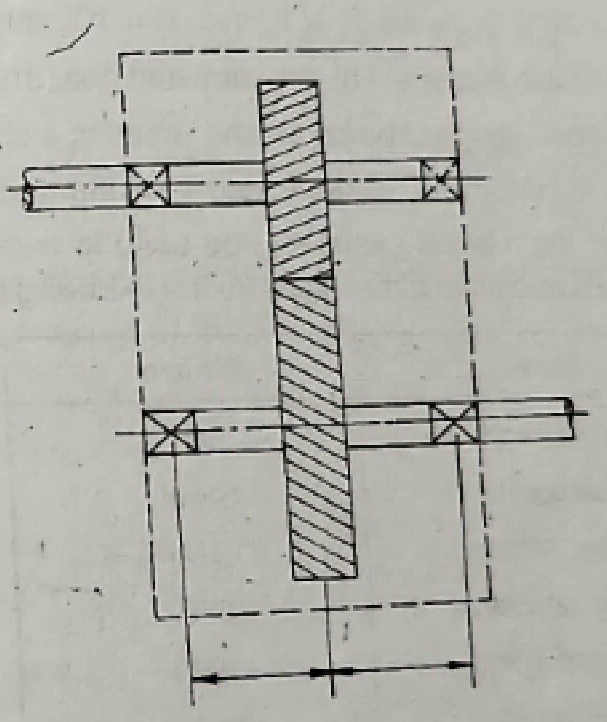


Fig. 19.8

19.9 : A pair of helical gears mounted on parallel shaft have a normal module of 6 mm. $\phi_n = 20^\circ$ and face width of 75 mm. The 32 teeth pinion and 44 teeth gear are made of SAE 2320 steel. The pinion is hardened to 400 Bhn and gear to 350 Bhn respectively. The centre distance is approximately 250 mm.

(a) What helix angle is required ?

(b) Estimate the power that can be transmitted if the pinion rotates at 960 rpm and the load involves medium shock.

19.10 : A helical gear speed reduce is used to transmit 25 kW from an electric motor running at 1440 rpm to a centrifugal blower running at 300 rpm. the gears are to be hobbed with 20° full depth hobs. The pinion should have 22 teeth. Design the pair of gears for continuous duty and give all the specifications including the heat treatment. Assume other data.

Chapter 20

WORM AND BEVEL GEAR DRIVES

20.1 : The engine of a truck develops its maximum power at 1200 rpm and 90 percent of this power is transmitted to the worm of the worm gear drive on the rear axle. The speed reduction in the transmission is 3 : 1 and that in the worm of the worm and gear 15:1. The worm has a triple thread of 90 mm lead, a pitch diameter of 67.5 mm and pressure angle of 30°. Determine the power of the engine that can be transmitted through the drive considering (a) strength alone, (b) permissible wear and (c) heat dissipating capacity. The worm is made of hardened steel and the gear is of phosphor bronze and has a face 50 mm wide.

20.2 : A traction type elevator is operated by a 40 kW, 960 rpm electric motor through a worm gear drive. The worm has 4 teeth with 20° pressure angle and a pitch diameter of 113 mm. The worm gear 52 teeth of 45 mm pitch and has a 100 mm face. The worm bearings are 450 mm centre to centre and the gear bearings are 225 mm centre to centre. Determine (a) the load on each bearing (b) the efficiency of the drive and (c) the temp. rise.

20.3 : A worm gear speed reducer driven by an 1440 rpm motor has a phosphor bronze gear and a harndened steel worm. $t_w = 3$, $t_t = 42$, $P_c = 12$ mm, C = 106 mm, b = 20 mm and $¢_n = 20^\circ$.

(a) Determine the pitch diameters of the worm and gear and the length of the worm.

(b) Is the worm large enough to be bored and keyed to a shaft ?

(c) Estimate the efficiency and the temp. rise if the input to worm is 1.5 kW.

20.4 : A pair straight tooth bevel gears is to be used to transmit 15 kW continuous duty. Determine the required module and other dimensions for the following specifications :

Sr.No.	Item	Pinion	Gear
1.	No. of teeth	21	60
2.	Material	Steel	C.I.
3.	Basic stress	210 MPa	105 Mpa
4.	Brinell hardness	250	160
5.	Speed, rpm	1200	420
6.	Tooth profile	20° FD	20°FD

20.5 : Two steel bevel gears with a 14 1/2° full depth tooth profile have been designed for strength to transmit 20 kW at 1250 rpm of the pinion. The no. of teeth on the pinion is 23 and on the gear 32. The module is 6 mm and the face width is 40 mm. Select proper steel and the suitable heat treatment for continuous duty.

20.6 : A pair of straight tooth bevel gears mounted on perpendicular shafts transmit 45 kW at 1500 rpm of the 30 tooth pinion. The gear has 60 teeth. Face width is 62.5 mm and.module 5 mm. The pressure angle is 20°. The gear is mounted in the centre of a simply supported shaft with 150 mm span between bearings. Thrust is taken by the bearing having the lighter radial load. Suggest suitable diameter for the gear shaft and proper size of deep groove ball bearing.

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Table I - 1

Properties of Sections

Subscript p for polar properties

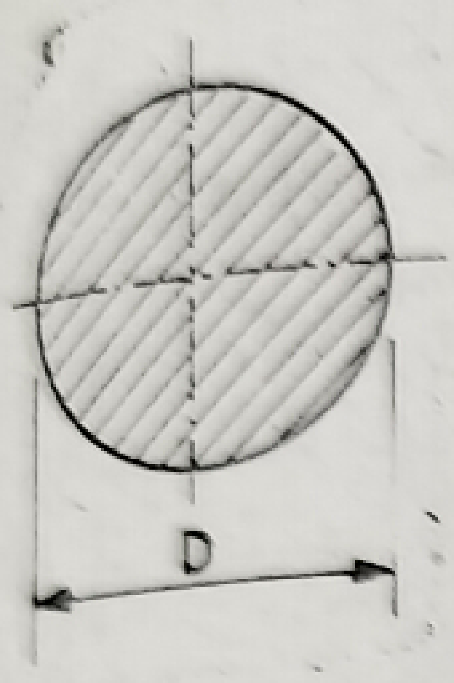
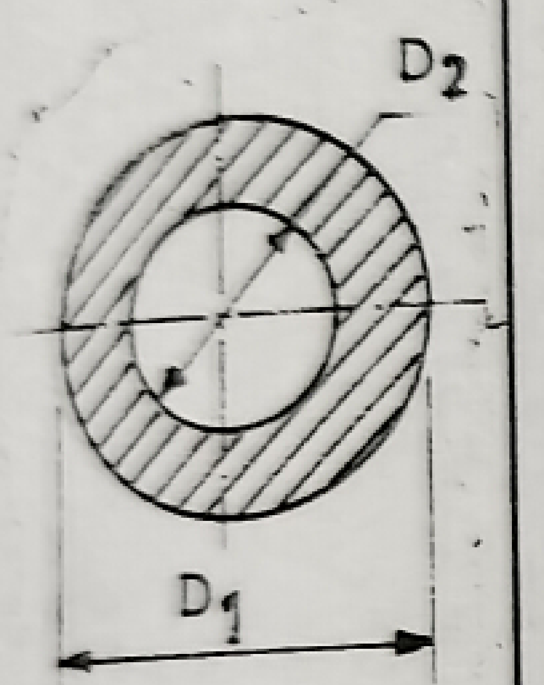
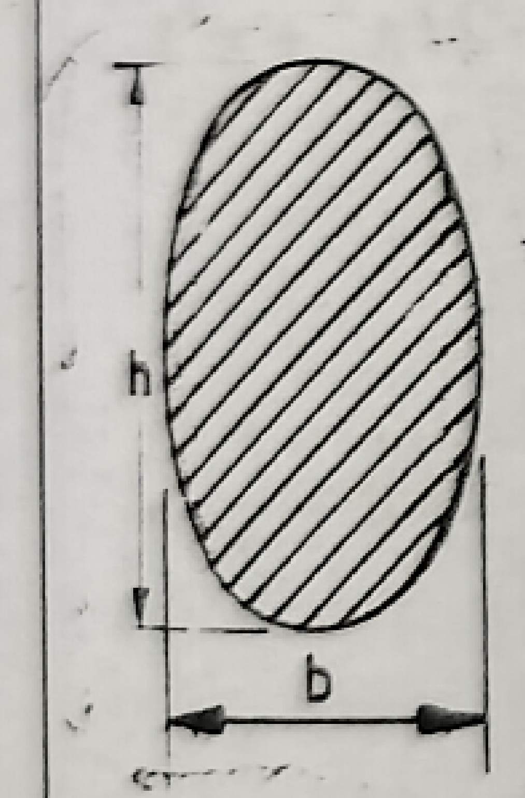
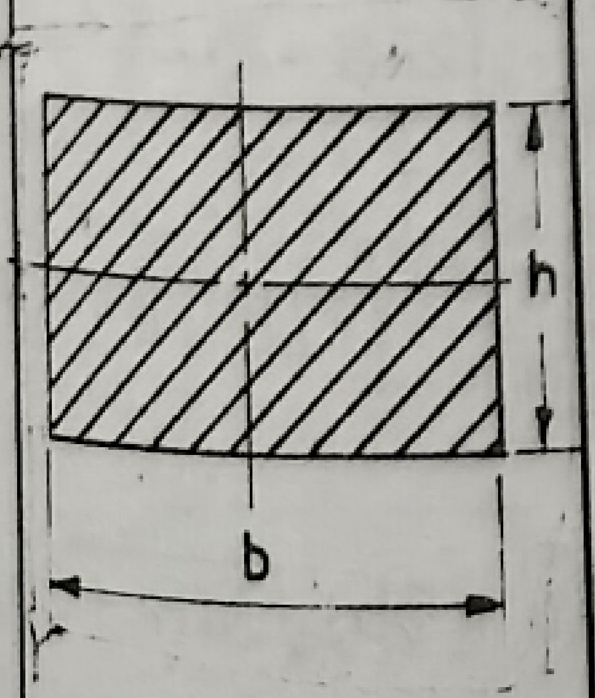
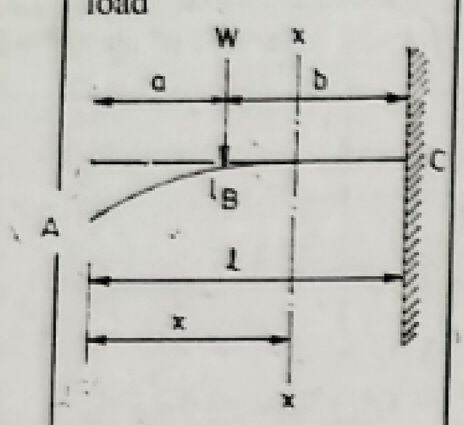
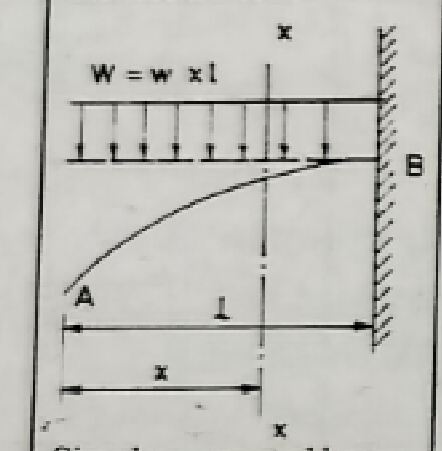
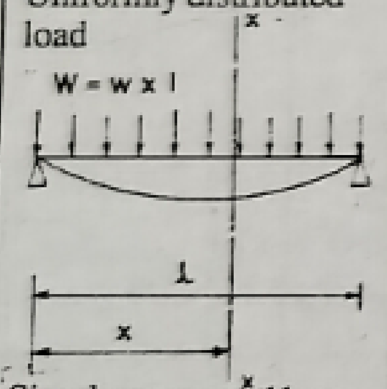
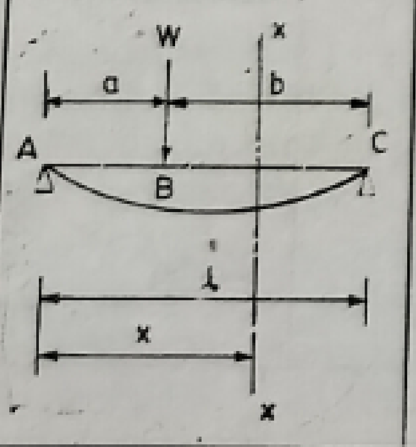
S.No.	Section	Moment of inertia	Section modulus	Radius of gyradion
1.		$I = \frac{\pi D^4}{64} = \frac{\pi R^4}{4}$ $I_P = \frac{\pi D^4}{32}$	$Z = \frac{\pi D^3}{32}$ $Z_P = \frac{\pi D^3}{16}$	$K = \frac{D}{4} = \frac{R}{2}$ $K_P = \frac{D}{\sqrt{8}}$
2.		$I = \frac{\pi}{64}\left(D_1^4 - D_2^4\right)$ $I_P = \frac{\pi}{32}\left(D_1^4 - D_2^4\right)$	$Z = \frac{\pi}{32}\frac{\left(D_1^4 - D_2^4\right)}{D_1}$ $Z_P = \frac{\pi}{16}\frac{\left(D_1^4 - D_2^4\right)}{D_1}$	$K = \frac{\sqrt{D_1^2 - D_2^2}}{4}$ $K_P = \sqrt{\frac{D_1^2 - D_2^2}{8}}$
3.		$I = \frac{\pi bh^3}{64}$ $I_P = \frac{\pi bh^3}{32}$	$Z = \frac{\pi bh^2}{32}$ $Z_P = \frac{\pi bh^2}{16}$	$K = \frac{h}{4}$ $K_P = \frac{1}{4}\sqrt{b^2 + h^2}$
4.		$I = \frac{bh^3}{12}$ $I_P = \frac{bh\left(b^2 + h^2\right)}{12}$	$Z = \frac{bh^2}{6}$ $Z_P = \frac{bh^2}{3 + 1.8\frac{h}{b}}$	$K = 0.289\,h$ $K_P = \sqrt{\frac{b^2 + h^2}{12}}$

Table I -2

Bending Moment and Deflection of Beams

S. No.	Figure	Bending Moment	Deflection
1.	Cantilever Intermediate load 	A to B : $M=0$ B to C : $M=W(x-a)$ $M_{max}=Wb$ at C	A to B : $y=\frac{W}{6EI}\left(b^3-3b^2l+3b^2x\right)$ B to C : $y=\frac{W}{6EI}\left[(x-a)^3-3b^2x-a)+2b^3\right]$ $y_{max}=\frac{Wb^2}{6EI}(3l-b)$ at A
2.	Cantilever, Uniformly distributed load 	$M=\frac{Wx^2}{2l}$ $M_{max}=\frac{Wl}{2}$ at B	$y=\frac{W}{24EIl}\left(x^4-4l^3x+3l^4\right)$ $y_{max}=\frac{Wl^3}{8EI}$
3.	Simply supported beam Uniformly distributed load 	$M=\frac{W}{2}\left(x-\frac{x^2}{l}\right)$ $M_{max}=\frac{Wl}{8}$ at $\frac{l}{2}$	$y=\frac{Wx}{24EIl}(l^3-2lx^2+x^3)$ $y_{max}=\frac{5}{384}\frac{Wl^3}{EI}$ at $x=\frac{l}{2}$
4.	Simply supported beam Intermediate load 	A to B : $M=\frac{Wbx}{l}$ B to C : $M=\frac{Wa}{l}(l-x)$ $M_{max}=\frac{Wab}{l}$ at B	A to B : $y=\frac{Wbx}{6EIl}\left[2l(l-x)-b^2-(l-x)^2\right]$ B to C : $y=\frac{Wa(l-x)}{6EI}\left[2lb-b^2-(l-x)^2\right]$ $y_{max}=\frac{Wab}{27EIl}(a+2b)\sqrt{3a(a+2b)}$ at $x=\sqrt{1/3\,a(a+2b)}$ When $a>b$

(Contd..next page)

Table I - 2 (Contd...)

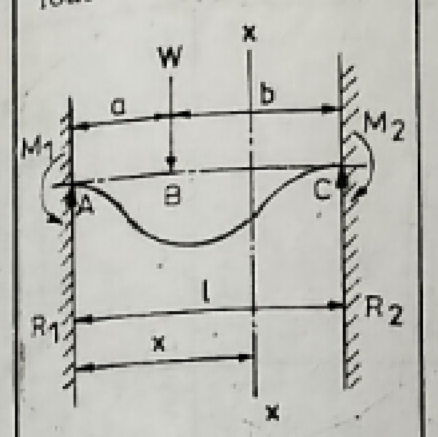
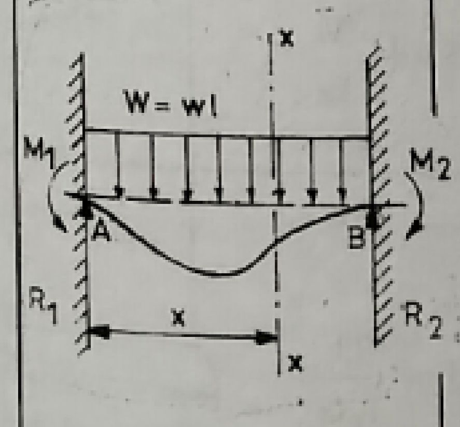
S. No.	Figure	Bending Moment	Deflection
5.	Fixed beam intermediate load  W x a b M1 M2 A B C l R1 R2 x x	A to B : $M=\frac{Wab^2}{l^2}-R_1x$ B to C : $M=\frac{Wab^2}{l^2}-R_1x+W(x-a)$ $M_{max}=\frac{Wab^2}{l^2}-R_1a$ at B $R_1=\frac{Wb^2}{l^3}(3a+b)$ $R_2=\frac{Wa^2}{l^3}(3b-a)$ $M_1=\frac{Wab^2}{l^2}$, $M_2=\frac{Wa^2b}{l^2}$	A to B : $y=\frac{Wb^2x^2}{6EIl^3}(3ax+bx-3al)$ B to C : $y=\frac{Wa^2(l-x)^2}{6EIl^3}\left[(3b+a)(l-x)-3bl\right]$ $y_{max}=\frac{2W}{3EI}\frac{a^3b^2}{(3a+b)^2}$ at $x=\frac{2a}{(3a+b)}$ if $a>b$
6.	Both ends fixed Uniformly distributed load 	$M=\frac{W}{2}\left[x-\frac{x^2}{l}-\frac{l}{6}\right]$ $M_{max}=\frac{Wl}{24}$ at $x=\frac{l}{2}$ $M_1=M_2=\frac{Wl}{12}$	$y=\frac{Wx^2}{24EI}(2lx-l^2-x^2)$ $y_{max}=\frac{Wl^3}{384EI}$ at $x=\frac{l}{2}$

Table I - 4

Simple Stresses Under Static Loading

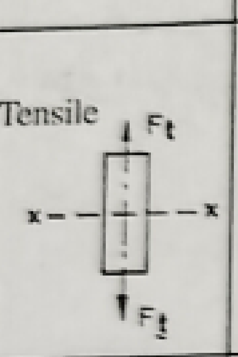
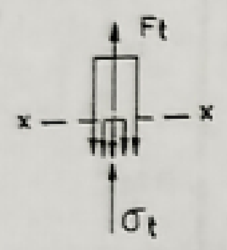
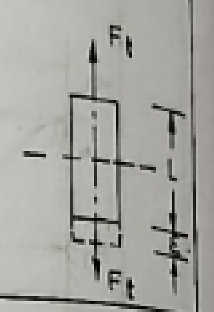
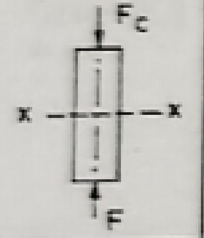
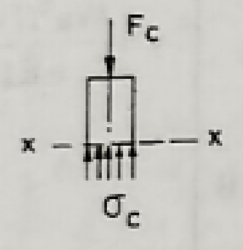
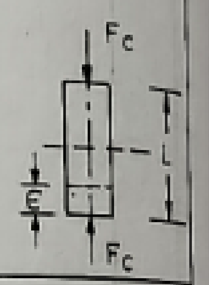
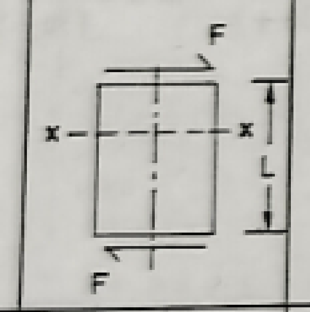
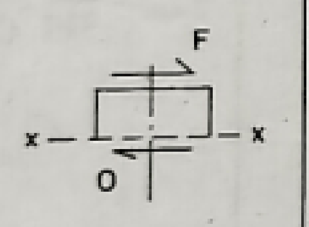
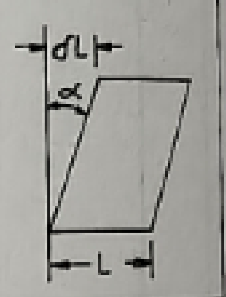
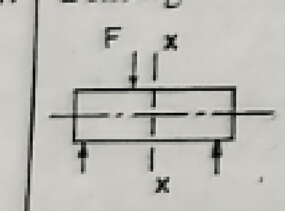
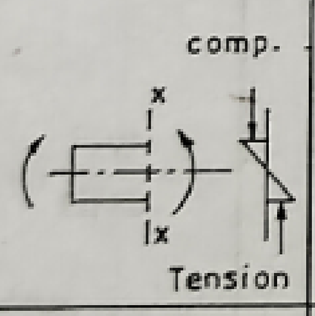
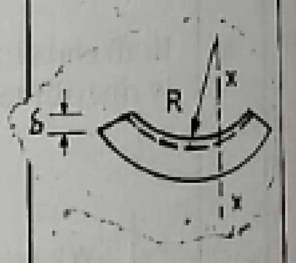
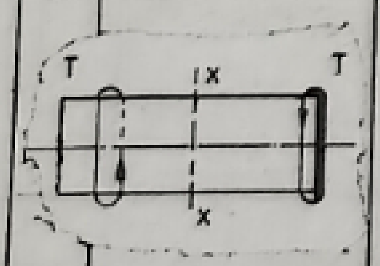
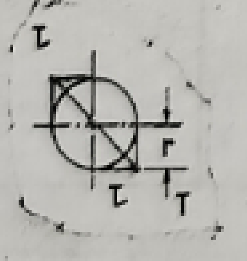
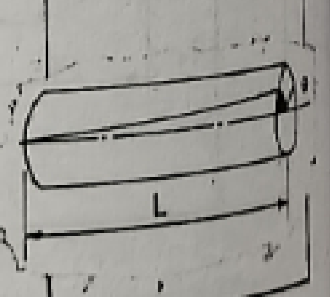
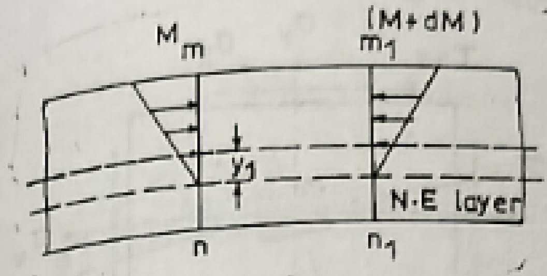
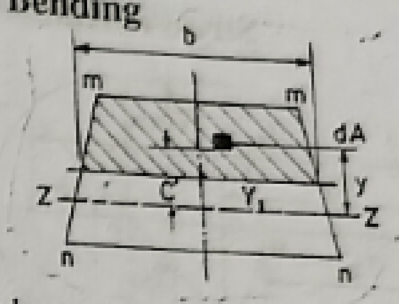
S. No.	Type of load	Stress distribution	Stress formulae	Deformation / Deflection	
1.	Tensile 		$\sigma_t = \frac{F_t}{A}$ A = C . S. area	$\varepsilon = \frac{F_t L}{AE}$	
2.	Compression 		$\sigma_C = \frac{F_C}{A}$ A = C . S. area	$\varepsilon = \frac{F_C L}{AE}$	
3.	Direct shear 		$\tau = \frac{F}{A}$ A = Surface area	$\alpha = \frac{\delta L}{L}$ $\delta L = \frac{FL}{AG}$	
4.	Bending 	 comp. x x Tension	$\sigma_b = \frac{M}{Z}$ $\frac{M}{I} = \frac{E}{R} = \frac{\sigma_b}{y}$	$\delta = \iint \frac{M}{EI}$	
5.	Torsion 		$\tau = \frac{T}{Z_p}$ $\frac{T}{I_p} = \frac{\tau}{r} = \frac{G\theta}{L}$ $Z_p = \frac{I_p}{r}$	$\theta = \frac{TL}{I_p G}$	

Table I - 5

Shear Stress in Bending


M
m
(M+dM)
m1
y1
N.E layer
n
n1


b
m
m
dA
y
Z
C'
Y1
Z
n
n

$$\tau = \frac{FA'c'}{I_z\, b}$$ where

A' = Cross hatched area

C' = Distance of C.G. of A'

I_z = M.I. about Z-Z (N.A) of whole section

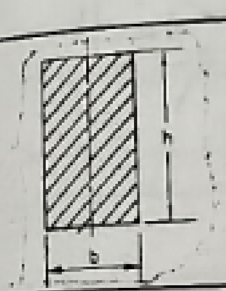
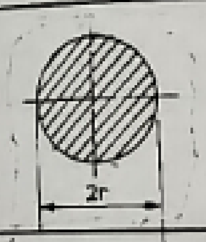
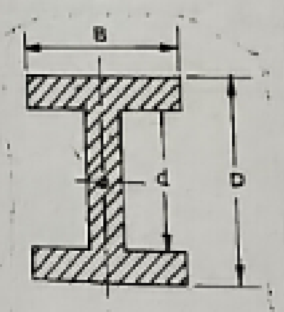
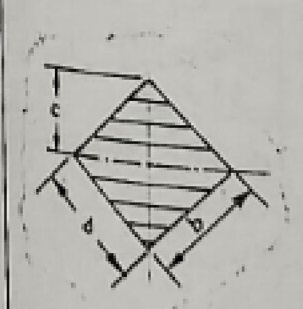
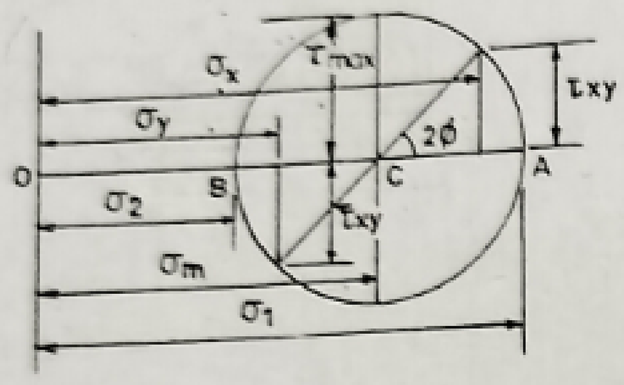
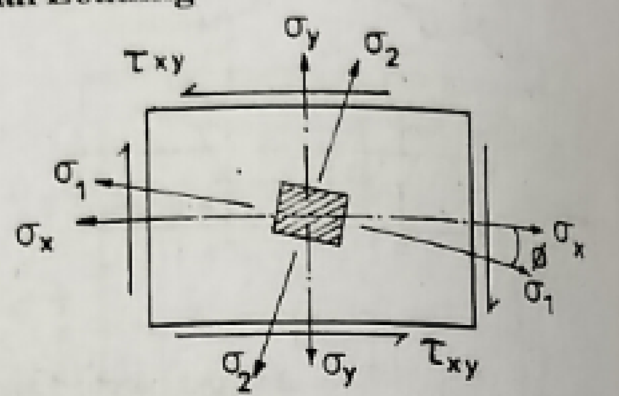
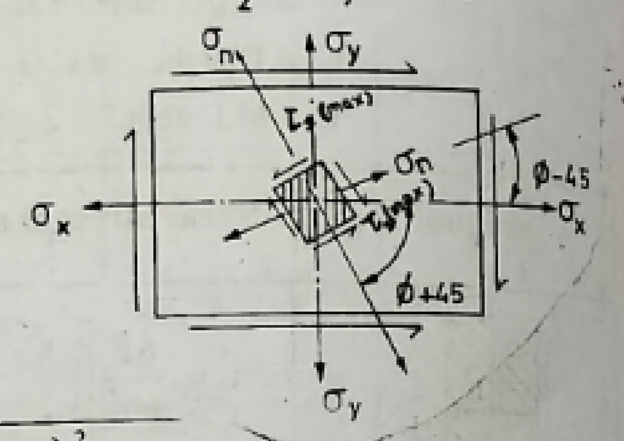
Section	Shear stress at a distance Y from N.A	Maximum shear stress
	$\tau = \frac{6F}{bh^3}\left[\left(\frac{h}{2}\right)^2 - y^2\right]$	$\tau_{(max)} = \frac{3}{2} \cdot \frac{F}{bh}$ For Y = 0
	$\tau = \frac{4}{3} \cdot \frac{F}{\pi r^2}\left(r^2 - y^2\right)$	$\tau_{(max)} = \frac{4}{3} \cdot \frac{F}{\pi r^2}$ For Y = 0
	Flange (y > d/2) $\tau = \frac{F}{2I_Z}\left(\frac{D^2}{4} - y^2\right)$ Web (y < d/2) $\tau = \frac{F}{2I_Z}\left\{\frac{B}{4b}(D^2 - d^2) + \left(\frac{d^2}{4} - y^2\right)\right\}$	$\tau_{(max)} =$ $\frac{F}{8I_Z}\left\{\frac{B}{b}(D^2 - d^2) + d^2\right\}$ at Y = 0
	$\tau = \frac{F\sqrt{2}}{b^2}\left[1 + \frac{Y\sqrt{2}}{b} - 4\left(\frac{y}{b}\right)^2\right]$	$\tau_{(max)} = 1.591\frac{F}{A}$ for Y = c/4

Table I - 6

Stress Formulae For Biaxial Loading


σ_x
τ_{max}
τ_{xy}
σ_y
2ϕ
O
B
C
A
σ_2
τ_{xy}
σ_m
σ_1


τ_{xy}
σ_y
σ_2
σ_1
σ_x
σ_x
ϕ
σ_1
σ_2
σ_y
τ_{xy}


σ_n
σ_y
$\tau_{(max)}$
σ_n
$\tau_{(max)}$
$\phi - 45$
σ_x
σ_x
$\phi + 45$
σ_y

1. **Principle stresses**

$$\sigma_1 = \overline{OA} = \overline{OC} + \overline{CD} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \overline{OB} = \overline{OC} - \overline{CD} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\phi = \frac{2\tau}{(\sigma_x - \sigma_y)}$$

2. **Maximum shear stresses**

Shear stress is maximum at an angle θ from x-axis.

$$\theta = \phi \pm 45^0$$

$$\tau_{(max)} = \frac{\sigma_1 - \sigma_2}{2}$$

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Table I - 7

Stresses Due to Impact Loading

In the equations for stress and elongation :

$$n = \frac{(1+am)}{(1+bm)^2}$$ when considering inertia effects

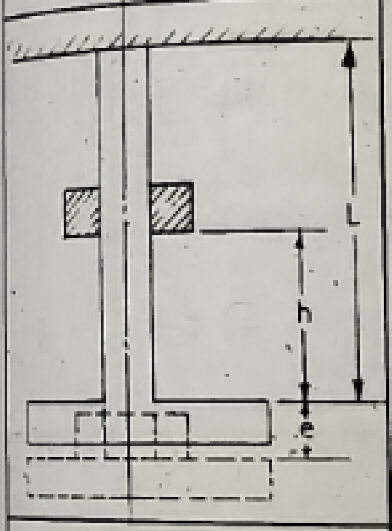
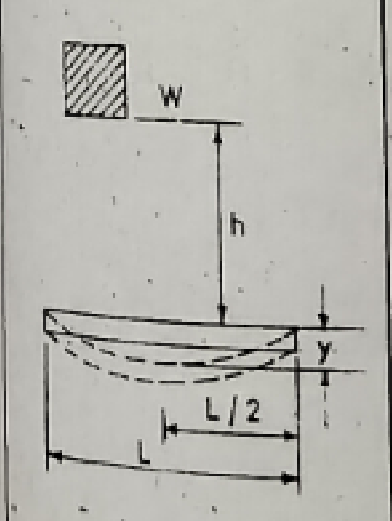
where $m = w' / w$

w' = self weight of the member

w = weight of the striking body

a, b = constants given in the table

n = 1, 0 when inertia effect is to be neglected.

Type of impact	Stress (maximum)	Elongation/deflection	a	b
	$\sigma = \frac{W}{A}\left[1 + \sqrt{1 + 2hn\frac{AE}{WL}}\right]$	$e = \frac{WL}{AE}\left[1 + \sqrt{1 + 2h\frac{AE}{WL}n}\right]$	1/3	1/2
	$\sigma = \frac{WLc}{4I}\left[1 + \sqrt{1 + \frac{96hnEI}{WL^3}}\right]$	$y = \frac{WL^3}{48EI}\left[1 + \sqrt{1 + \frac{96EIhn}{WL^3}}\right]$	13/35	1/2

(Contd..next page)

Table I - 7 (Contd..)

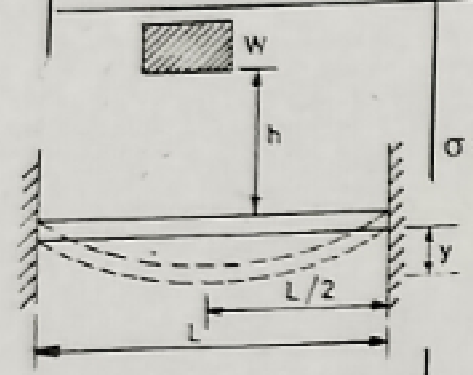
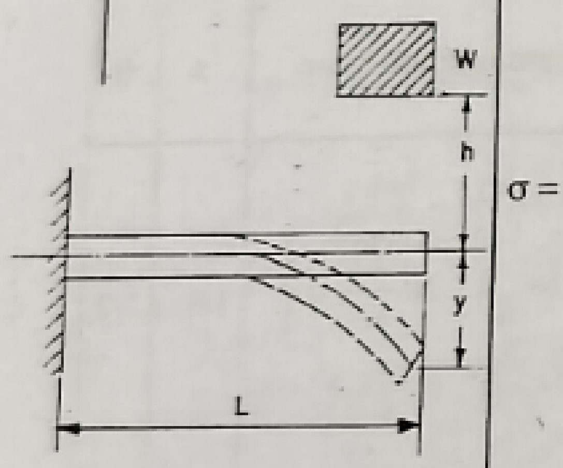
Type of impact	Stress (maximum)	Elongation/deflection	a	b
	$\sigma = \frac{WLc}{8I}\left[1+\sqrt{1+\frac{384\,hnEI}{WL^3}}\right]$	$y = \frac{WL^3}{192EI}\left[1+\sqrt{1+\frac{384EI\,hn}{WL^3}}\right]$	13/35	1/2
	$\sigma = \frac{WLc}{I}\left[1+\sqrt{1+\frac{6\,hnEI}{WL^3}}\right]$	$y = \frac{WL^3}{3EI}\left[1+\sqrt{1+\frac{6\,hn\,EI}{WL^3}}\right]$	4/17	3/8

Table I - 8

Moduli of Resilience

Energy stored = (Volume of member) × modulus of resilience, R

Maximum stress induced = S; (type of stress depends upon type of load)

S.No.	Type of loading	R
1.	Tension or Compression	$\frac{\sigma^2}{2E}$
2.	Shear, transverse	$\frac{\tau^2}{2E}$
3.	Beam, simply supported ends a) Concentrated central load	
	i) Rectangular cross section	$\frac{\sigma^2}{18\,E}$
	ii) Circular cross section	$\frac{\sigma^2}{24\,E}$
	b) Uniform load	
	i) Rectangular Cross Section	$\frac{3\sigma^2}{56\,E}$
4.	Cantilever beam a) Concentrated load	
	i) Rectangular cross section	$\frac{\sigma^2}{18\,E}$
	ii) Circular cross section	$\frac{\sigma^2}{24\,E}$
	b) Uniformly distributed load	
	i) Rectangular cross section	$\frac{\sigma^2}{30\,E}$
	ii) Circular cross section	$\frac{\sigma^2}{40\,E}$
5.	Beam with both ends fixed a) Concentrated load	
	i) Rectangular cross section	$\frac{\sigma^2}{18\,E}$

(Contd..next page)

Table I - 8 (Contd..)

S.No.	Type of loading	R
	ii) Circular cross section	$\frac{\sigma^2}{24\,E}$
	b) Uniformly distributed load	
	i) Rectangular corss section	$\frac{\sigma^2}{30\,E}$
	ii) Circular cross section	$\frac{\sigma^2}{40\,E}$
6.	Torsion	
	i) Solid circular bar	$\frac{\tau^2}{4\,G}$
	ii) Hollow circular bar	$\left[1+\left(\frac{D_o}{D_i}\right)^2\right]\frac{\tau^2}{4G}$

where D_o = Outside diameter

D_i = Inside diameter

Table I - 9

Formulae For Thermal Stresses in Plates & Cylinders

Notation :

θ = Temperature ; °C

μ = Poisson's ratio

α = Coefficient of linear expansion, mm/mm, °C

K = Thermal conductivity, $\frac{J}{m^2,\ hr.,\ ^0C}$

q = Heat flow, $\frac{J}{m^2 - hr}$

t = Thickness, mm

δ = Deflection, mm.

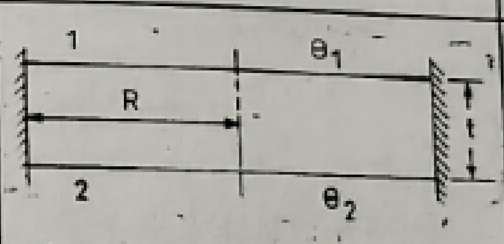
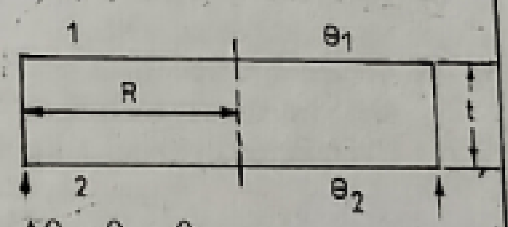
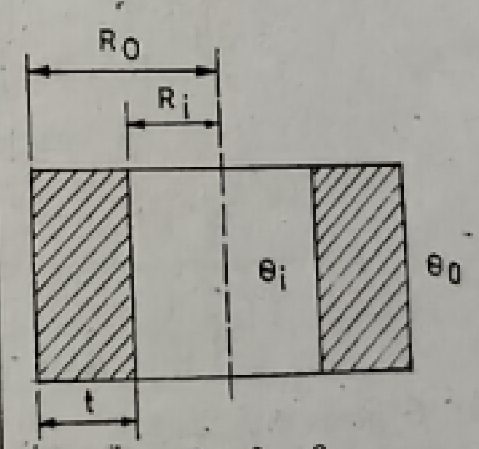
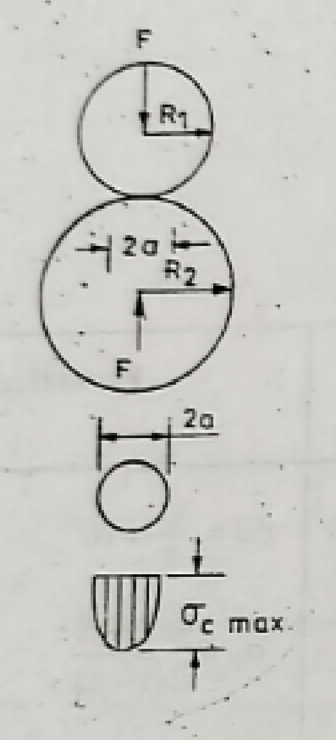
S. No.	Shape & Edge Conditions	Figure	Formulae
1.	Circular plate clamped at edge with heat flow q across the plate.	 $\theta_1 > \theta_2 \quad \Delta\theta = \theta_1 - \theta_2$	$\Delta\theta = \frac{qt}{K}\times 10^{-3}$ $\sigma_1 = \frac{\alpha(\Delta\theta)E}{2(1-\mu)}$ $\sigma_2 = -\sigma_1$ $\delta = 0$
2.	Circular plate with edge simply supported & heat flow, q across the plate	 $\Delta\theta = \theta_1 - \theta_2$	$\sigma_1 = \sigma_2 = 0$ $\delta = \frac{4\alpha(\Delta\theta)R^2}{8t}$
3.	Cylinder with heat flow q	 $\theta_i > \theta_0 ; \Delta\theta = \theta_i - \theta_0$ $1+M = \frac{R_0}{R_i}$	$\sigma_0 = \frac{\alpha E(\Delta\theta)\left(1-\frac{M}{3}\right)}{2(1-\mu)}$ $\sigma_1 = -\frac{\alpha E(\Delta\theta)\left(1+\frac{M}{3}\right)}{2(1-\mu)}$ $\Delta\theta = \frac{qt}{K}\times 10^{-3}$

Table I - 10
Contact Stresses

Notations : a = Radius or width of contact surface
δ = Displacement under load
μ = Poissions ratio - taken as 0.3

1.



Case :

i) Sphere on a sphere

$$\sigma_{C\max} = 0.616\left[\frac{F\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^2}{\left(\frac{1}{E_1}+\frac{1}{E_2}\right)^2}\right]^{1/3}$$

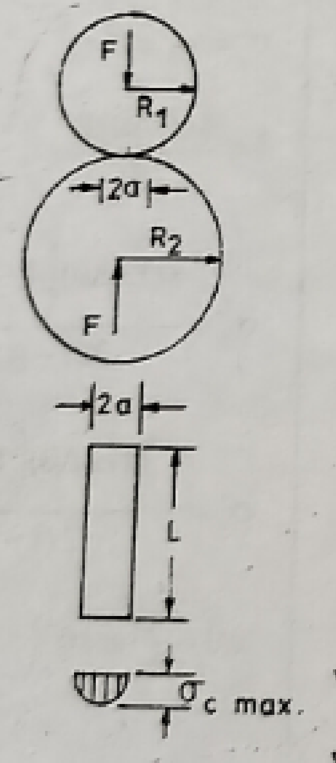
$$a = 0.880\left[\frac{F\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}\right]^{1/3}$$

$$\delta = 0.775\left[F^2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\left(\frac{1}{E_1}+\frac{1}{E_2}\right)^2\right]^{1/3}$$

ii) Sphere on a plane
same as case (i) with $R_2 = \infty$

iii) Sphere in a Spherical cavity
same as case (i) with $R_2 = -ve$

2.



iv) Cylinder on cylinder length L

$$\sigma_{C\max} = \frac{0.59\sqrt{F\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}}{\sqrt{L\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}}$$

$$a = \frac{1.076\sqrt{F\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}}{\sqrt{L\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}}$$

v) Cylinder on a plane
same as case (iv) with $R_2 = \infty$

vi) Cylinder in a cylinderical cavity
same as case (iv) with $R_2 = -ve$

Table I - 12
Endurance limit of parts

Endurance limit of a part (S_e) :

$$S_e = \frac{A \cdot K_R \cdot K_\theta \cdot K_Z \cdot K_S}{K_f} \times S_{eb}$$

where S_{eb} is the endurance limit of a specimen of 12.5 mm. size in reversed bending. (The values are given in chapter II).

1. Factor for type of loading (A)
   - a) For Bending load, $A = 1.0$
   - b) For Axial load, $A = 0.7$
   - c) For Torsion load, $A = 1/\sqrt{3}$

2. Reliability Factor (K_R)

Percent Reliability	Standardised Z_r	Reliability factor K_R
50	0	1.000
90	1.28	0.897
95	1.65	0.868
99	2.33	0.814
99.9	3.09	0.753
99.99	3.72	0.702

3. Temperature factor K_θ
   If operating temperature T is less than 70°C, $K_\theta = 1$

   If operating temperature T is higher than 70°C, $K_\theta = \frac{343}{273+T}$
4. Size factor (K_Z) - Refer table I-13
5. Surface finish factor (K_S) – Refer Fig. I-7
6. Fatigue stress concentration factor (K_f)
   $K_f = 1 + q\,(K_t - 1)$
   K_t = Theoretical stress concentration factor
   Refer Fig. I-1 to I-6
   q = Notch sensitivity factor. Refer Table I-15 & I-16
   For K_f of steel bolts and keyways, refer table I-17 and I-18

Table I - 13

Size Factor, K_z

Size	K_z
$d \le 12.5$	$K_z = 1$
$75 > d > 12.5$	$K_z = 1 - \left[\frac{d - 12.5}{75 - 12.5}(1 - K'_z)\right]$
$d \ge 75$	$K_z = K'_z$

K'_z as obtained from table I-14

Table I - 14

Size Factor For 75mm Specimen, K'_z

S. No.	Material	Untreated	Annealed and drawn at		
			750°C	575°C	500°C
1.	Aluminium	0.93	-	-	-
2.	Bronze	0.90	-	-	-
3.	Monel metal	0.80	-	-	-
4.	Ductile iron	0.80	0.98	-	-
5.	Low carbon steel	0.84	-	-	-
6.	Medium carbon steel	-	0.85	0.59	0.53
7.	Nickel steel	-	0.86	0.71	-
8.	Ni - Cr steel	-	0.80	0.70	0.65
9.	Cast iron I.S.15	0.55	-	-	-
10.	Cast iron I.S.20	0.73	-	-	-
11.	Cast iron I.S.25	0.60	-	-	-
12.	Wrought iron	0.55	-	-	-

Table I - 16

Notch Sensitivity Factor q For Repeated Loading

S.No.	Material	Annealed	Heat treated & drawn at 650°C	Heat treated & drawn at 480°C
1.	Carbon steel			
	0.10 % C	0.05 - 0.10	-	-
	0.20 % C (also cast steel)	0.10	-	-
	0.30 % C	0.18	0.35	0.45
	0.50 % C	0.26	0.40	0.50
	0.85 % C	-	0.45	0.57
2.	Spring steel 0.56 % C 0.23 % Si	-	0.38	-
3.	Ni Cr steel 0.37 % C 0.60 % Cr 1.3 % Ni	0.25	0.45	-
4.	Ni Cr steel 0.80 % Cr 3.5 % Ni	-	0.25	0.70
5.	Stainless steel 0.30 % C 8.3 % Cr 19.7 % Ni	0.16	-	-
6.	Cast Iron	0.05	-	-
7.	Copper	0.07	-	-
8.	Duraluminium	0.05 - 0.13	-	-

Table I - 17

Fatigue stress concentration factors K_f for steel bolts

Steel	Rolled threads	Cut threads
Annealed (less than 200 Bhn)	2.2	2.8
Quenched and Drawn (over 200 Bhn)	3.0	3.8

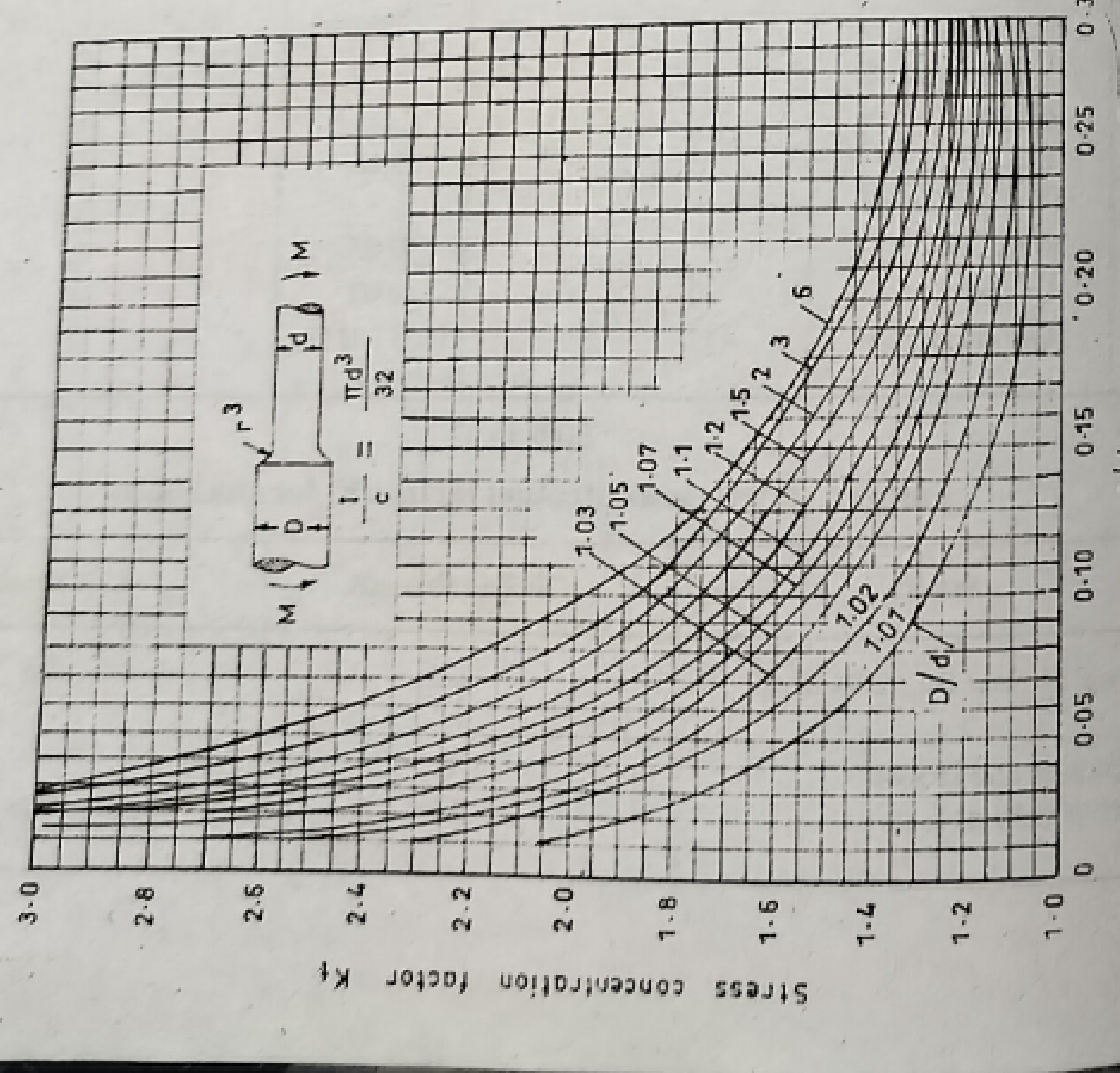


Fig. (1.1) Stress concentration factor, K_t for stepped bar in bending

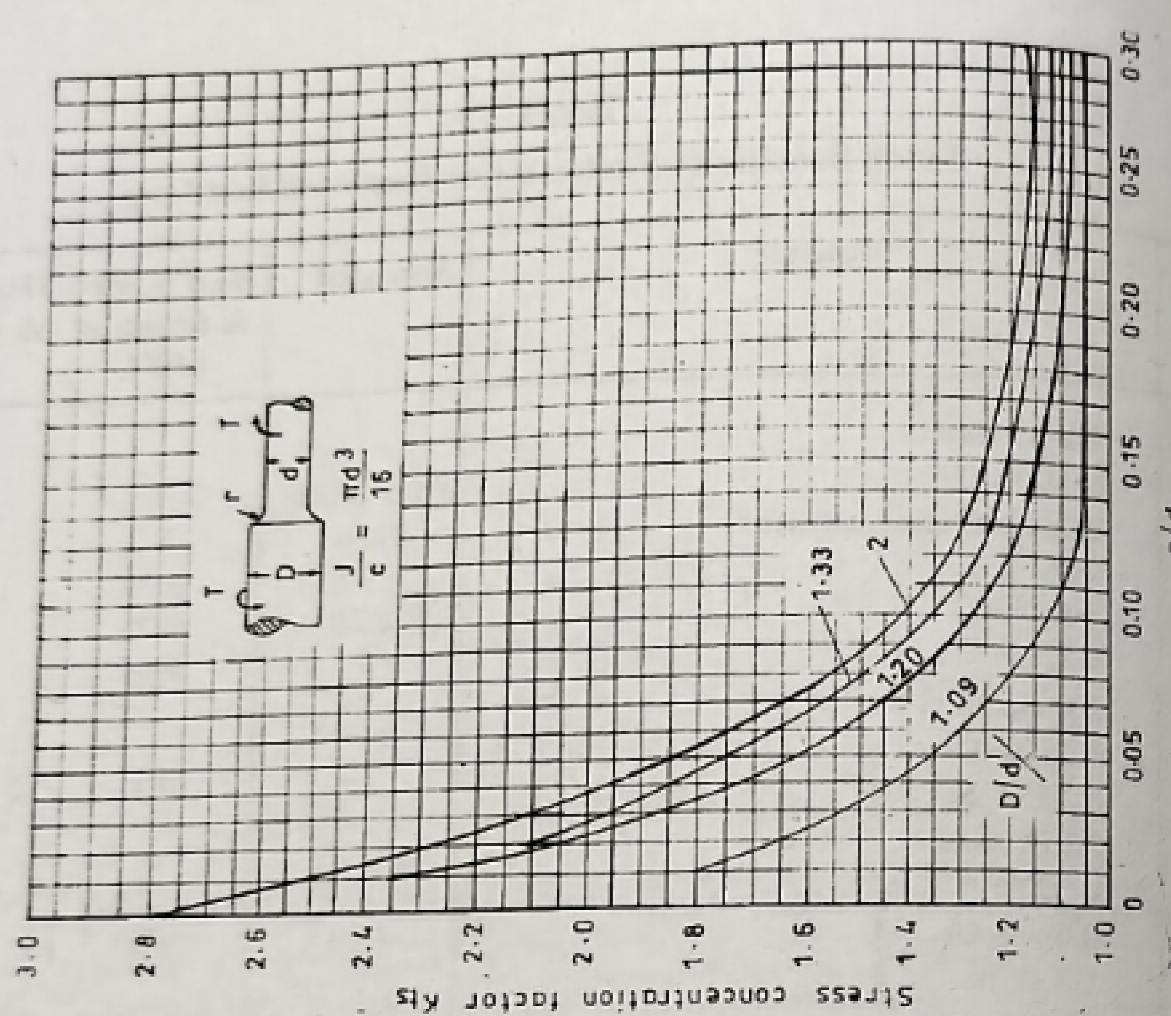


Fig. (1.2) Stress concentration factor, K_{ts} for stepped bar n torsion

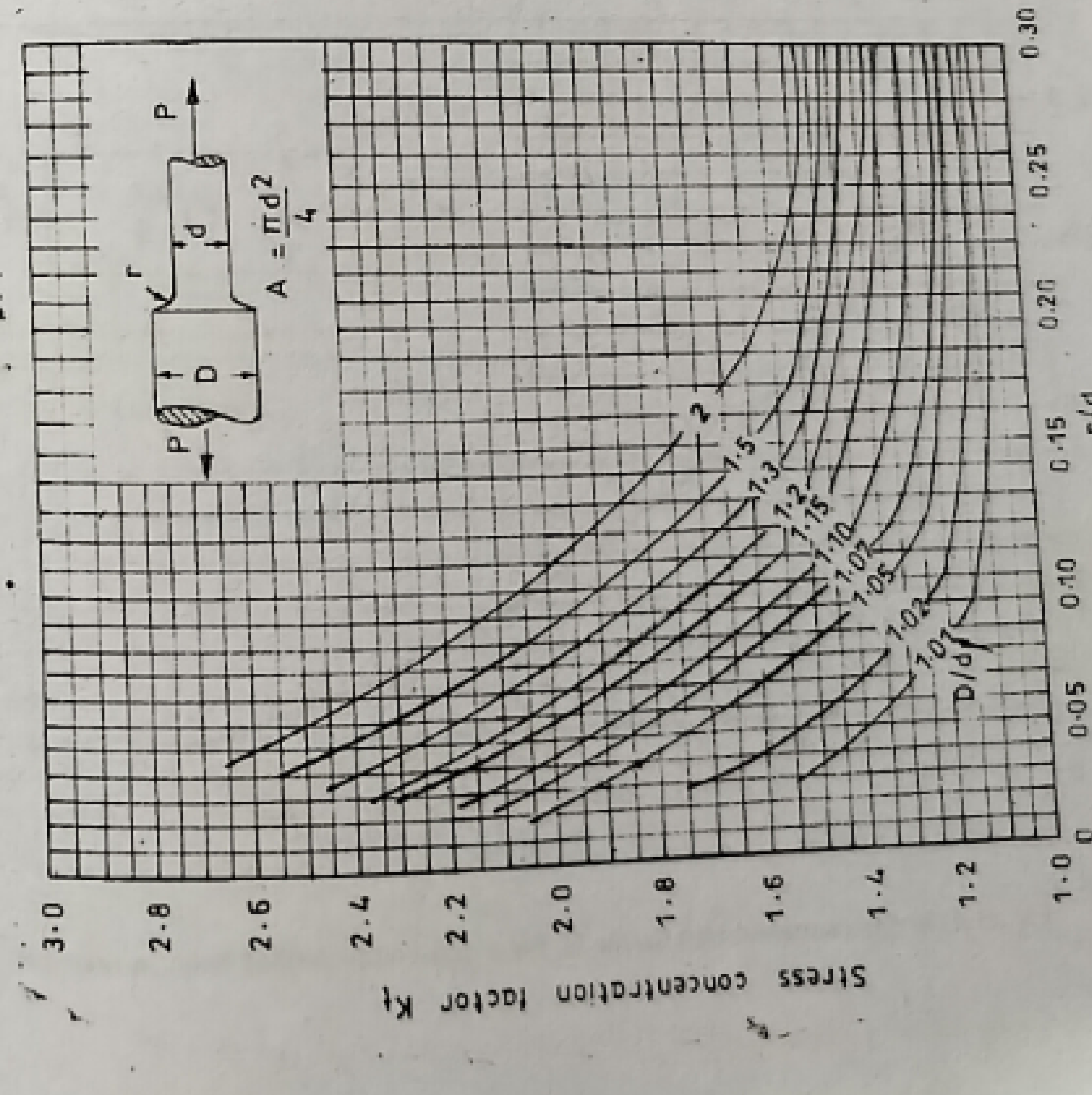


Fig. (1.3) Stress concentration factor K_t for stepped bar in tension

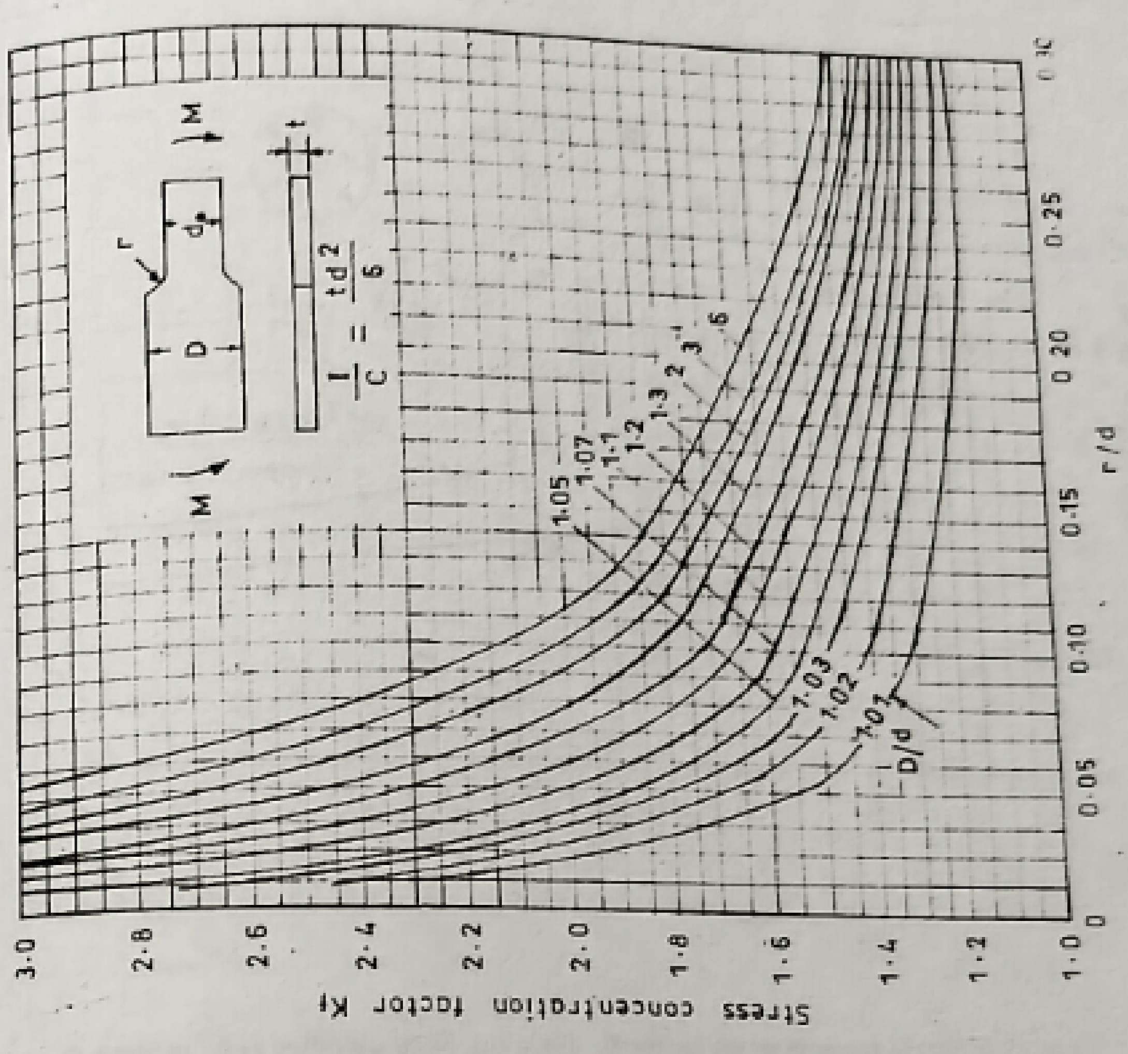


Fig. (1.4) Stress concentration factor K_t for stepped flat plate in bending

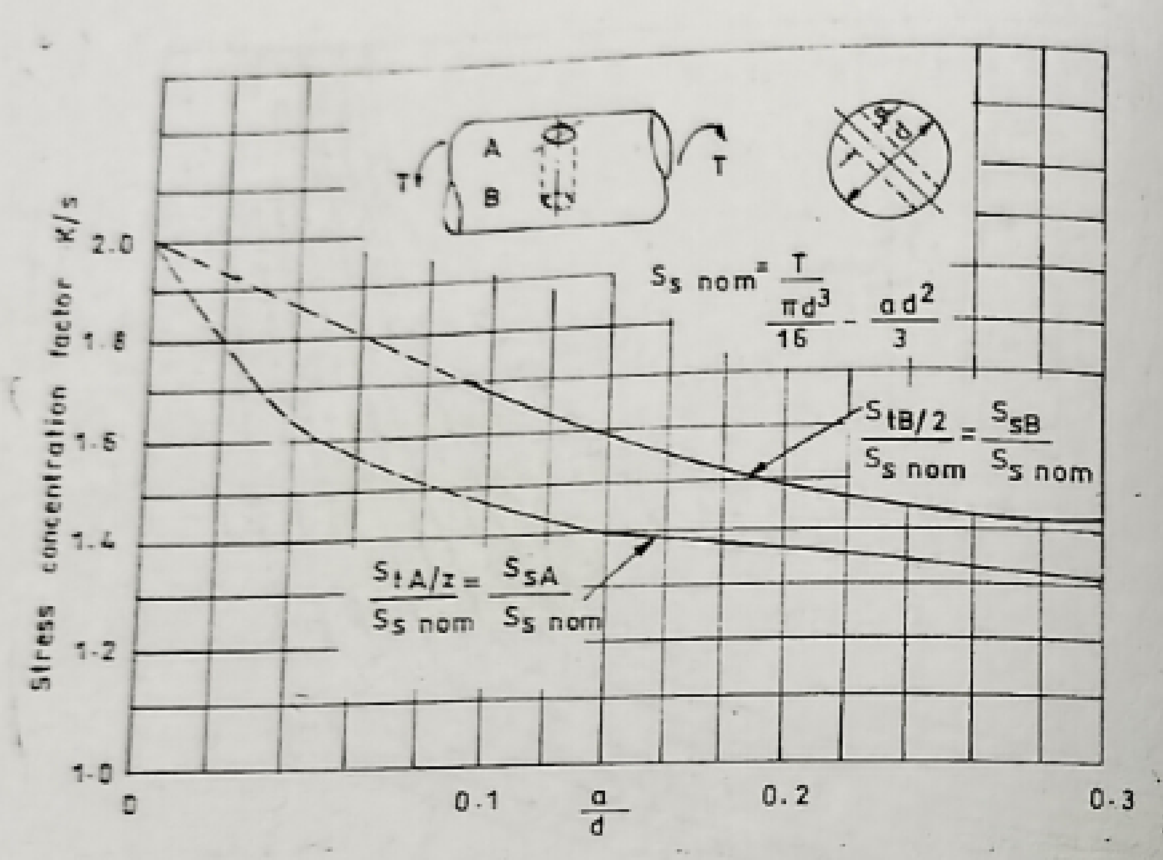


Fig. (1.5) Stress concentration factor K_u for a bar with a drilled hole in torsion

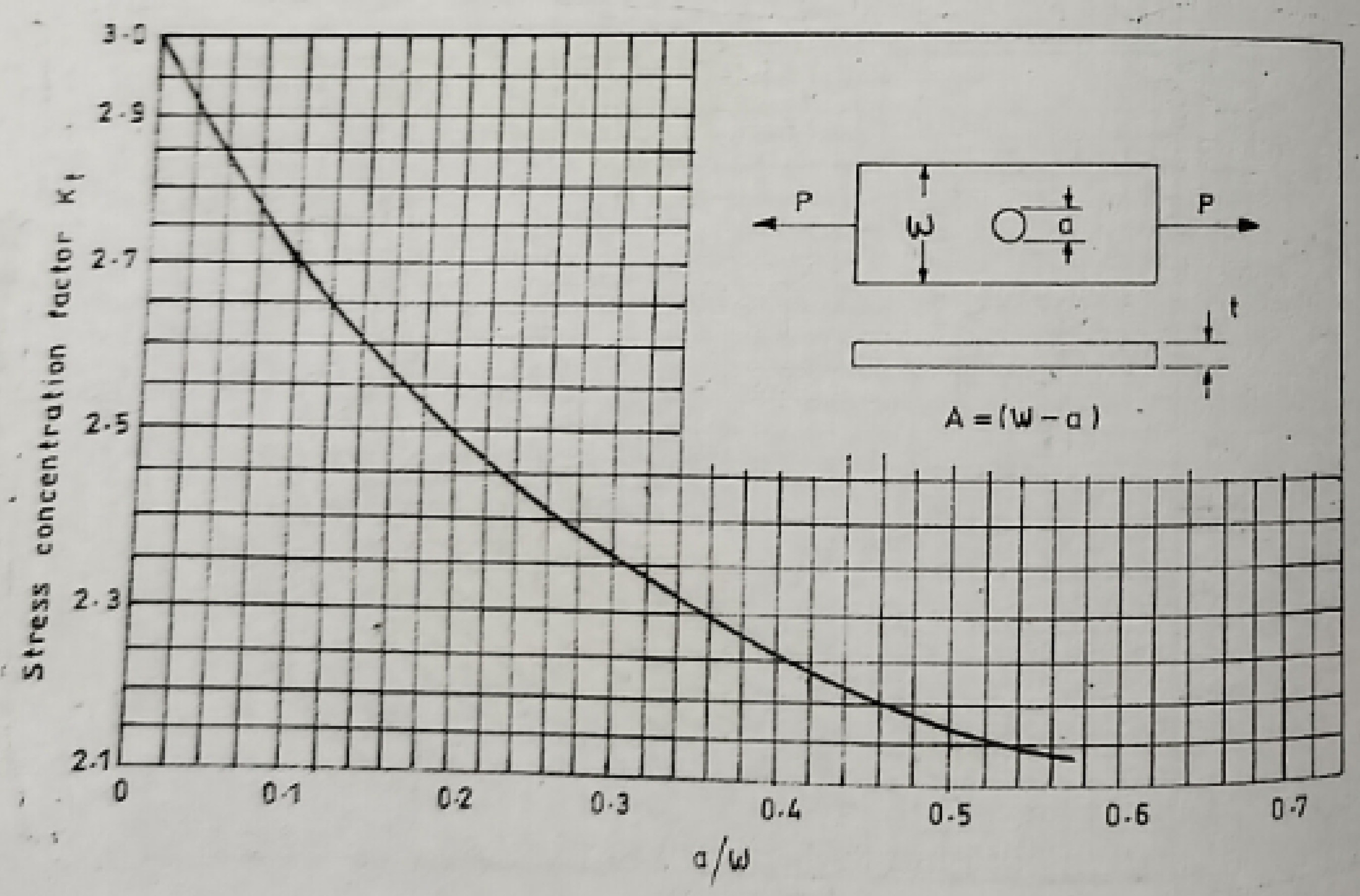


Fig. (1.6) Stress concentration factor K_t for a plate with drilled hole, in tension

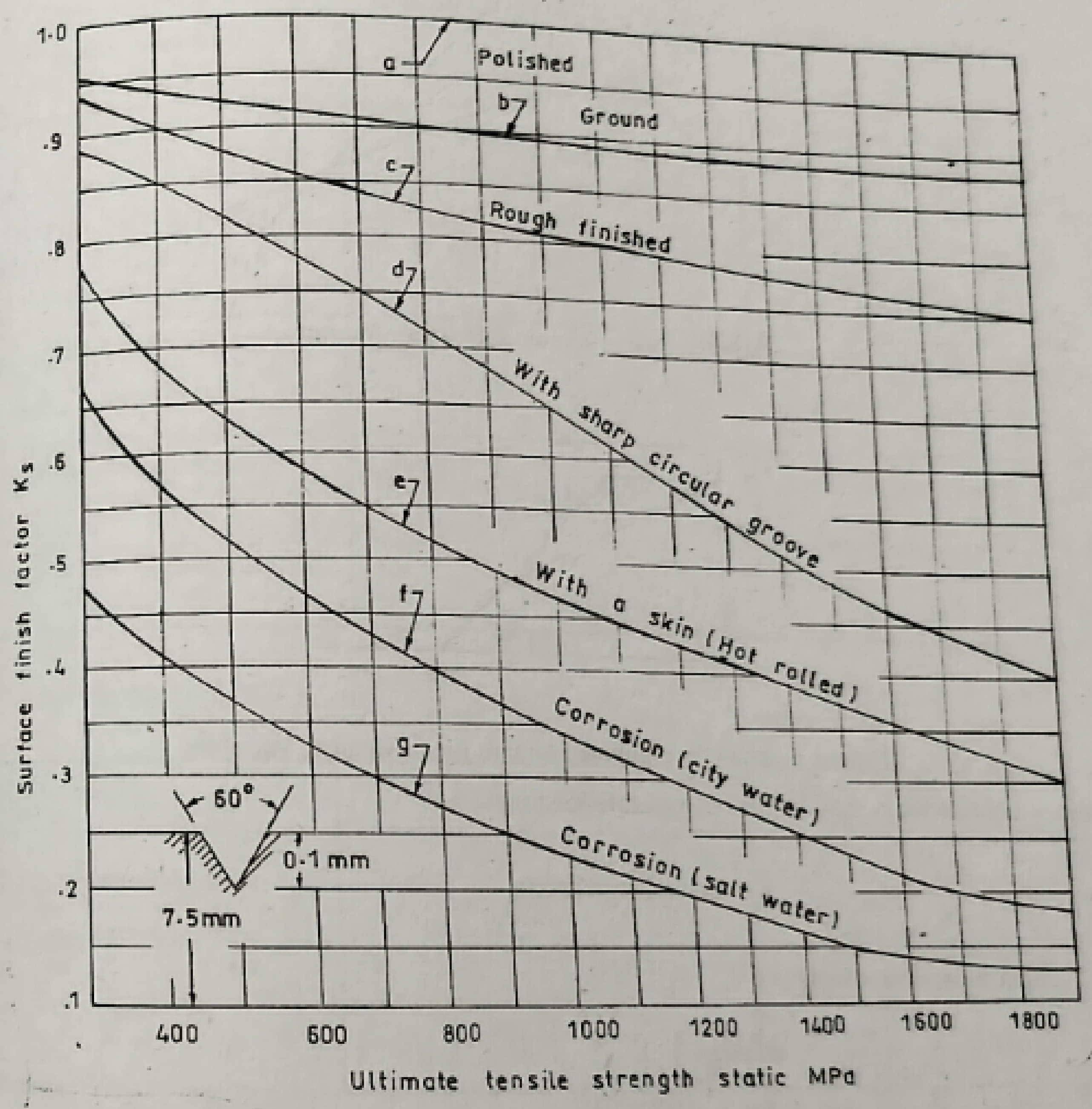
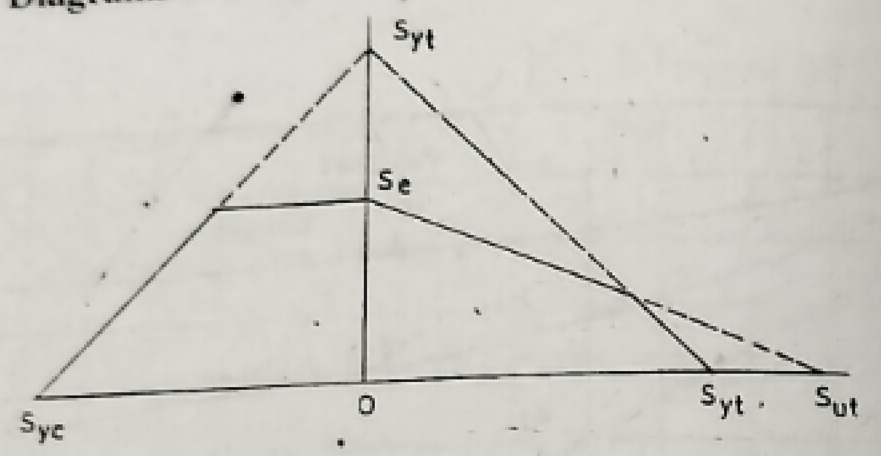


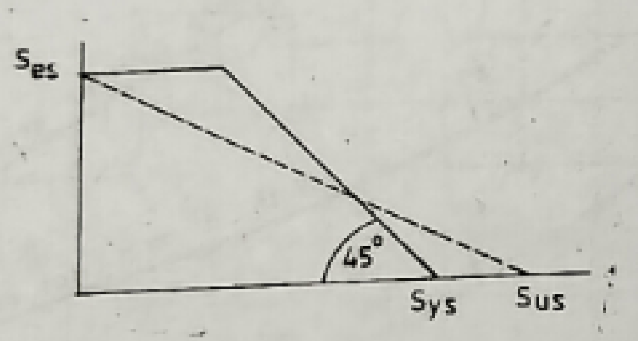
Fig. (1.7) Surface finish factor, K_s

Table I - 20

Fatigue Diagrams & Design Equations For Variable Loading



Approximate maximum safe stress diagram



Fatigue diagram for combined torsion load (Mean & Variable)

Design equations for combined biaxial variable loading :

$$\sigma_{eq} = \frac{S_Y}{N} = \sigma_m + \frac{\sigma_V}{S_e} S_Y - \text{Soderberg criteria}$$

1. Max Normal stress theory

$$\frac{1}{N} = \left[\sigma_{1m} + \frac{\sigma_{1V}}{S_e} S_Y \right] \frac{1}{S_Y}$$

2. Max Shear stress theory

$$\frac{1}{N} = \left[\sigma_{1m} + \frac{\sigma_{1V}}{S_e} S_Y \right] \frac{1}{(\sigma_{1m} - \sigma_{2m})}$$

3. Max Distortion energy theory

$$\frac{1}{N} = \left[\sigma_{1m} + \frac{\sigma_{1V}}{S_e} S_Y \right] \frac{1}{\sqrt{\sigma_{1m^2} - \sigma_{1m}\sigma_{2m} + \sigma_{2m^2}}}$$

Table I - 21

Theories of Failure

Failure of materials :

- The material is ductile if % elongation in simple tension test is greater than 5%.

A. **Ductile Materials**

Theory	Failure Criteria	Remarks
Maximum Normal stress theory (Rankine's theory)	a. $\sigma_1 > \sigma_2 \cdot \sigma_1 = S_Y$ b. $\sigma_2 > \sigma_1 \cdot \sigma_2 = S_Y$	Only for pure tension and compression. Not in pure torsion.
Maximum Shear stress theory (Guest's theory)	$S_Y = \sigma_1 - \sigma_2$	Gives always safe results
Distortion energy theory (Hencky Von-Mises theory)	$S_Y = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$	Accurate when part is to be designed without actual testing.

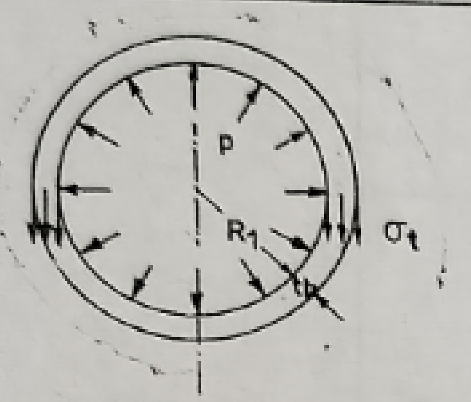
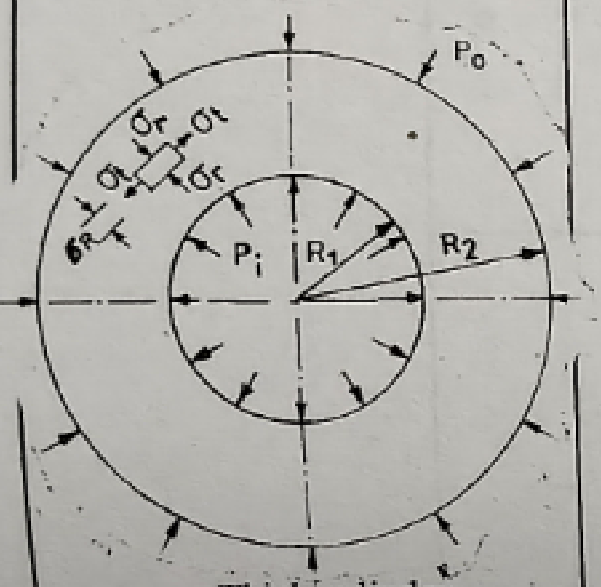
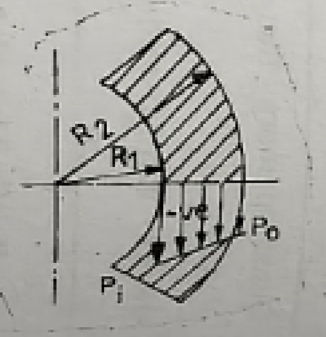
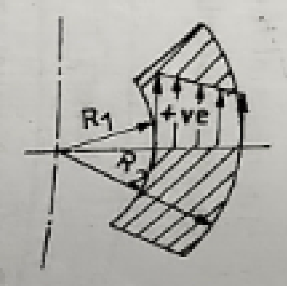
B. **Brittle Materials**

Theory	Failure Criteria	Remarks
Maximum Normal stress theory	$\sigma_1 > \sigma_2$ $\sigma_1 = \sigma_{ut}$ $\sigma_1 = \sigma_{uc}$	Only for pure tension and compression

Table I - 22

Formulae for stresses in Cylinders

If $t_b / R_1 > 0.2$, cylinder is thick If $t_b / R_1 < 0.2$, cylinder is thin

S. No.	Type of cylinder and loading	Stress at radius R Radial	Stress distribution graph Tangential
1.	 Thin Cylinder		$\sigma_t = \frac{pR_1}{t_b}$
2.	 p_o σ_r σ_t σ_t σ_r δR P_i R_1 R_2 Thick cylinder	$\sigma_R = \frac{R_1^2 p_i - R_2^2 p_o}{R_2^2 - R_1^2} - \frac{R_1^2 R_2^2 (p_i - p_o)}{R^2 (R_2^2 - R_1^2)}$ $\sigma_R = p_i$ at $R = R_1$ $\sigma_R = p_o$ at $R = R_2$ 	$\sigma_t = \frac{R_1^2 p_i - R_2^2 p_o}{R_2^2 - R_1^2} + \frac{R_1^2 R_2^2 (p_i - p_o)}{R^2 (R_2^2 - R_1^2)}$ $\sigma_{t(max)} = \frac{p_i (R_1^2 + R_2^2) - 2 p_o R_2^2}{R_2^2 - R_1^2}$ at $R = R_1$ 

(Contd., next page)

Table I - 22 (Contd..)

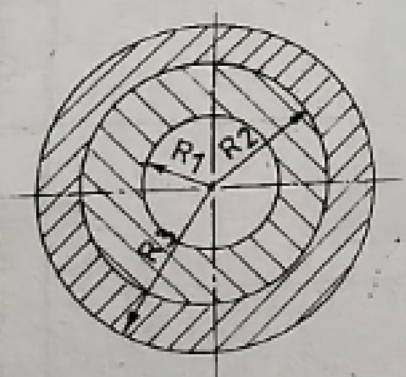
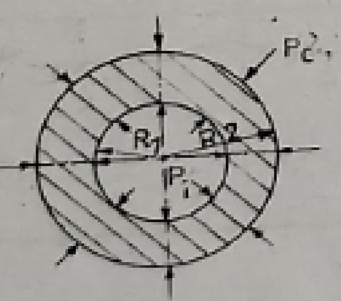
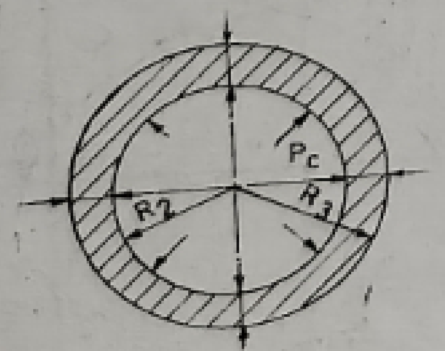
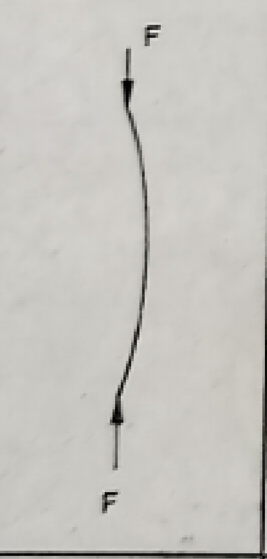
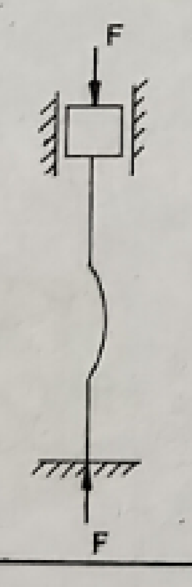
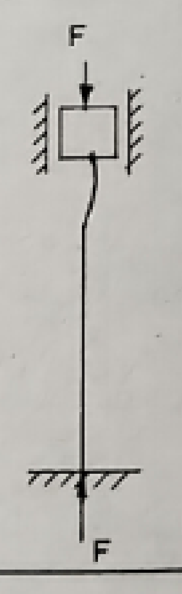
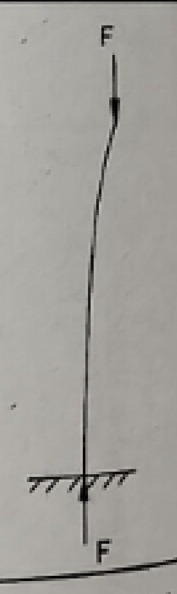
S. No.	Type of cylinder and loading	Stress at radius R Radial	Stress distribution graph Tangential
3.	Compound cylinders   	Total deformation δ $\delta = \delta_o - \delta_i = \frac{R_2 P_C}{E_o}\left[\frac{R_3^2 + R_2^2}{R_3^2 - R_2^2} + \mu_o\right] + \frac{R_2 P_C}{E_i}\left[\frac{R_2^2 + R_1^2}{R_2^2 - R_1^2} - \mu_i\right]$ If both cylinders of same material $[E_o = E_i \ \& \ \mu_o = \mu_i]$ $P_C = \frac{E\delta}{R_2}\left[\frac{(R_3^2 - R_2^2)(R_2^2 - R_1^2)}{2R_2^2 (R_3^2 - R_1^2)}\right]$ Inner cylinder : Analyse as case II by substituting $P_o = P_C$, $P_i = 0$ Outer cylinder : Analyse as case II by substituting $P_i = P_C$, $P_o = 0$	

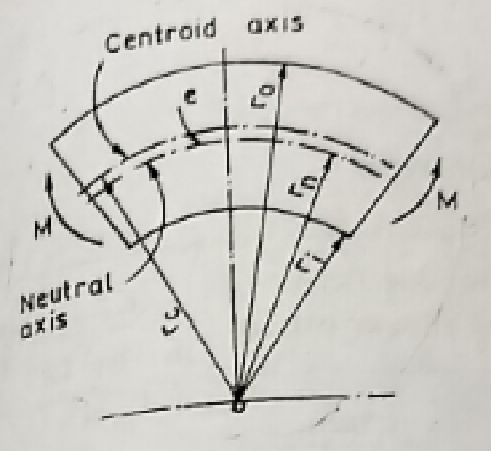
Table I - 24

Formulae for Columns

S. No.	Name of formula	Application	Formula
1.	Euler's Formula	$\frac{L}{K} > 90$	$F_{Cr} = \frac{n\pi^2 EI}{L^2} = \frac{n\pi^2 EA}{(L/K)^2}$
2.	Johnson's Parabolic Formula	$10 < \frac{L}{K} < 90$	$F_{Cr} = AS_Y\left[1 - \frac{S_Y}{4n\pi^2 E}\left(\frac{L}{K}\right)^2\right]$
3.	Rankine's Formula	$10 < \frac{L}{K} < 90$	$S_Y = \frac{F_{Cr}}{A}\left[1 + \frac{1}{an}\left(\frac{L}{K}\right)^2\right]$ where, $a = \frac{\pi^2 E}{S_Y}$
4.	Secant Formula (For beam columns or columns with eccentric load)	$10 < \frac{L}{K} < 90$	$S = \frac{F_{Cr}}{A}\left[1 + \frac{ec}{K^2}\,\text{Sec}\left(\frac{L}{2K}\sqrt{\frac{F}{EA}}\right)\right]$ e = eccentricity of load.

Column Constant 'n'

Figure				
End conditions	Both ends hinged	Both ends fixed	One end fixed other end guided	One end fixed other end free
n	1	4	2	0.25



Centre of Curvature

Table I - 25

Formulae For Bending Of Curved Beams

Where,

r = Radius at the section, M = Moment applied.

A = Area of cross section, e = As show in fig.

r_c = Radius to centroid surface.

r_n = Radius to neutral surface.

$$\sigma_Y = \frac{M}{Ae}\left(\frac{y}{r_n + y}\right)$$

σ_o = Stress at the outer fibre $= \frac{MC_o}{Aer_o}$

σ_i = Stress at the inner fibre $= \frac{MC_i}{Aer_i}$

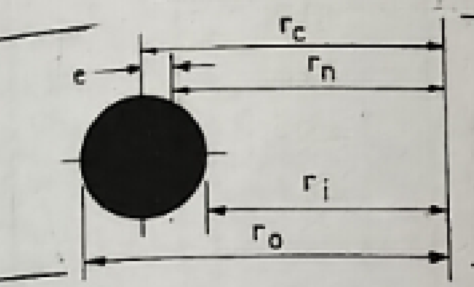
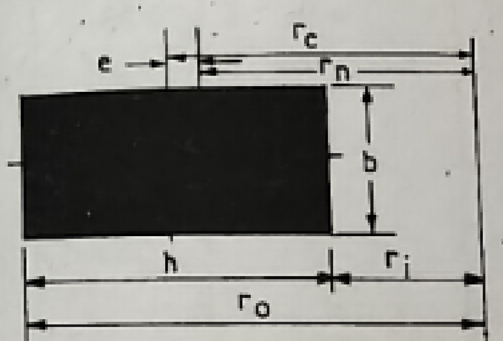
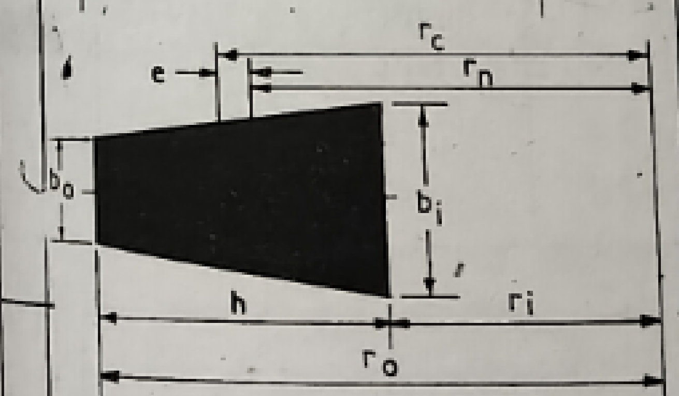
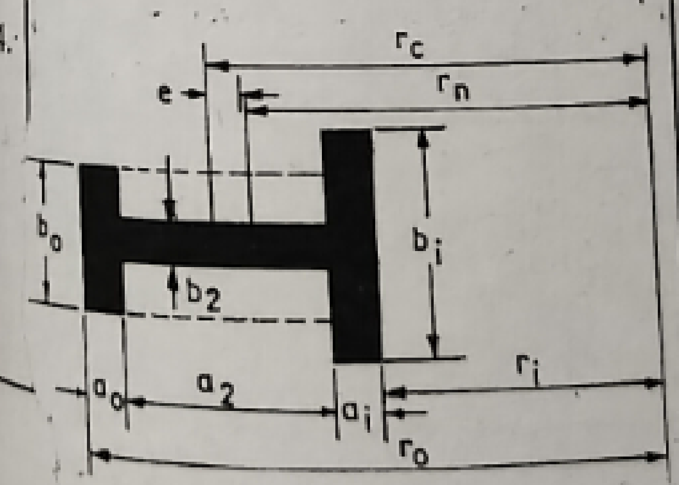
No.	Section	Radius of neutral surface r_n
1.		$r_n = \frac{\left(\sqrt{r_o} - \sqrt{r_i}\right)}{4}$
2.		$r_n = \frac{h}{\log_e (r_o/r_i)}$
3.	 r_c e r_n b_o b_i h r_i r_o	$r_n = \frac{1/2\,h(b_i + b_o)}{\frac{b_i r_o - b_o r_i}{h}\log_e (r_o - r_i)(b_i - b_o)}$ If $b_o = 0$, this section reduces to a triangle.
4.	 r_c e r_n b_o b_2 b_i a_o a_2 a_i r_i r_o	$r_n = \frac{A}{b_i \log\frac{r_i + a_i}{r_i} + b_2 \log\frac{r_o - a_o}{r_i + a_i} + b_o \log\frac{r_o}{r_o - a_o}}$ If $a_o = 0$, the section reduces to a T section; r_n is the same for a box section shown in dotted lines with each side panel 1/2 b_2 thick.

Table II-2
Physical Constants of Materials

S. No.	Material	E MPa × 10^3	G MPa × 10^3	Possion's Ratio μ	Specific gravity	Coefficient of linear expansion : $\propto \times 10^{-6}$ $\frac{mm}{mm, deg^0C}$	$\frac{KJ}{m-hr^0C}$	Specific heat $\frac{J}{Kg^0C}$
1.	Aluminium (all alloys)	72.5	26.7	0.334	2.71	24	623	866
2.	Brass	12 6	49.1	0.285	8.21	18.7	383	376
3.	Carbon steel	200	77.2	0.292	7.8	10.7	160	460
4.	Cast Iron (Gray)	100	42.1	0.271	7.2	10.6	166	544
5.	Inconel	218	77.2	0.290	8.49	13.1	-	-
6.	Magnesium	45.6	16.9	0.350	1.80	25.2	-	-
7.	Molybdenum	337	119	0.337	10.2	-	-	-
8.	Monel Metal	183	66.6	0.32	8.82	14.9	93	531
9.	Nickel Silver	130	49.1	0.322	8.75	-	-	-
10.	Nickel Steel	200	77.2	0.291	7.75	-	-	-
11.	Phosphor Bronze	113	42.1	0.349	8.16	-	-	-
12.	Stainless Steel	196	74.5	0.305	7.75	17.6	383	427
13.	Rubber (Soft)	01.1	-	0.33	0.97-1.25	20 for 0°C to 60°C	-	-
14.	Concrete	13.5	-	0.10-0.25	2.32	-	-	-
15.	Wood	10.0	-	-	0.4-1.0	2.30	-	-

Table II - 3
Properties & Uses of Typical Cast Irons & Cast Steels

S. No.	Material	Ultimate Strength S_{ut}	S_{uc}	S_{eb}	Modulus of Elasticity E	G	Percentage elongation	Brinell hardness Number BHN	Remarks & Common Uses
1.	Cast Iron SAE 20 I.S. Grade 15	140	560	63	77	28	1-2	110	Machine Beds. Frames, Rollers, Handwheels
2.	Cast Iron SAE 30 I.S. Grade 20	271	700	98	91	35	1-2	170	Wheels, Pulleys, Flywheels.
3.	Cast Iron High Grade SAE 50 I.S. Grade 35	350	1000	155	126	49	1-2	250	Gears, Pulleys, Flywheels.
4.	Malleable Cast Iron	490	-	210	190	77	1-2	210	Frames, Gears, Shafts, Dies, for press-forming.
5.	Nickel Cast Iron Ni 1.75	310	-	112	126	53	2	180	Tough casting as Automobile crank shafts.
6.	Cast Steel								
i)	IS-CS-65	650	-	4	200	79	22	190	Castings of complicated shapes.
ii)	IS-CS-85	850	-	340	200	82	18	248	
iii)	IS-CS-125	1250	-	500	200	89	15	363	

Table II - 6A
Approximate Relationship Between Strengths

S. No.	Material	$\frac{S_{yt}}{S_{ut}}$	$\frac{S_{eb}}{S_{ut}}$	$\frac{S_{eb}}{S_{yt}}$	$\frac{S_{ea}}{S_{eb}}$	$\frac{S_{es}}{S_{eb}}$	$\frac{S_{us}}{S_{ut}}$
1.	Steel	0.6 - 0.7	0.5 - 0.6	0.8 - 0.9	0.65	0.55	0.65 - 0.75
2.	Cast Iron	-	0.4	-	1.0	0.92	-

Table II - 7

Properties and Uses of Typical Plain Cabron-steels & Wrought Iron

S. No.	Material	S_{ut}	S_{yt} S_{yc}	S_{ys}	S_{eb}	E	G	e	BHN	Remarks & Common Uses
A.	Carbon Steel									
	I.S. C-10 Hot rolled (SAE 1010)	379	218	140	168	206	80	36	110	Stampings, Sheets, wires, Rivets.
	Cold drawn	455	374	-	-	206	80	25	137	Cold drawn parts.
	I.S. C-15 Hot rolled	415	310	-	-	206	80	39	125	Wires, Sheets
	(SAE 1015) Colddrawn	503	421	-	-	206	80	24	143	
	I.S.C-20 Hot rolled	435	246	154	183	250	80	30	125	Structural works, Machine parts, Gears, levers; can be carburized.
	I.S. C-25 Annealed (SAE 1025) Water quenched & drawn 700°C.	455 531	232 280	136 163	198 293	204 204	80 80	26 35	120 159	Gears, levers.
	I.S. C-30 Soft (SAE 1030)	527	296	183	225	204	79	26	150	Gears, Shafts, levers, welded tubes.
	I.S. C-35 Hot rolled (SAE 1035)	580	367	-	-	204	79	39	126	Bolts & Screws.
	I.S. C-40 Annealed (SAE 1040)	632	350	210	260	203	78	32	180	Axles, Crane, hooks, Seamless tubes.
	I.S. C-45 Annealed (SAE 1045) Oil quenched & drawn 700° C.	579 780	306 545	170 306	286 443	203 203	78 78	20 16	140 235	Axles, Crank shafts.
	I.S. C-50 Annealed (SAE 1050)	670	365	246	295	202	78	20	190	Shafts, Gears, Studs, Valves, Clutch discs; Can be heatreated.
	I.S. C-95 Annealed (SAE 1095) Oil quenched	840 913	420 463	253 273	323 477	202 202	78 78	20 16	240 300	Springs, Cutting tools.
B.	Free Cutting Steels									
	SAE1112 Cold drawn	575	494	-	-	206	80	15	170	Machine Screws, Power Screws.
	SAE 1113 Cold drawn	58.0	500	-	-	206	80	14	170	
	SAE 1117 Hot drawn	490	309	-	-	206	80	33	137	
	SAE 1120 Cold drawn	440	240	150	177	204	80	20	156	
	SAE 1137 Hot rolled	675	395	-	-	204	78	27	192	Universal Joints, Crank shafts.
C.	Wrought Iron	337	190	210	176	183	68	30 to 40	100	Forgings.

Table IV-I

Standard Classes of Fits and Their Details

S. No.	Type of fit	Method of Assembly	Allowance	Hole Tolerance	Shaft Tolerance	Fields of Use
1.	Loose	Strictly interchangeable	$-0.0075\sqrt[3]{D^2}$	$+0.02\sqrt[3]{D}$	$-0.02\sqrt[3]{D}$	Agriculture, Mining & General purpose machinery.
2.	Free	-do-	$-0.0040\sqrt[3]{D^2}$	$+0.01\sqrt[3]{D}$	$-0.01\sqrt[3]{D}$	Shafts of generators, motors, engines & some automotive parts.
3.	Medium	-do-	$-0.0025\sqrt[3]{D^2}$	$+0.007\sqrt[3]{D}$	$-0.007\sqrt[3]{D}$	More accurate machine tools & Automotive parts.
4.	Snug	-do-	-0.0000	$+0.005\sqrt[3]{D}$	$-0.0035\sqrt[3]{D}$	Closest fit to be assembled by hand, cases where no perceptible shake is permissible.
5.	Wringing	Selective Assembly	+ 0.0000	$+0.005\sqrt[3]{D}$	$+0.0035\sqrt[3]{D}$	
6.	Tight	-do-	+ 0.00025D	$+0.005\sqrt[3]{D}$	$+0.005\sqrt[3]{D}$	Semi-permanent assembly, long fits in heavy sections, drive fit in thin sections, shrink fit in light sections.
7.	Medium Force	-do-	+ 0.0005D	$+0.005\sqrt[3]{D}$	$+0.005\sqrt[3]{D}$	Locomotive wheels, Generator & motor alternators, crank dics.
8.	Heavy Force or Shrink	-do-	+ 0.001D	$+0.005\sqrt[3]{D}$	$+0.005\sqrt[3]{D}$	Steel rims on C.I. wheel, Aluminium heads on steel cylinders, built up large bore guns.

Where $D' =$ Nominal diameter, in mm.

Table IV - 2
Commonly Used Fits & Their Uses

Type of Fit	Class of Shafts	Width Holes				Remarks & Uses
		H_6	H_7	H_8	H_{11}	
Clearance	a	-	-	-	$^{*}a_{11}$	Large Clearance fit not widely used.
	b	-	-	-	-	
	c	-	c_8	$^{*}c_9$	c_{11}	Slack running fit.
	d	-	d_8	$^{*}d_8$ d_{10}	d_{11}	Loose running fit.
	e	e_7	e_8	$^{*}e_8$ e_9	-	Easy running fit..
	f	f_6	$^{*}f_7$	$^{*}f_8$	-	Normal running fit.
	g	$^{*}g_5$	$^{*}g_6$	g_7	-	Close running fit or sliding fit also spigot & location fit.
	h	$^{*}h_5$	$^{*}h_6$	$^{*}h_7$	h_{11}	Precision sliding fit, also fine spigot and location fit.
Transition	j	$^{*}j_5$	$^{*}j_6$	$^{*}j_7$	-	Push fit for very accurate location with easy assembly & dismantling.
	k	$^{*}k_5$	$^{*}k_6$	$^{*}k_7$	-	Light keying fit (true transistion fit for keyed shaft, non running loaded pins etc.)
	m	$^{*}m_5$	$^{*}m_6$	m_7	-	Medium keying fit.
	n	n_5	$^{*}n_6$	n_7	-	Heavy keying fit (for tight assembly of mating surfaces.)
Interference	p	p_5	$^{*}p_6$	-	-	Light press fit with easy dismantling for non-ferrous parts, standard press fit with easy dismantling for ferrous and non-ferrous parts assembly.
	r	r_5	$^{*}r_6$	-	-	Medium drive fits with easy dismantiling for ferrous part assembly. Light drive fit with easy dismantling for non-ferrous part assembly.
	s	s_5	$^{*}s_6$	s_7	-	Heavy drive fit for ferrous parts permanent or semi-permanent assembly. Standard press fit for non-ferrous parts.
	t	t_5	t_6	t_7^{*}	-	Force fits on ferrous parts for parmanent assembly.
	u	u_5	u_6	u_7^{*}	-	Heavy force fit or shrink fit.
	x	-	-	-	-	Very large interferance fit, not recommend for general use.
	y					
	z					

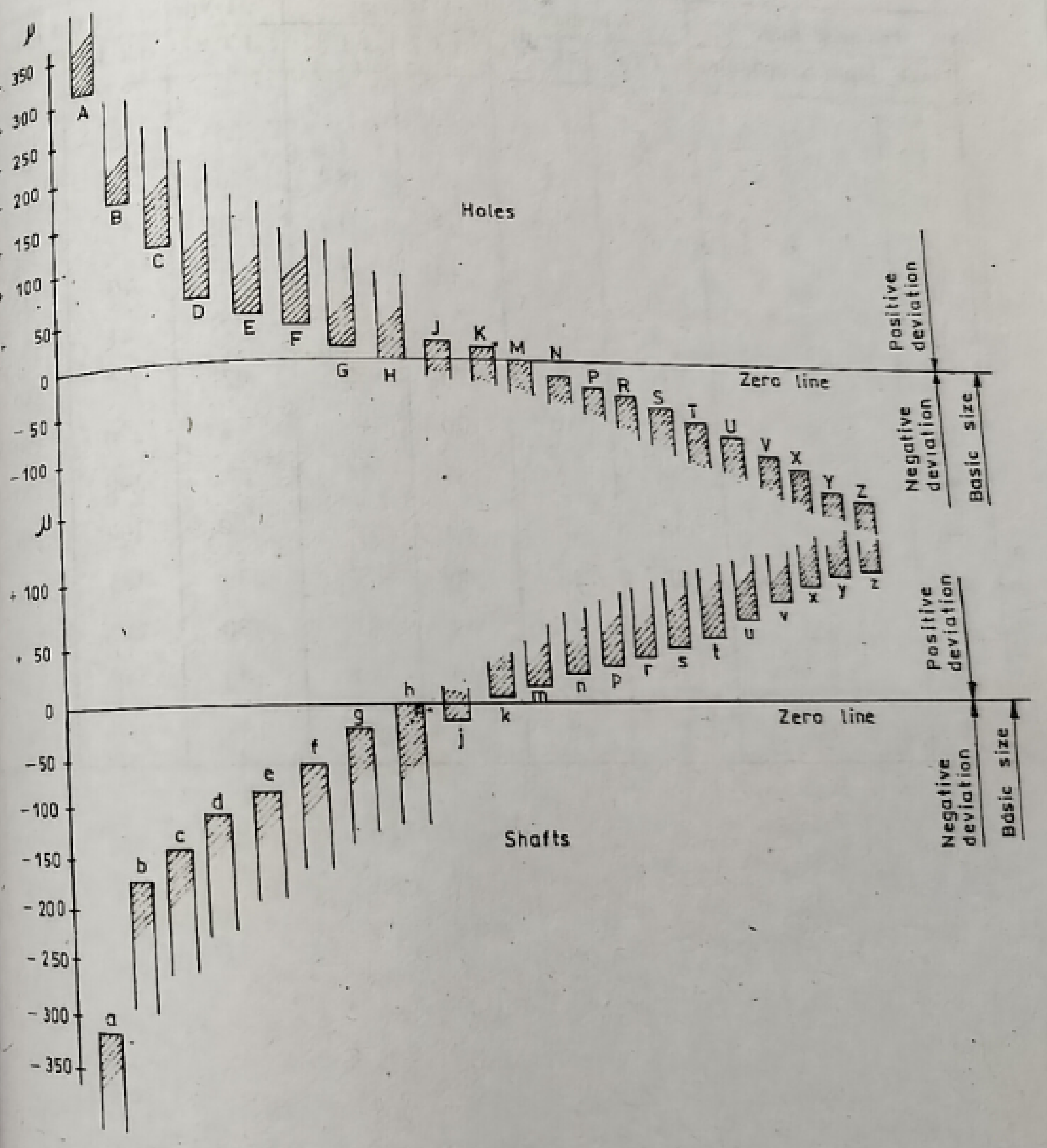

μ
+ 350
+ 300
+ 250
+ 200
+ 150
+ 100
50
0
- 50
- 100
A
B
C
D
E
F
G
H
J
K
M
N
P
R
S
T
U
V
X
Y
Z
Holes
Zero line
Positive deviation
Negative deviation
Basic size
μ
+ 100
+ 50
0
- 50
- 100
- 150
- 200
- 250
- 300
- 350
a
b
c
d
e
f
g
h
j
k
m
n
p
r
s
t
u
v
x
y
z
Zero line
Shafts
Positive deviation
Negative deviation
Basic size

Fig. 4.1 Representative fundamental deviations for shafts and holes

Table IV - 6

Tolerances For Grades Used In Fits

Diameter mm.		Tolerance		Micron		(1 Micron = 0.001 mm)		
over	upto & including	I T 5	I T 6	I T 7	I T 8	I T 9	I T 10	IT 11
1	3	5	7	9	14	25	40	60
3	6	5	8	12	18	30	48	75
6	10	6	9	15	22	36	58	90
10	18	8	11	18	27	43	70	110
18	30	9	13	21	33	52	84	130
30	50	11	16	25	39	62	100	160
50	80	13	19	30	46	74	120	190
80	120	15	22	35	54	87	140	220
120	180	18	25	40	63	100	160	250
180	250	20	28	46	72	115	185	290
250	315	23	32	52	81	130	210	320
315	400	25	36	57	89	140	230	360
400	500	27	40	63	97	155	250	400

Table IV - 7

Fundamental Deviations For Shafts

Note : Use same values with opposite sign for holes (refer table IV 4)

Diameter mm		Upper deviation							
over	upto & including	a	b	c	d	e	f	g	h
		All grades							
1	3	-270	-140	-60	-20	-14	-7	-3	0
3	6	-270	-140	-70	-30	-20	-10	-4	0
6	10	-280	-150	-80	-40	-25	-13	-5	0
10	14	-290	-150	-95	-50	-32	-16	-6	0
14	18	-290	-150	-95	-50	-32	-16	-6	0
18	24	-300	-160	-110	-65	-40	-20	-7	0
24	30	-300	-160	-110	-65	-40	-20	-7	0
30	40	-310	-170	-120	-80	-50	-25	-9	0
40	50	-320	-180	-130	-80	-50	-25	-9	0
50	65	-340	-190	-140	-100	-60	-30	-10	0
65	80	-360	-200	-150	-100	-60	-30	-10	0
80	100	-380	-220	-170	-120	-72	-36	-12	0
100	120	-410	-240	-180	-120	-72	-36	-12	0
120	140	-460	-260	-200	-145	-85	-43	-14	0
140	160	-520	-280	-210	-145	-85	-43	-14	0
160	180	-580	-310	-230	-145	-85	-43	-14	0
180	200	-660	-340	-240	-170	-100	-50	-15	0
200	225	-740	-380	-260	-170	-100	-50	-15	0
225	250	-820	-420	-280	-170	-100	-50	-15	0
250	280	-920	-480	-300	-190	-110	-56	-17	0
280	315	-1050	-540	-330	-190	-110	-56	-17	0
315	355	-1200	-600	-360	-210	-125	-62	-18	0
355	400	-1350	-680	-400	-210	-125	-62	-18	0
400	450	-1500	-760	-440	-230	-135	-68	-20	0
450	500	-1650	-840	-480	-230	-135	-68	-20	0

(Contd..next page)

Table IV - 7 (Contd...)

Diameter mm		Lower deviation											
Over	upto & including	j			k		m	n	p	r	s	t	
		5p					All grades						
1	3	-1	-2	Deviation = $\frac{IT}{2}$	0	0	+2	+6	+9	+12	+15	-	+18
3	6	-1	-3		+1	0	+4	+8	8	+15	+19	-	+23
6	10	-2	-5		+1	0	+6	+10	+15	+19	+23	-	+28
10	14	-3	-6		+1	0	+7	+12	+18	+23	+28	-	+33
14	18	-3	-6		+1	0	+7	+12	+18	+23	+28	-	+33
18	24	-4	-8		+2	0	+8	+15	+22	+28	+35	-	+41
24	30	-4	-8		+2	0	+8	+15	+22	+28	+35	+41	+48
30	40	-5	-10		+2	0	+9	+17	+26	+34	+43	+48	+60
40	50	-5	-10		+2	0	+9	+17	+26	+34	+43	+54	+70
50	65	-7	-12		+2	0	+11	+20	+32	+41	+53	+66	+87
65	80	-7	-12		+2	0	+11	+20	+32	+43	+59	+75	+102
80	100	-9	-15		+3	0	+13	+23	+37	+51	+71	+91	+124
100	120	-9	-15		+3	0	+13	+23	+37	+54	+79	+104	+144
120	140	-11	-18		+3	0	+15	+27	+43	+63	+92	+122	+170
140	160	-11	-18		+3	0	+15	+27	+43	+65	+100	+134	+190
160	180	-11	-18		+3	0	+15	+27	+43	+68	+108	+146	+210
180	200	-13	-21		+4	0	+17	+31	+50	+77	+122	+166	+236
200	225	-13	-21		+4	0	+17	+31	+50	+80	+130	+180	+258
225	250	-13	-21		+4	0	+17	+31	+50	+84	+140	+196	+284
250	280	-16	-26		+4	0	+20	+34	+56	+94	+158	+218	+315
280	315	-16	-26		+4	0	+20	+34	+56	+98	+170	+240	+350
315	355	-18	-28		+4	0	+21	+37	+62	+108	+190	+268	+390
355	400	-18	-28		+4	0	+21	+37	+62	+114	+208	+294	+435
400	450	-20	-32		+5	0	+23	+40	+68	+126	+232	+330	+490
450	500	-20	-32		+5	0	+23	+40	+68	+132	+252	+360	+540

Table V - 1
Type of Rivetted Joints & Their Efficiencies

S.No.	Type of Joints	Efficiency range	S.No.	Type of Joint	Efficiency range
1.	Lap joints		2.	Butt joints (Two cover plates)	
i)	Single row	40 - 60	i)	Single row	60 - 72
ii)	Double row	60 - 72	ii)	Double row	72 - 82
iii)	Triple row	72 - 82	iii)	Triple row	80 - 90
			iv)	Quadruple row	85 - 94

Table V - 2
Normal Proportions of Rivetted Joints

S.No.	Item	Proportion
1.	Diameter of rivets, d, mm a) Pressure vessels	
	i) Lap Joint	$d = 2t$ for $t < 10$ mm $d = t + 2$ for $t > 10$ mm
	ii) Butt Joint	$d = t + 6$
	iii) Lap & Butt Joints Prof. Unwins formula)	$d = 6\sqrt{t}$ to $7\sqrt{t}$
	b) Structural Joints	$d = 7\sqrt{t} - 2$
2.	Types & proportions of Rivet heads	Refer Fig. 5.2
3.	Pitch P mm. i) Multirow rivetted joint, largest pitch	$P = \frac{\pi}{4} d^2 \frac{(2n_2 + n_1) s_s}{t s_t} + d$. where, n_2 = Number of rivets in double shear n_1 = Number of rivets in single shear s_s = Shear strength of rivets, MPa s_t = Tensile, strength of plate, MPa
	ii) Minimum pitch geometrically possible	$P = 2.25$ d
	iii) For leak proof joints	$P < 6d$

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Table V - 2 (Contd..)

S.No.	Item	Proportion
4.	Transverse Pitch. P_t , mm. i) Chain Rivetting ii) Staggered Rivetting iii) As per ASME Boiler code	$P_t > 2d$ $P_t > (0.33\ P + 0.67\ d)$ $P_t = 1.75\ d$ if $\frac{P}{d} \leq 4$ $P_t = 1.75\,d + 0.1\,(P - d)$ if $\frac{P}{d} > 4$
5.	Diagonal Pitch, P_d, mm Staggered Joints	$P_d \geq \frac{P+d}{2}$ $P_d = (2.25 \text{ to } 2.5)\ d$ $P_d = \frac{2\,P+d}{3}$
6.	Margin; m(mm), Minimum distance of rivet centre from edge	$m = 1.5\ d$
7.	Thickness of Cover plates	
	a) Single Butt strap	
	i) Ordinary rivetting	$t_i = 1.125\ t$
	ii) Alternate rivet omitted in outer rows	$t_i = 1.125\,t\,\frac{(P-d)}{(P-2d)}$
	b) Double Butt strap	
	i) Ordinary rivetting	$t_i = t_o = 0.625\ t$
	ii) Alternate rivet omitted in outer rows	$t_i = t_o = 0.625\,t\,\frac{(P-d)}{(P-2d)}$
	c) Double Butt strap Unequal width of cover plates	$t_o = 0.625\ t$ $t_i = 0.750\ t$
8.	Rivet length L mm i) Ordinary rivets ii) Counter sunk rivets	 $L = t + (1.5 \text{ to } 1.7)\ d$ $L = t + 0.75 \times$ dia. of counter sunk hole

Table V - 3

Selection of Rivetted Joints (For Pressure Vessels)

S.No.	Type of Joint	Shell dia. (mm)	Shell Thickness
1.	Double Rivetted	600 - 1600	6 - 12
2.	Triple Rivetted	900 - 2100	8 - 15
3.	Quadruple Rivetted	1500 - 2700	10 - 45

Table V - 4

Allowable Stresses in Rivetted Joints

S. No.	Equipment	Material	Use	Factor of safety	S_t	S_c	S_s
1.	Boilers	i) Wrought Iron	Rivets -	5	80	135	63
		ii) SAE 1010	Rivets & plates	5	73	94	48
		iii) SAE 1020	Plates	5	82	106	51
2.	Structural	A. Steel					
		i) SAE 1010	Rivets (Hand Rivetting)	3	-	115	70
			Rivets (Power Rivetting)	3	-	170	95
		ii) SAE 1020	Plates	3	130	-	-
		B. Aluminium					
		i) Alcoa 25 (Pure aluminum cold as received)	- do -	1.5	-	50	20
		ii) Aloca 17S cold immediately after quenching	- do -	1.5	-	190	70
		iii) Alcoa 17S hot 500°C	- do -	1.5	-	130	65
		iv) Alcoa 61 S -TS cold as received	- do -	1.5	-	107	57
		v) Alcoa 53 S hot 530° C	- do -	-	-	107	43

Table V - 5

Strength of Shielded Arc Flush Steel Welds

S. No.	Type of Stress	Limit stress MPa			Recommended Design stress MPa		
		Basic Metal Elastic Limit	Deposited metal		(Factor of Safety 2) Steady load	(Factor of Safety 2) Repeated load	(Factor of Safety 2.75) Reversed load
			Elastic Limit	Endurance Limit			
1.	Tension	230	280	155	115	103	54
2.	Compression	250	315	-	130	115	54
3.	Bending	250	315	185	130	115	68
4.	Shear	140	170	-	80	70	35
5.	Shear & tension	-	-	-	80	70	35

Table V - 6

Actual Stress Concentration Factors For Welded Joints (K_f)

S. No.	Types of weld	K_f
1.	Reinforced Butt weld	1.2
2.	Toe of Transverse fillet weld	1.5
3.	End of longitudinal weld	2.7
4.	T - Butt Joint with sharp corners	2.0

Table V - 9

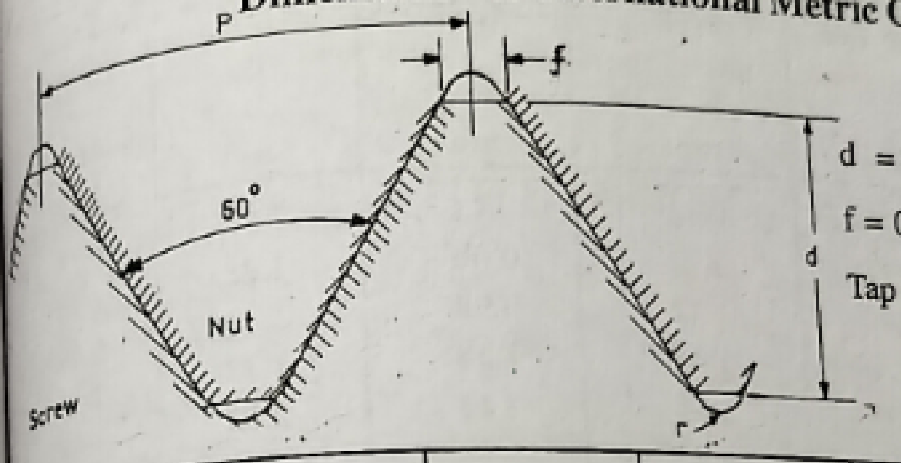
Minimum Size of Fillet Welds

(For various Plate thickness)

S. No.	Plate thickness, mm	Minimum weld size, mm	S. No.	Plate thickness,mm	Minimum weld
1.	3 - 4.5	3	4.	18 - 25	9
2.	6 - 8.0	4.5	5.	28 - 35	12
3.	9 - 15	6	6.	above 40	18

Table VI - 1

Dimensions For International Metric Coarse Threads



d = 0.7035 P to 0.6855 P

f = 0.125 P

Tap drill Dia. = (Major Dia - P)

Major Dia.	Pitch of Threads	Effective or Pitch Dia.	Minor or Core Dia. for screw	Thread Depth	Nut	
					Major Día.	Minor or Core Dia.
mm	mm	mm	mm	mm	mm	mm
6	1.00	5.350	4.59	0.703	6.11	4.70
7	1.00	6.350	5.59	0.703	7.11	5.70
8	1.25	7.188	6.24	0.879	8.14	8.38
9	1.25	8.188	7.24	0.879	8.14	7.38
10	1.50	9.026	7.89	1.055	10.16	8.05
12	1.75	10.863	9.54	1.231	12.19	9.73
14	2.00	12.701	11.19	1.410	14.22	11.40
16	2.00	14.701	13.19	1.410	10.22	13.40
18	2.50	16.376	14.48	1.759	18.27	14.75
20	2.50	18.376	16.48	1.759	20.27	16.75
22	2.50	20.376	18.48	1.759	22.27	18.75
24	3.00	22.051	19.78	2.110	24.32	20.10
27	3.00	25.051	22.78	2.110	27.32	23.10
30	3.50	27.727	25.07	2.462	30.78	25.45
33	3.50	30.727	28.07	2.462	33.38	28.45
36	4.00	33.402	30.37	2.814	36.43	30.80
39	4.00	36.402	33.37	2.814	39.43	32.80
42	4.50	39.077	35.67	3.165	42.49	36.15
45	4.50	42.077	38.67	3.165	45.49	39.15
48	5.00	44.752	40.96	3.518	48.54	41.50

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Table VI - 2

Dimensions For International Metric Fine Threads

Major Dia. mm	Pitch mm	Effective or Pitch Dia. mm	Minor or Core Dia. for screw mm	Core Area mm	Nut Major Dia. mm	Nut Minor or Core Dia. mm
1.0	0.20	0.870	0.72	0.41	1.02	0.74
1.2	0.20	1.070	0.92	0.66	1.22	0.94
1.4	0.20	1.270	1.12	0.98	1.42	1.14
1.7	0.20	1.570	1.42	1.58	1.72	1.14
2.0	0.25	1.838	1.65	2.13	2.03	1.68
2.3	0.25	2.138	1.95	2.98	2.33	1.98
2.6	0.35	2.373	2.11	3.49	2.64	2.15
3.0	0.35	2.773	2.51	4.94	3.04	2.55
3.5	0.35	3.273	3.01	7.10	3.54	3.05
4.0	0.50	3.675	3.30	8.53	4.05	3.35
4.5	0.50	4.175	3.60	11.32	4.55	3.85
5.0	0.50	4.875	4.30	14.50	5.05	4.35
6.0	0.75	5.513	4.94	19.20	6.08	5.03
7.0	0.75	8.513	5.94	27.75	7.08	6.03
8.0	1.00	7.350	6.59	34.14	8.11	6.70
9.0	1.00	8.350	7.59	45.28	9.11	7.70
10.0	1.00	9.350	8.59	57.99	10.11	8.70
12.0	1.50	11.028	9.89	76.81	12.16	10.05
14.0	1.50	13.026	11.89	111.00	14.16	12.05
16.0	1.50	15.026	13.89	152.00	16.16	14.05
18.0	1.50	17.026	15.89	188.00	18.16	16.05
20.0	1.50	19.026	17.89	251.00	20.16	18.05
22.0	1.50	21.026	18.89	311.00	22.16	20.05
24.0	2.00	22.701	21.19	353.00	24.22	21.40
27.0	2.00	25.701	24.19	459.00	27.22	24.40
30.0	2.00	28.701	27.19	580.00	30.00	27.40
33.0	2.00	31.701	30.19	716.00	33.22	30.40
36.0	3.00	34.051	31.78	713.00	36.32	32.10
39.0	3.00	37.051	34.78	950.00	39.32	35.10
42.0	3.00	40.051	37.78	1121.00	42.32	38.10
45.0	3.00	43.051	40.78	1306.00	45.32	41.10
48.0	3.00	46.051	43.78	1505.00	48.32	44.10

Table VI - 5

Standard Pitches For Metric Square Threads

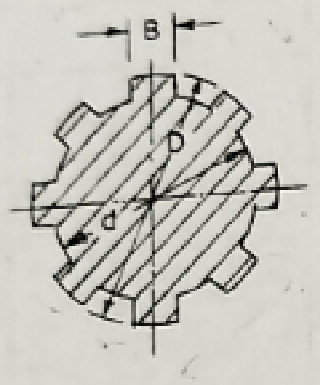
Nominal Diameter or major diameter, for bolt, d	Pitch; mm. Fine Series	Pitch; mm. Normal Series	Pitch; mm. Coarse Series
10, 12, 14, 16, 18, 20	2	--	--
22, 24, 26, 28	3	5	8
30, 32, 36	3	6	10
40, 44, 48, 50, 52	3	7	12
55, 60	3	8	14
65, 70, 75, 80	4	9	16
85, 90, 95	4	10	18
100, 110	4	12	20

Table VI - 7

Properties of Materials For Bolts & Nuts

S. No.	Material	Heat Treatment	Ultimate Tensile Strength S_{ut}	Yield Strength S_{yt}	Elongation %
1.	Free cutting screw Stock SAE 1120	Annealed	440	240	28
2.	Nickel Steel SAE 2330 Rolled	--	570	300	23
3.	Nickel Steel SAE 2330 Forged	Annealed, Oil-tempered	580	380	25
4.	Nickel Chromium Steel SAE 3140 (Forged)	Annealed Oil-tempered	700	485	21
5.	Gunmetal - (Cu 88%, Sn 10%, Zn 2%) Cast	--	290	85	25
6.	Phosphor bronze, hot rolled	--	460	250	23
7.	Monel metal, hot rolled	--	600	250	40

Table VII - 2
Dimensions For Straight Splines



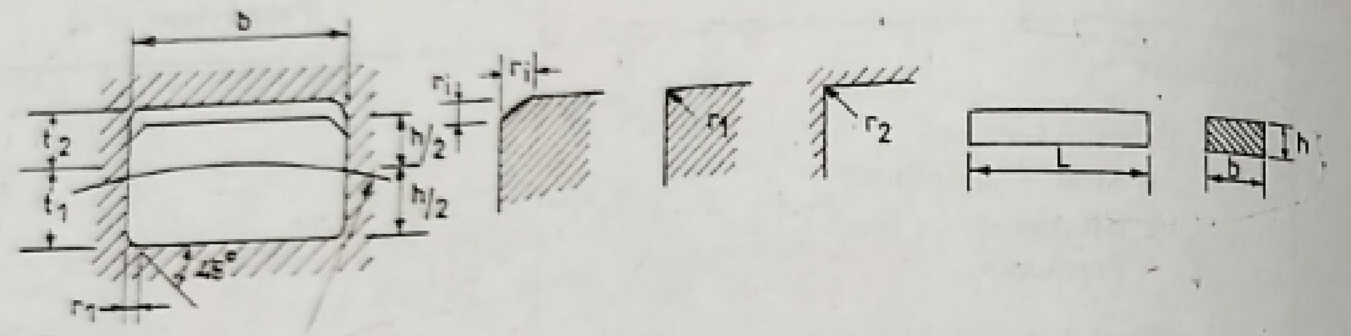
Light duty series					Medium duty series				
Nominal Size N × d × D	No. of Splines N	Minor Dia. d,mm.	Major Dia. D,mm.	Width B mm.	Nominal Size N × d × D	No. of Splines N	Minor Dia. d,mm.	Major Dia. D,mm.	Width B mm.
6 × 23 × 26	6	23	26	6	6 × 11 × 14	6	11	14	3
6 × 26 × 30	6	26	30	6	6 × 13 × 16	6	13	16	3.5
6 × 26 × 32	6	28	32	7	6 × 16 × 20	6	16	20	4
					6 × 18 × 22	6	18	22	5
					6 × 21 × 25	6	21	25	5
					6 × 23 × 28	6	23	28	6
					6 × 26 × 32	6	26	32	6
					6 × 28 × 34	6	28	34	7
8 × 32 × 36	8	32	36	6	8 × 32 × 38	8	32	38	8
8 × 36 × 40	8	36	40	7	8 × 36 × 42	8	36	42	7
8 × 42 × 46	8	42	46	8	8 × 42 × 48	8	42	48	8
8 × 46 × 50	8	46	50	9	8 × 46 × 54	8	46	54	9
8 × 52 × 58	8	52	58	10	8 × 52 × 60	8	52	60	10
8 × 56 × 62	8	56	62	10	8 × 56 × 65	8	56	65	10
8 × 62 × 68	8	62	68	12	8 × 62 × 72	8	62	72	12
10 × 72 × 78	10	72	78	12	10 × 72 × 82	10	72	82	12
10 × 82 × 88	10	82	88	12	10 × 82 × 92	10	82	92	12
10 × 92 × 98	10	92	98	14	10 × 92 × 102	10	92	102	14
10 × 102 × 108	10	102	108	16	10 × 102 × 112	10	102	112	16
10 × 112 × 120	10	112	120	18	10 × 112 × 125	10	112	125	18

Table VII - 3
Design Data For Splines

S.No.	Item	Proportion
1.	Straight Splines	
	i) Bearing Pressure	
	a) Sliding under load	7 MPa
	b) Sliding on no load	15 MPa
	c) Permanent	20 MPa
	ii) Stress concentration factor	
	a) In torsion , K_{ts}	1.8 Based on Major diameter
	b) In bending, K_t	1.4 Based on Major diameter [Based on usual fillet radius = (.05 – .02) d]
2.	Involute Splines	
	i) Bearing Pressure	
	a) Light duty	15 MPa
	b) Medium duty	20 MPa
	ii) Stress concentration factor	
	a) In torsion, K_{ts}	1.5 } Based on major diameter
	b) In bending, K_t	1.2 } Based on major diameter
	iii) No. of Splines	5 to 50

Table VII - 5

Dimensions of Parallel & Taper Keys & Keyways

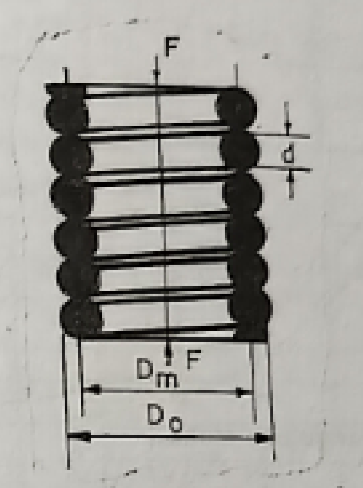
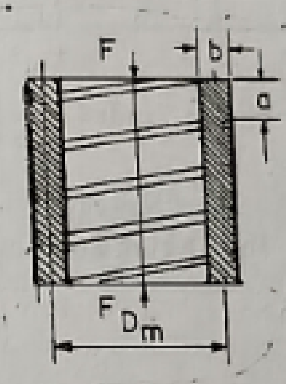
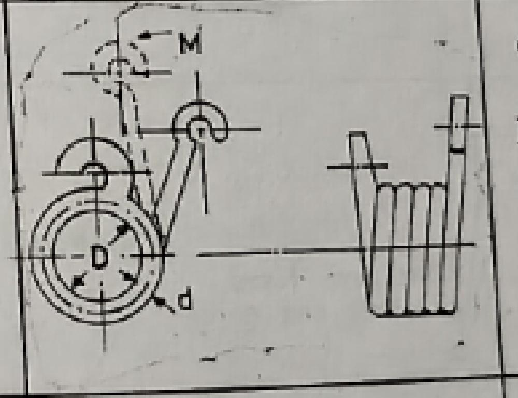
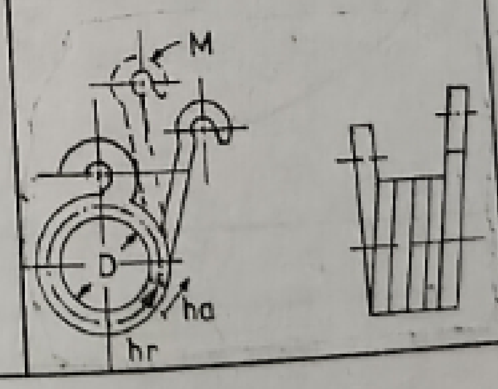


Taper keys will have a taper of 1 in 1000 on face along hub

For shaft diam.		Key cross section		Keyway	depth	Length of key	
above	upto	b	h	in shaft t_1	in hub t_2	Min.	Max.
6	8	2	2	1.2	1	6	20
8	10	3	3	1.8	1.4	6	36
10	12	4	4	2.5	1.8	8	45
12	17	5	5	3	2.3	10	56
17	22	6	6	3.5	2.8	14	71
22	30	8	7	4	3.3	18	90
30	38	10	8	5	3.3	22	110
38	44	12	8	5	3.3	28	140
44	50	14	9	5.5	3.8	36	160
50	58	16	10	6	4.3	45	180
58	65	18	11	7	4.4	50	200
65	75	20	12	7.5	4.9	56	220
75	85	22	13	8.5	5.0	63	250
85	95	25	14	9	5.4	71	280
95	110	28	16	10	6.4	80	320
110	130	32	18	11	7.4	90	360
130	150	36	20	12	8.4	100	400
150	170	40	22	13	9.4	110	400
170	200	45	25	15	10.4	125	400
200	230	50	28	17	11.4	140	400
230	260	56	32	19	13.4	160	400
260	290	63	33	20	13.4	180	400
290	330	70	36	22	14.4	200	400
330	380	80	40	25	15.4	220	400
380	440	90	45	28	17.4	250	400

Table VIII - 1

Stress & Deflection Formulae For Helical Coil Springs

S. No	Wire cross section & type of load	Figure	Stress	Deflection
1.	Round Section Compressive Load		$\tau = K\dfrac{8FD_m}{\pi d^3}$ Where, K = Wahl's Factor $= \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$ C = Spring index $= \dfrac{D_m}{d}$	$\delta = \dfrac{8FD_m^3 n}{Gd^4}$ Where, n = number of effective coils.
2.	Rectangular Section Compressive Load		$\tau = \dfrac{FD_m}{K_1 ab^2}$ For constant K_1, refer table VIII -2	$\delta = \dfrac{FD_m n}{K_2 ab^3 G}$ Where, n = number of effective coils For K_2, refer table VIII-2.
3.	Round Section Torsional Load		$\sigma_b = K_3 \cdot \dfrac{10.2\,M}{d^3}$ For K_3, refer fig. 8.1	$\theta = \dfrac{2.04\,\pi M D_m n}{Ed^4}$ (radians) Where, n = number of effective coils.
	Rectangular Section Torsional Load		$\sigma_b = K_4 \cdot \dfrac{6M}{h_a h_r^2}$ For K_4, refer fig. 8.1	$\theta = \dfrac{12\,\pi M D_m n}{E h_r^3 h_a}$ (radians) Where, n = number of effective coils.

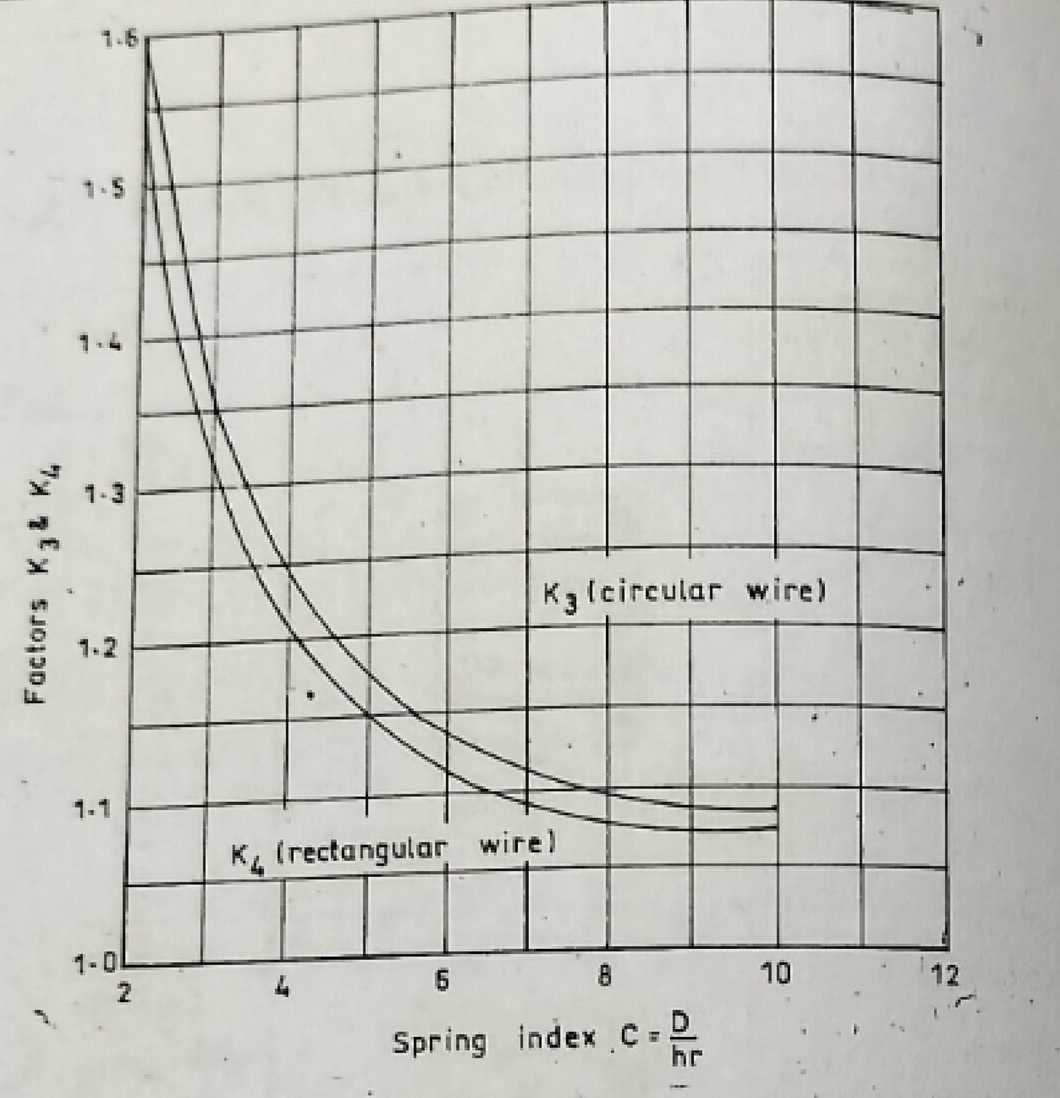


Fig 8-1. Constants for helical coil spring in torsion.

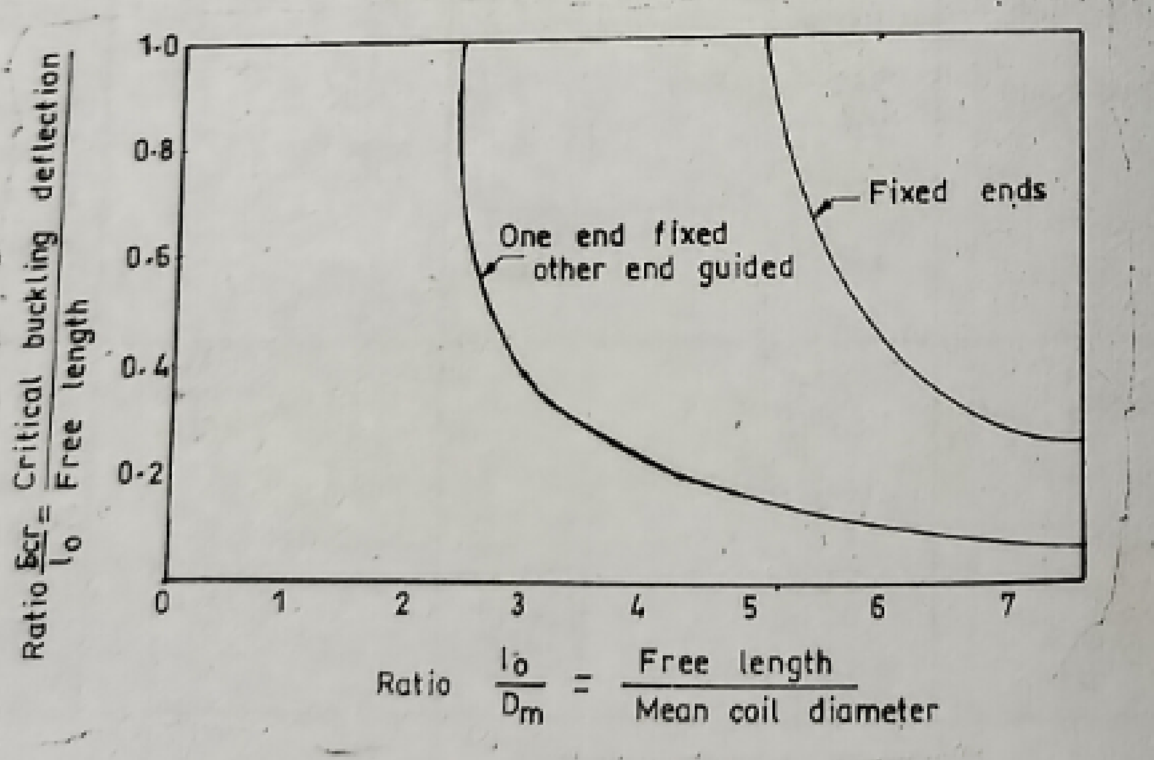


Fig. 8.2 Bulking of Round, Wire Helical Coil Compression Springs

Table VIII - 2

Constant For Rectangular Bar Helical Springs

Ratio a/b	1.0	1.2	1.5	2.0	2.5	3.0	5.0	10.0	∞
Constant K_1	0.416	0.438	0.462	0.492	0.516	0.534	0.582	0.624	0.666
K_2	0.180	0.212	0.250	0.292	0.317	0.335	0.371	0.398	0.424

Table VIII - 3

Round Wire Helical Coil Spring, Further Details

S.No.	Item	Relation
1.	Relation between active turns (n) & total turns (n_1)	
	i) Squared and ground ends	$n = n_1 - 1.75$
	ii) Plain ends	$n = n_1 - 0.5$
	iii) Plain & ground ends	$n = n_1 - 1.0$
2.	Change in diameter of compressiion springs under load, (ΔD)	
	i) Ends free to unwind	$\Delta D = 0.1\left(\frac{P^2 - 0.8\,Pd - 0.2d^2}{D_m}\right)$ Where, P = Pitch of coils in free condition.
	ii) Ends prevented from unwinding	$\Delta D = 0.05\left(\frac{P^2 - d^2}{D_m}\right)$
3.	Buckling of compression springs	Refer figure 8.2
4.	Bending stiffness (EI) of Helical Springs.	$EI = \frac{E\,d\,L}{32\,D_m n(2+\mu)}$
5.	Modulus of resilience R	$R = \frac{\tau^2}{4G}$

(Contd... next page)

Table VIII - 3 (Contd..)

S.No.	Item	Relation
6.	Stress due to eccentric loading with eccentricity e, τ	$\tau = \frac{8 F D_m K}{\pi d^3}\left(1 + \frac{2e}{m}\right)$ Where, K = Wahl's factor
7.	Stress in end loops of tension springs	$\tau = \frac{16 F D_m}{\pi d^3} K_1 + \frac{4F}{\pi d^2}$ Where, $K_1 = \frac{r_m}{r_i}$ r_m = Mean radius of the Bend. r_i = Inside radius of the Bend.
8.	Natural Frequency (f_n)	
	i) Both ends clamped	$f_n = \frac{2 d \times 10^5}{\pi D_m^2 n}\sqrt{\frac{G}{32\rho}}$
	ii) One end free & other end fixed	$f_n = \frac{d \times 10^5}{\pi D_m^2 n}\sqrt{\frac{G}{32\rho}}$ Where, ρ = density of spring material Refer fig. 8.3
9.	Optimum Design	
	i) Condition for Minimum weight	$\frac{\pi \tau D_o^2}{8 F_i} = \frac{2K^2 c(c+1)^2(3c+1)}{K(3c+1) - c(c+1)^2 \frac{dk}{dc}}$
	ii) Condition for Minimum length & minimum space.	$\frac{\pi \tau D_o^2}{8 F_i} = \frac{(4c+3)K^2(c+1)^2}{K + \frac{2K}{c} - (c+1)\frac{dk}{dc}}$ Where, F_i = Initial load K = Wahl's factor

Table VIII - 4
Stress & Deflection Formulae For Leaf Springs

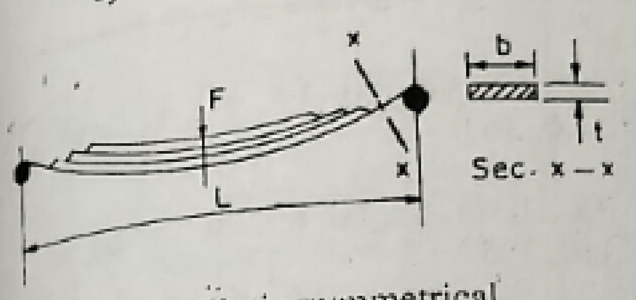
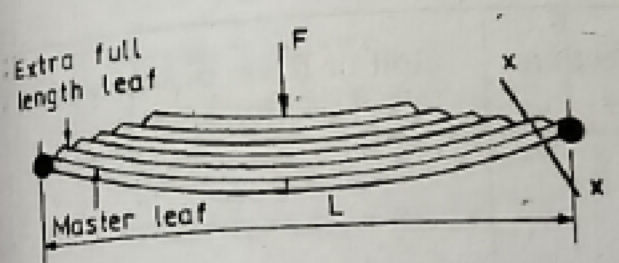
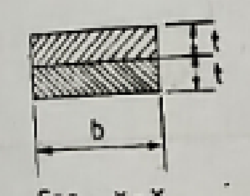
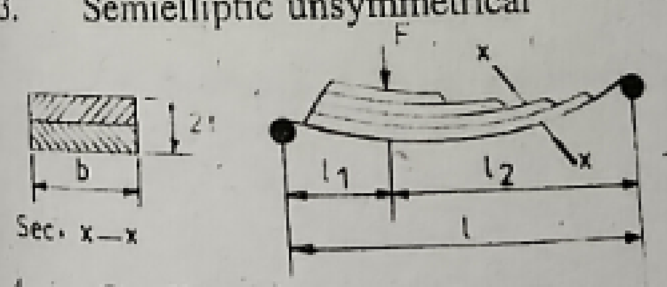
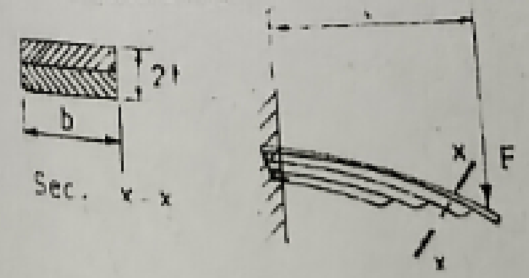
No.	Type of Spring	Stress	Deflection
1.	Semielliptic symmetrical  	$\sigma_b = \frac{3FL}{2nbt^2}$ Where, n = total no. of leaves including master leaf.	$\delta = \frac{3FL^3}{8Enbt^3}$
2.	Semielliptic symmetrical with extra full length leaves.   	In full length leaves, $\sigma_b = \frac{3FL}{2n_g bt^2}\left[\frac{1.5}{1 + 1.5\frac{n_f}{n_g}}\right]$ In graduated leaves $\sigma_b = \frac{3FL}{2n_g bt^2}\left[\frac{1}{1 + 1.5\frac{n_f}{n_g}}\right]$ Where, n_g = no. of graduated leaves including master leaf. n_f = no. of extra full length leaves.	$\delta = \frac{3FL^3}{8En_g bt^2}\left[\frac{1}{1 + 1.5\frac{n_f}{n_g}}\right]$
3.	Semielliptic unsymmetrical  	$\sigma_b = \frac{6FL_1L_2}{nbt^2L}$	$\delta = \frac{6FL_1^2L_2^2}{Enbt^3L}$
4.	Cantilever  	$\sigma_b = \frac{6FL}{nbt^2}$	$\delta_b = \frac{6FL^3}{Enbt^3}$

Table VIII - 5

Leaf Springs, Further Details

S.No.	Item	Relation
1.	Standard leaf thickness, mm	3.2, 4.5, 5.0, 6.0, 7.5, 9.0, 11.0, 12.0, 14.0, 16.0
2.	Standard leaf width, mm	32, 41, 45, 50, 60, 65, 70, 75, 80, 90, 110, 125
3.	Standard proportions of centre bolts and clips	

Leaf width mm	Centre bolt dia. mm	Clip cross section	Bolt or Rivet diameter on clip, mm
[32, 41, 45, 50]	8 - 10	20 × 4	6
[55, 60, 65, 70, 75, 80]	8 - 10 12-16	20 × 4 25 × 6	8 10
90, 110, 125	12 - 16	32 × 6	10

Table VIII - 9

Properties of Spring Materials

S. No.	Material	Ultimate Strength Tension S_{ut}	Elastic limit		Endurance limit		Modulus of elasticity	
			Bending S_{yb}	Shear S_{ys}	S_{eb}	S_{es}	Direct E	Transverse G
1.	Music wire	2040	1760	1060	950	560	210	30
2.	Cr - V Steel, SAE 6150	1690	1340	770	630	390	220	85
3.	Si-Mn Steel, SAE 9250	1550	1130	670	560	340	211	80
4.	Carbon Steel, SAE 1095	1410	930	600	490	300	170	80
5.	Carbon Steel, SAE 1050	840	530	320	400	240	210	79
6.	Monel Metal Cold Rolled	840	490	300	350	210	180	68
7.	Phosphor Bronze	690	350	210	320	140	110	44
8.	Brass wire, SAE 80	670	350	200	140	84	102	39
9.	Brass, SAE 70 Hard Drawn	630	320	190	110	73	102	39
10.	Duraluminum, SAE 26	455	240	140	95	56	73	27

.. Multiply these values by size factor, S_z give below :

For Leaf Springs $S_Z = 0.8 + \frac{2.5}{t}$ where t = leaf thickness, mm.

For Round wire $S_Z = 0.86 + \frac{1.75}{d_w}$ for all materials, except for monel metal

$S_Z = 0.986 + \frac{0.1075}{d_w}$ for monel metal.

Where, d_w = wire diameter, mm.

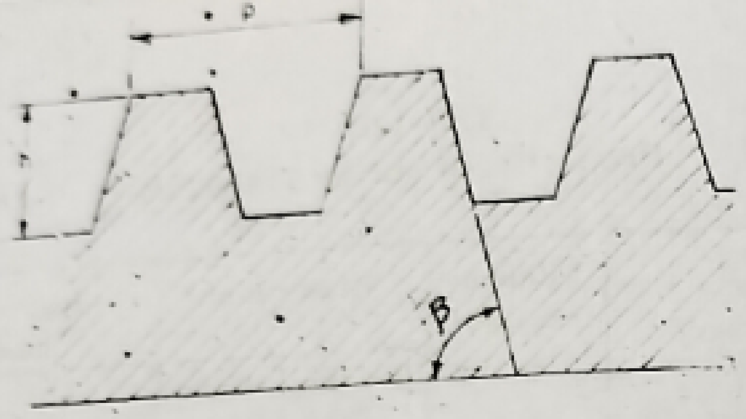


Table IX - 1

Formulae For Power Screws

S. No.	Item	Formula
1.	Torque required for lifting, T_1	$T_1 = \frac{WD_m}{2} \tan(\alpha + \phi)$
2.	Torque required for lowering, T_2	$T_2 = \frac{WD_m}{2} \tan(\alpha - \phi)$
3.	Efficiency of power screw, η	$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}$
4.	Torque due to collar friction, T_C	$T_C = \frac{WD_C \mu_C}{2}$
5.	Efficiency considering collar friction, η	$\eta = \frac{D_m \tan(\alpha)}{D_m \tan(\alpha+\phi) + D_C \mu_C}$

Where,

α = Helix angle $= \tan^{-1} \frac{l}{\pi D_m}$

l = Lead = pitch for single start

D_m = Mean screw diameter

ϕ = Angle of friction at screw $= \tan^{-1}\left(\frac{\mu}{\sin\beta}\right)$

β = angle made by screw face at axis (Also see fig. 9.1)

μ = Coefficient of friction at screw and nut

μ_C = Coefficient of friction at collar

D_C = Mean collar diameter

$= \frac{2}{3}$ Do for flat pivot or conical pivot

Table IX - 2

Design Data For Power Screws

S. No.	Applications	Material		Lubrication	Safe bearing pressure MPa	Coefficient of friction μ	Operating speed
		Screw	Nut				
1.	Hand operated press	Steel	Bronze	Well Lubricated	17 - 24	0.081-0.101	Low speed
				Intermittent Lubrication	15 - 20	0.11-0.17	
2.	Screw Jack	Steel	Cast Iron	Intermittent Lubrication	12.5-17.5	0.10-0.16	Low speed (below 25 m/min)
		Steel	Bronze	Intermittent Lubrication	11-17.5	0.15-0.20	Low speed (below 3 m/min)
3.	Hoisting Screw	Steel	Cast Iron	Bad Lubrication	4-7	0.15-0.20	Medium speed (6-12m/min)
				Intermittent Lubrication	10-15	0.15-0.20	
		Steel	Bronze	Bad Lubrication	5.5-10	0.15-0.20	
				Intermittent Lubrication	10-15	0.10-16	
4.	Lead Screw	Steel	Bronze	Well Lubricated	10-17	0.081-0.101	High speed (15 m/min & above)

Table IX - 3

Approximate Data For Wire Ropes (Plow steel, Hemp Core)

S.No.	Type of Rope	Rope diameters available, Dr. mm.	Breaking Strength Fut, KN	Weight 10^{-3} N/m	Wire diameter dw, mm	Cross section area of metal A, mm^2	Sheave Size in mm Average	Sheave Size in mm Minimum	Uses
1. docks.	Standard hoisting rope 6×19	8, 10, 12, 14, 18, 20, 22, 25, 29, 32, 35, 38, 41, 44, 48, 51, 54, 57.	1.20 Dr^2	37 Dr^2	0.063 Dr	0.38 Dr^2	45 Dr	30 Dr	Most commonly used Rope, Mine hoist, Derricks, Quarries, Dredges.Ore Tramways, Cargo hoist, Well drilling, Car pullers, Cranes, Elevators.
2.	Special flexible hoisting rope 6×37	- do -	1.12 Dr^2	35 Dr^2	0.045 Dr	0.38 Dr^2	27 Dr	18 Dr	Cranes, High speed elevators, Service where sheave diameter is limited.
3.	Extra flexible 8×19	8, 10, 12, 19, 16, 18, 20, 22, 25.	1.08 Dr^2	33 Dr^2	0.050 Dr	0.35 Dr^2	31 Dr	21 Dr	Space limitation for sheave.
4.	Standard coarse laid rope 6×7	8, 9, 10, 11, 12, 14, 16, 18, 20, 22, 24, 26, 28, 32, 36, 40.	1.12 Dr^2	31 Dr^2	0.106 Dr	0.38 Dr^2	72 Dr	42 Dr	Standard Haulage rope, Mine haulage, Factory yard haulage, Incline planes, Tramways, Power transmission. Guy wires.

Table IX - 4

Wire Rope Materials

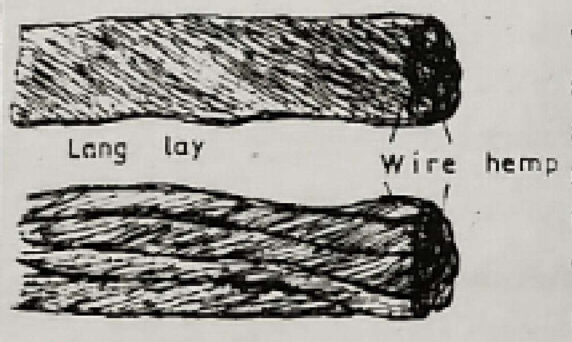
S.No.	Material	Ultimate Strength S_{ut} MPa
1.	High Carbon steel	
	a) Plow steel (P.S.)	1500 - 1700
	b) Improved plow Steel (I.P.S.)	1700 - 2100
	c) Mild Plow Steel (M.P.S.)	1300 - 1500
	d) Very High Strength Steel (VHS)	2000 - 2400
2.	Low Carbon steel (0.1 % C)	700

Table IX - 5

Recommended Minimum Safety Factors For Wire Ropes

S. No.	Application	Factor of Safety	S. No.	Application	Factor of Safety
1.	Track Cable	3.2	6.	Overhead & Gantry cranes	6.0
2.	Guys	3.5	7.	Jib & Pillar cranes	6.0
3.	Mine Shafts		8.	Derricks	6.0
	Upto 150 meters	8.0	9.	Small Electric & Air hoist	7.0
	300 - 600 meters	7.0	10.	Hot ladle cranes	8.0
	600 - 1000 meters	6.0	11.	Slings	8.0
	Above 1000 meters	5.0			
4.	Miscellaneous hoisting	5.0			
5.	Haulage Ropes	6.0			

Table IX - 7
Wire Ropes - Other Details

S.No.	Item	Details	
1.	Designation	20mm, 6 × 19 indicates	20mm rope diameter, No. of strands = 6, (First no.) No. of wires in each strand = 19. (Second no.)
2.	Lang lay rope Types Regular lay rope		Twist of wires & strands are in same direction. It is stronger than regular lay rope & has higher tendency to spin, twist. Twist of wires & strands are in opposite direction.
3.	Bending load	It is commonly used. Equivalent bending load, Fb $Fb = \left(84 \times 10^3 \times A \times \frac{dw}{D_S}\right)$, N Where, dw = Wire diameter, mm D_s = Sheave diameter, mm.	A = Area of metal, mm²
4.	Formula for fatigue life	Dr. $D_S = \frac{2\,(FS)\,F_t}{K\,S_{ut}}$ Where, (FS) = Factor of safety, F_t = Tensile force in rope, N	

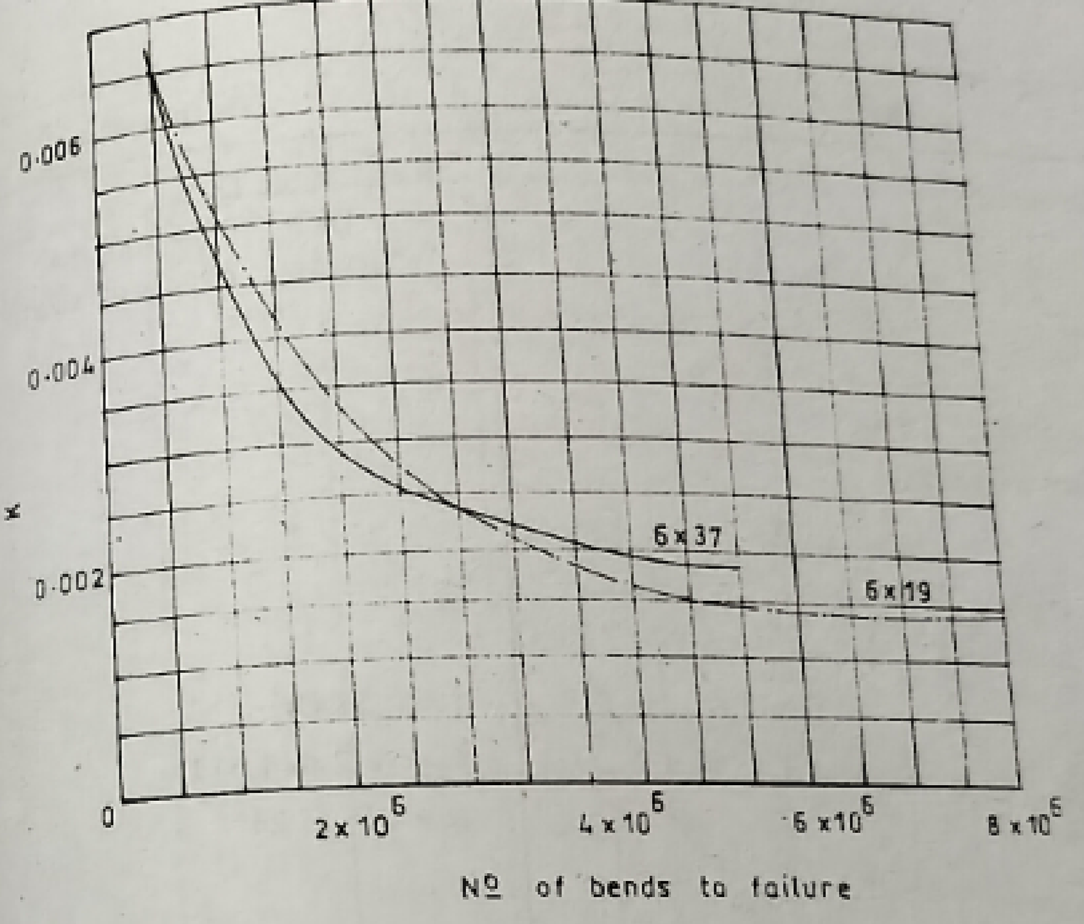


Fig. 9.2 Factor K for fatigue of Ropes

Table IX - 8

Rope Sheave & Drum Proportions

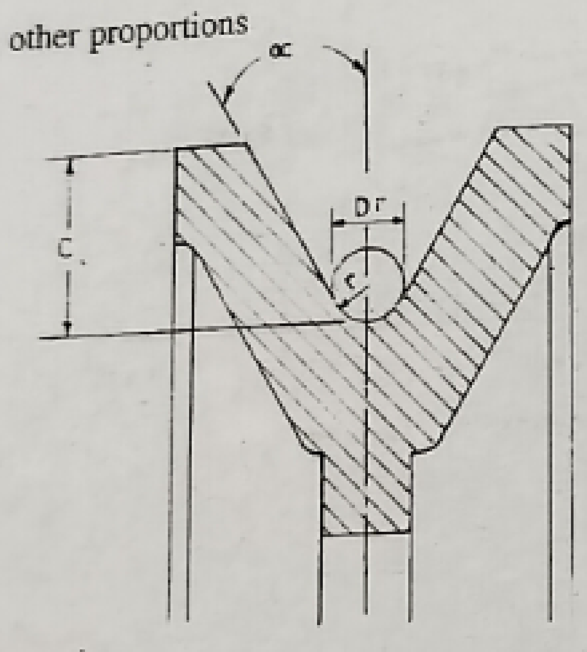
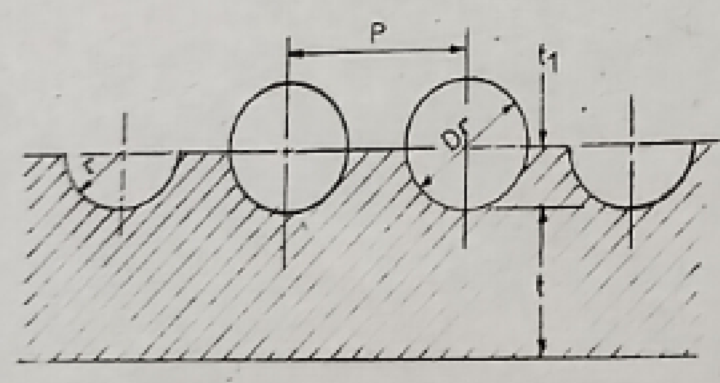
S.No.	Item	Proportion
1.	Sheave Diameter D_S	Refer Table IX-3 Also Ds ≥ (300 – 500) dw Where dw = large wire size in rope.
2.	Sheave, other proportions 	r = 0.53 Dr C = (1.2 to 1.5) Dr α = 21° to 24°
3.	Drum. proportions 	r = 0.53 Dr p = 1.15 Dr t_1 = 0.25 Dr t = Dr (check for strength)

Table X - 1

Wall Thickness For Pressure Vessels

S. No.	Type of Cylinder	Classification	Type of Material	End Condition	Wall thickness, t_b mm	Design stress S_d MPa
1.	Thin	$\frac{t_b}{D_i} < 0.1$ or $\frac{P}{S_d} < 0.2$	Any	No effect on design	$t_b = \frac{P D_i}{2S_d}$	For Ductile materials $S_d = 0.8\,S_{yt}$ For Brittle materials $S_d = \frac{S_{ut}}{8}$
2.	Thick	$\frac{t_b}{D_i} > 0.1$	Brittle	No effect on Design	Lame's Equation. $t_b = \frac{D_i}{2}\left[\sqrt{\frac{S_d + P}{S_d - P}} - 1\right]$	$S_d = \frac{S_{ut}}{8}$
		$\frac{P}{S_d} > 0.2$	Ductile	Open ends	Birnie's Equation $t_b = \frac{D_i}{2}\left[\sqrt{\frac{S_d + (1-\mu)P}{S_d - (1+\mu)P}} - 1\right]$	$S_d = 0.8\,S_{yt}$
			Ductile	Closed ends	Clavirano's Equation $t_b = \frac{D_i}{2}\left[\sqrt{\frac{S_d + (1-2\mu)P}{S_d - (1+\mu)P}} - 1\right]$	$S_d = 0.8\,S_{yt}$
3.	Pipes	$L > 8\,D_i$	Any	No effect on design	Barlow's Equation $t_b = \frac{P D_o}{2S_d}$, for steel $t_b = \frac{PD_o}{2S_d} + 6\,\text{mm}$, for C.I.	For Ductile material $S_d = 0.8\,S_{yt}$ For Brittle material $S_d = \frac{S_{ut}}{8}$
4.	Low pressure cylinders for Engines, Pumps etc.		Gray Cast Iron	No effect	$t_b = \frac{PD_i}{2S_d} + 0.75$	$S_d = 10$ MPa

Internal diameter = D_i, mm   Internal pressure = P, N / mm^2
External diameter = D_o. mm

Table X - 2 Flange & Bolt Details

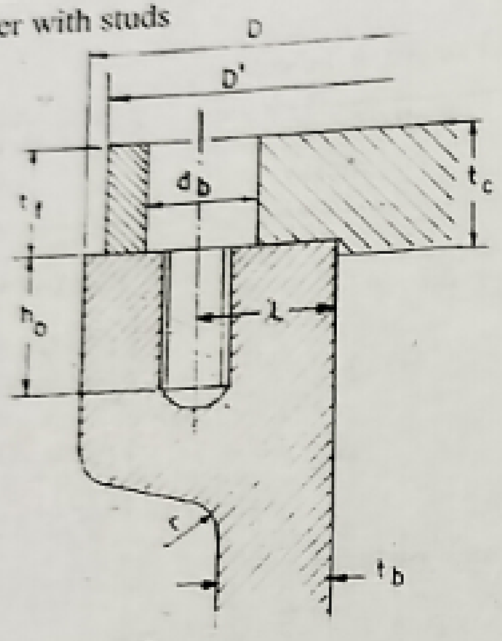
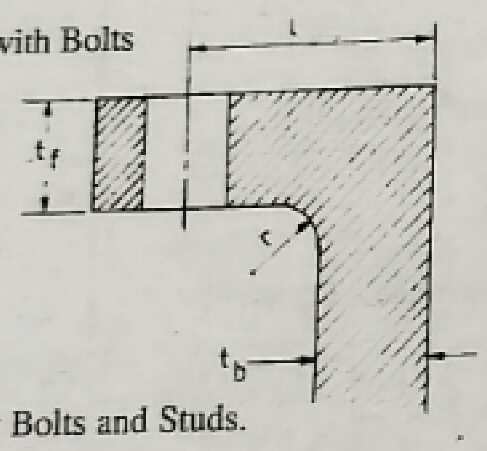
S.No.	Figure	Relation
1.	Cover with studs  D D' d_b t_f t_c h_o l c t_b	$t_f = (1.25 \text{ to } 1.5)\ d_b$ where d_b = Stud diameter, nominal. $L = (1.25 \text{ to } 1.5)\ d_b$ $h_o = (1.5 \text{ to } 1.75)\ d_b$, Cast Iron. $= 1.25\ d_b$, Steel. $= (1.75 \text{ to } 2.0)\ d_b$ Al and other non ferrus materials.
2.	Cover with Bolts 	$l = t_b + d_b + 5$ $l = t_b + 0.5\ d_b + 5$ – Nut spot faced $t_f = (1.25 \text{ to } 1.5)\ d_b$ or $(1.1 \text{ to } 1.25)\ t_b$
3.	Pitch for Bolts and Studs. P mm.	$P = 7\ d_b$ For pressure, < 0.35 MPa $P = 3.5\ d_b$ For Pressure, 1.4 MPa & above. $P = 5\ d_b$ - normally
4.	Initial - Tightening load F_i N/per bolt	$F_i = 3200\ d_b$ -Metal to Metal contact $F_i = 1600\ d_b$ - For soft gasket joint.
5.	Final load, F, N	$F = C\,Fe + F_i$, F_e = External load per bolt C = From table X-3.
6.	Design stress S_d, MPa	$S_d = 1.1\ (d_b^2 + 25) \times 10^{-2}$ For rough joints $S_d = 1.8\ (1.2\ d_b^2 + 25) \times 10^{-2}$ For faced joints For sizes below 20 mm. (Initial tightening considered
7.	Bolt capacity, for external loads only Fe, N	$F_e = \frac{S_{yt}}{150}(A_r)^{3/2}$ for $d_b < 40$ $F_e = \frac{S_{yt}}{4}(A_r)$ for $d_b > 40$ Where, A_r = root area of the bolt, mm^2

Table X - 3
Gasket Factor C

S. No	Type of Gasket and Joint	C	
1.	Soft with studs	1.00	Where, $C = \frac{t A_b E_b}{t A_b E_b + l A_g E_g}$
2.	Soft with Bolts	0.80	
3.	Copper Asbestos	0.60	
4.	Soft Copper corrugated	0.40	Where,
5.	Lead with studs	0.10	t = Gasket thickness
6.	Narrow copper ring	0.01	l = Length of bolt
7.	Metal to metal joint	0.00	A = Area E = Elasticity b & g = Suffixes for Bolt & Gasket, resp.

Table X - 4

Facing & Yielding Width

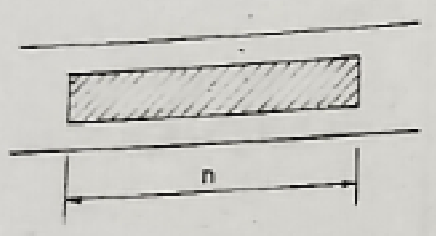
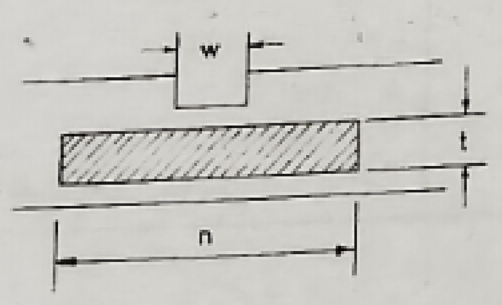
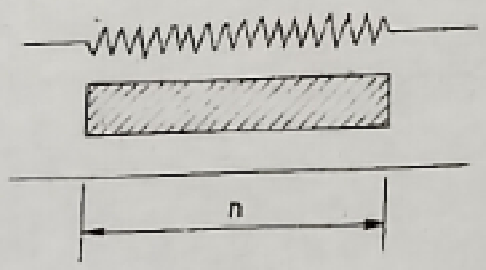
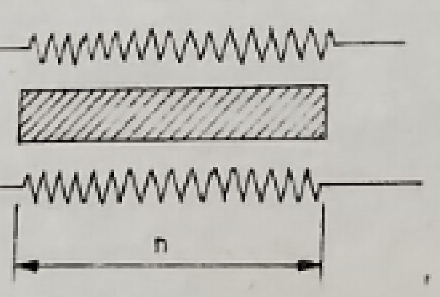
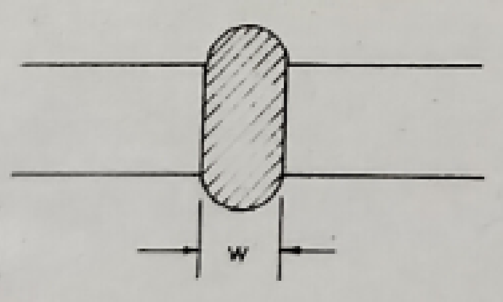
S.No.	Facing sketch	Basic Gasket Yielding Width, b_o Column I	Basic Gasket Yielding Width, b_o Column II
1a.		$\frac{n}{2}$	$\frac{n}{2}$
1b.		$\frac{w+t}{2}\left(\frac{w+n}{4}\right)$	$\frac{w+t}{2}\left(\frac{w+n}{4}\right)$
2.		$\frac{3n}{8}$	$\frac{7n}{16}$
3.		$\frac{n}{4}$	$\frac{3n}{8}$
4.		$\frac{w}{8}$	–

Table X - 5

Design Data For Gasket Selection & Design

S. No.	Gasket Material		Gasket factor m	Yield value, MPa	Facing sketch from table X-4	Yielding width b_o from table X-4 use column.
1.	Gum Rubber Sheet		0.50	0.00	1a	II
2.	Hard Rubber Sheet		1.00	1.30	1b	
3.	Cloth Inserted Soft Rubber		0.75	.35	2	
4.	Cloth Inserted Hard rubber		1.25	3	3	
5.	Vegetable Fibre sheet (Hemp or jute)		1.75	8		
6.	Asbestos composition or Compressed Asbestos	3mm thick	2.00	11	1a	
		1.5 mm thick	2.75	26	1b	
		0.75 mm thick	3.50	46	3	
7.	Corrugated Metal	Soft Aluminium	2.70	26	1a	II
		Soft copper or Brass	3.00	32	1b	
		Iron or Mild steel	3.25	39		
		Monel metal	3.50	46		
		11% Cr Steel	3.75	54		
8.	Solid Metal (3 mm thick)	Soft Aluminimum	4.00	63		
		Soft Copper or Brass	4.75	93		
		Iron or Soft Steel	5.50	130	All	I
		Monel	6.00	155		
		11% Cr Steel	6.50	185		

Notations Used :

$$m = \text{Gasket factor} = \frac{\text{Sealing pressure}}{\text{Internal Pressure}} = \frac{P}{P_i}$$

y = Yield value of Gasket, MPa

b = Effective gasket yielding width.

$= b_o$ for $b_o < 6\text{mm}$

$= 2.5\sqrt{b_o}$ for $b_o > 6\text{mm}$

(Contd..next page)

Table X - 5 (Contd..)

P_c = Minimum contact pressure necessary to form leak-proof joint, MPa

$P_C = P_i\left[\frac{G}{4b} + m\right]$ and $P_C > y$

G = load attack diameter, fig. 10.1 = (O.D. of gasket or P.C.D. of bolts which ever is smaller - 2b)

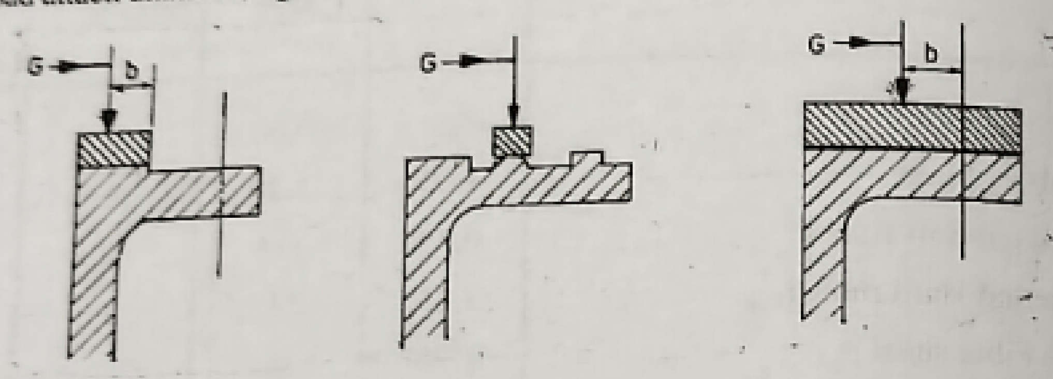


Table X - 6

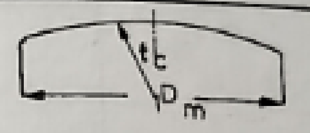
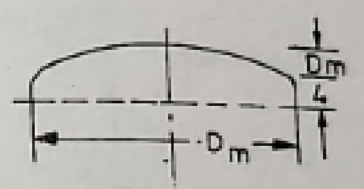
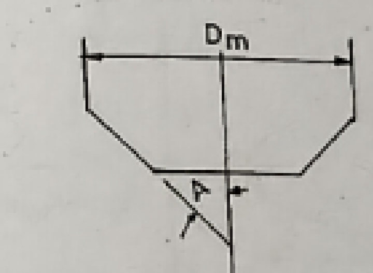
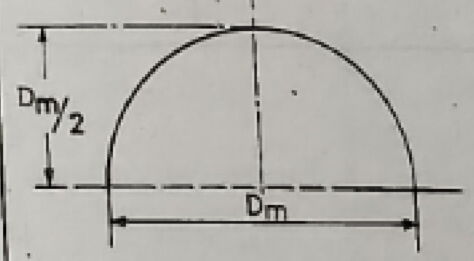
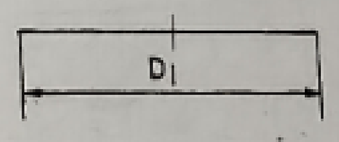
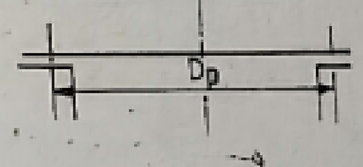
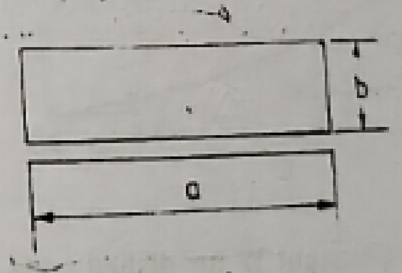
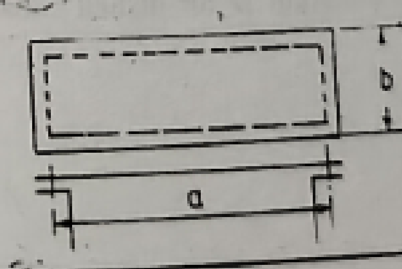
Selection of Gasket Materials

Internal pressure P_i, kgf/cm²	Max. operating Temp.	Suitable material & shape
Upto 10	70° C	Any material, any shape
Upto 20	250° C	Compressed Asbestos Metallic Reinforced Asbestos
Upto 20	370° C	Corrugated Metal Plain Asbestos, Iron, Alluminum, Copper
Above 20	Above 370° C	Plain Metal Gaskets (Pressure is the deciding factor)

Table X - 7

Cover Thickness For Pressure Vessels

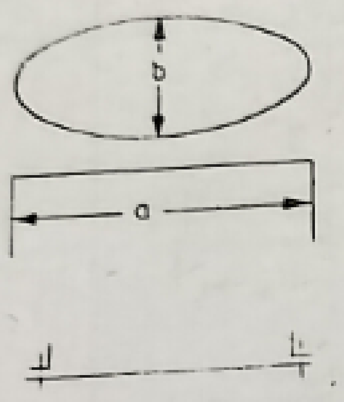
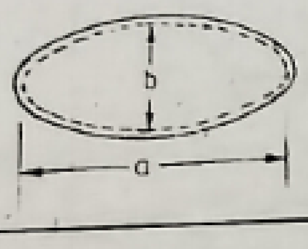
η = efficiency of the joint, Refer Table V-1 & V-7

S. No.	Type of Cover	Figure	Cover Thickness t_c, mm.	Remark
1.	Dished (Integral or Welded)		$t_c = \frac{P_i r_c W}{2 S_d \eta}$	W From Fig. 10.2
2.	Standard Ellipsoidal (Integral or Welded)		$t_c = \frac{P_i D_m}{2 S_d \eta}$	
3.	Conical (Integral or Welded)		$t_c = \frac{P_i D_m}{2 S_d \eta (\text{Cos } A)}$	
4.	Hemispherical (Integral or Welded)		$t_c = \frac{P_i D_m}{4 S_d \eta}$	
5.	Flat Circular Fixed Edges (Welded or Integral)		$t_c = K_1 D_l \sqrt{\frac{P_i}{S_d}}$	K_1 = 0.44 C.I. = 0.35 MS.
	Free Edges (Bolted)		$t_c = \frac{K_2 D_p \sqrt{P_i}}{\sqrt{S_d}}$	K_2 = 0.54 CI. = 0.42 M.S.
6.	Flat Rectangular Fixed Edges (Welded or Integral		$t_c = K_3 ab \sqrt{\frac{P_i}{S_d(a^2+b^2)}}$	K_3 = 0.62 C.I. = 0.49 M.S.
7.	Free Edges (Bolted)		$t_c = K_4 ab \sqrt{\frac{P_i}{S_d(a^2+b^2)}}$	K_4 = 0.75 C.I. = 0.60 M.S.

(Contd..next page)

Table X - 7 (Contd..)

η = efficiency of the joint, Refer V-1 & V-7

S. No.	Type of Cover	Figure	Cover Thickness t_c, mm.	Remark
8.	Elliptical Fixed Edges (Welded or Integral)		$t_c = K_5 ab \sqrt{\frac{P_i}{S_d(a^2+b^2)}}$	K_5 = 1.2 C.I. = 0.9 M.S.
9.	Elliptical Free Edges (Bolted)		$t_c = K_6 ab \sqrt{\frac{P_i}{S_d(a^2+b^2)}}$	K_6 = 1.5 C.I. = 1.2 M.S.

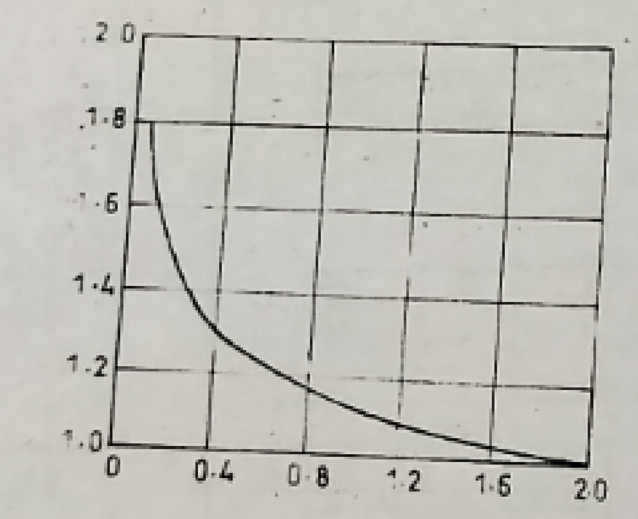


Fig. 10.2 Constant W for dished ends.

Table XI - 1
Design Formulae For Shafts

S.No.	Item		Formula
1.	Design Torque, T_d, N–m		$T_d = \frac{60 P \times K_l}{2 \pi N}$ K_l = Load factor from Table XI - 5
2.	Maximum Stress	Solid shaft	$\tau_{(max)} = \frac{16 \times 10^3}{\pi D^3} \sqrt{(M^2 + T_d^2)}$
		Hollow Shaft $\frac{D_i}{D_o} = K$	$\tau_{(max)} = \frac{16 \times 10^3}{\pi D_o^3 (1-K^4)} \sqrt{(M^2 + T_d^2)}$
		Solid Shaft with axial load F, N	$\tau_{(max)} = \frac{16 \times 10^3}{\pi D^3} \sqrt{T_d^2 + \left(M + \frac{\alpha F D}{8}\right)^2}$ For α, refer table XI - 2
3.	Maximum Stress by A.S.M.E. code	For Hollow Shaft $\frac{D_i}{D_o}$ = K and K = 0 For Solid Shaft	$\tau_{(max)} = \frac{16 \times 10^3}{\pi D_o^3 (1-K^4)} \sqrt{(K_t T)^2 + \left(K_b M + \frac{\alpha F D}{8}\right)^2}$ For α table XI-2 & For K_t, K_b Table XI - 3 $\tau_{(max)} < .30 S_{yt}$ or $< .18 S_{ut}$
4.	Theoretical Stress Concentration Factor due to Key way.		$K_t = 1 + \frac{0.2 w + 1.1 h}{D}$ Where, w = width of keyway h = depth of keyway in the shaft For standard keyway having $w = \frac{D}{4}$ and $h = \frac{D}{8}$ $K_t = 1.19$

(Contd.. next page)

Table XI - 1 (contd...)

S.No.	Item	Formula
5.	Torsional Deflection	$\theta = \frac{32 \times 10^3 TL}{\pi GD^4}$, Radians $\theta = \frac{584 \times 10^3 TL}{GD^4}$, degrees Where, T = torque transmitted, in N-m
6.	Criticial speed N_C-r.p.m. a) DO NOT CONSIDER forces which remain constant irrespective of deflection, such as belt tensions, gear tooth load etc. b) Consider dead weights, magnetic pulls for calculating deflection.	$N_C = \frac{945}{\sqrt{\delta_1 + \delta_2 + \delta_3 + \ldots\ldots + (\delta s / 1.27)}}$ δ_1 = Deflection at F_1 due to force F_1 mm. δ_s = Maximum deflection of shaft due to its own weight, mm.

Table XI - 2
Constant ∝ For Shaft Design

$\frac{\text{Column Length}}{\text{Rad. of gyration}} = \frac{l}{k}$	0	25	50	75	100	115
Constant ∝	1.00	1.12	1.28	1.49	1.78	2.02

Table XI - 3
Constant K_b & K_t For Shaft Design

Nature of loading	K_b	K_t
Stationary Shaft		
i) Gradually applied load	1.0	1.0
ii) Suddenly applied load	1.5 to 2.0	1.5 to 2.0
Rotating Shaft		
i) Gradually applied load	1.5	1.0
ii) Suddenly applied load (Minor shock)	1.5 to 2.0	1.0 to 2.0
iii) Suddenly applied load (Heavy shock)	2.0 to 3.0	1.5 to 3.0

Table XI - 4
Practical Design Considerations For Shafts

1.	Available Sizes mm. (Diameters available)	5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 22, 25, 28, 32, 36, 40, 45, 50, 56, 63, 71, 80, 90, 100, 110, 125, 140, 160, 180, 200.
2.	Materials (Refer Table II-4 & II-5) for properties)	SAE 1015, 1020, 1030, 1040, SAE 3245, Oil quenched. Forged steel { For integral flanges. For hollow shafts.
3.	Permissible transverse deflection, δ - mm	Ordinary Shafts } $\delta \leq 0.75$ mm/m length or $\delta \leq 0.06$ L, mm. where L = load to Bearing distance in meters. Shaft carrying gears, $\delta = \frac{3}{b}$, mm Where b = face width of gear in mm.
4.	Permissible Torsional deflection θ degrees.	Machine Tools $\theta \leq 0.25°$ / m Cam Shafts For I.C. engines } $\theta \leq 0.5°$ Total Normal Use $\theta \leq 1°$ per 20 D or $\theta \leq 2.25°$ per m. length.
5.	Critical speed, N_c R.P.M.	N_c at least 20% away from operating speed.

Table XI - 5
Load Factors For Various Machines - K_l

S.No.	Driving Machinery	Driven Machinery	Load Factor K_l $\frac{\text{Max. Torque}}{\text{Min. Torque}}$
1.	Steam turbines	Turbine blowers, Electric generator, Steady load.	1.00
		Centrifugal pump, Electric motor, uneven load.	1.25
		Induced draft fan, Line shaft with gear drive.	1.50
		Rolling Mill with gear drive	2.00
2.	Electric Motor	Turbine Blower, Lathe, Milling machine, Centrifugal pump, Wood working machine.	1.25 1.50
		Line shafts, ship propellers, Double acting pumps	1.75
		Cranes, Elevators, Triplex pump Single acting.	1.75
		Reciprocating compressors.	1.75
		Rolling mill, Rubber mill	2.20
3.	Steam Engines, Gas & Oil Engines.	(values of K_l for electric motor) × 1.2 - 1.6 depending upon wheel speed fluctuations.	

Table XI - 6
Flywheel Design

S.No.	Item	Relation
1.	Weight required, W, N	$\Delta E = \left[\left(\frac{W}{g} k^2\right) K_S w_m^2\right] \Big/ 100$ Where K_s = Coefficient of speed fluctuation. Table XI - 7. k = Radius of gyration $= \{0.125\,[D_o^2 + (D_o - 2h)^2]\}^{1/2}$ D_o = Outside diameter of flywheel w_m = mean velocity, Radians/second h = Rim thickness b = Rim width
2.	Surface speed V_s	$V_S = \pi D_o N$ $V_s < 1600$ m/min. for cast iron flywheel and $P < 75$ Kw. < 1800 m/min. for cast iron flywheel and $P. > 75$ Kw < 2100 m/min. for carefully C.I. flywheel < 3600 m/min. for cast steel flywheel < 6000 m/min. for forged steel flywheel.
3.	a) Stresses centrifugal, N/m² b) Stress due to bending of rim (due to centrifugal force) Resultant stress	$\sigma_1 = \rho V_s^2$ $\sigma_2 = \frac{\pi^2 V_S^2 \rho D_o}{i^2 h}$ where i = No. of Arms. $\sigma_3 = 0.75\,\sigma_1 + 0.25\,\sigma_2$
4.	Allowable Stresses in Rim	$\left.\begin{array}{l}\sigma_3 = 3.5 \text{ to } 40 \text{ MPa} \\ \sigma_1 = 8 \text{ MPa}\end{array}\right\}$ For Cast Iron $\left.\begin{array}{l}\sigma_3 = 55 \text{ to } 60 \text{ MPa} \\ \text{and} \\ \sigma_1 = 30 \text{ MPa}\end{array}\right\}$ For Cast Steel
5.	Allowable stresses in arms	$\sigma_b = 15$ MPa For cast iron and steady load $= 8$ MPa For cast iron and sudden load. $= 25$ MPa For 'cast steel'

Table XI - 8

Flywheel Construction Details

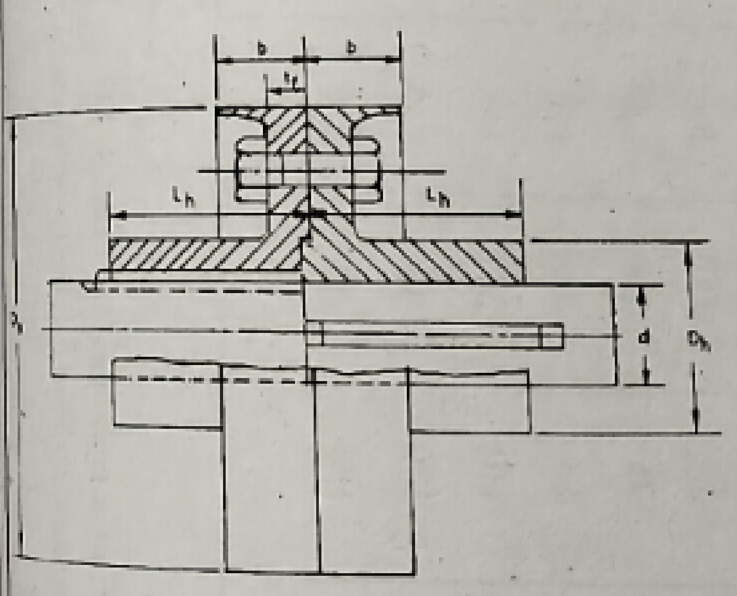
S.No.	Item	Formula
1.	Hub diameter D_h	$D_h = 2d$, where d = shaft diameter
2.	Hub length, L_h	$L_h = (2 \text{ to } 2.5)\ d$
3.	Type of construction i) Arms where $D_o > (7.5\ d + 80)$ mm	Taper 10 to 25% from hub to rim, section elliptical. Usually $\frac{\text{Major Diameter}}{\text{Minor Diameter}} = \frac{a}{b} = 2$ Number of Arms $\left.\right\}$ $i = 4$, $D_o < 400$ mm. $i = 6$, $400 < D_o < 3000$ mm. $i = 8$, $D_o > 3000$.
	ii) Web when $D_o < (7.5\ d + 80)$ mm.	Web Thickness, t_w $t_w = b/20$
	iii) Split flywheel, when $D_o > 2500$ mm.	Consider Joint's relative strength in designing. For Relative strength refer table XI - 9.
4.	Rim proportions	$\frac{b}{h} = 0.65$ to 2.0

Table XII - 1

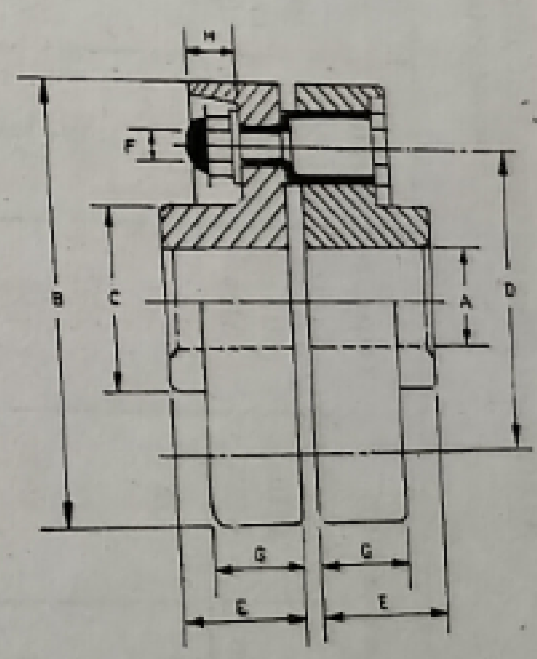
Standard Proportions For Rigid Flanged Couplings

S.No.	Item	Relation
1.	Hub diameter D_h , mm.	$D_h = 1.5\ d + 25$
2.	Hub length L_h , mm.	$L_h = 1.25\ d + 20$
3.	Number of Bolts, n.	$n = 0.02\ d + 3$
4.	Bolt diameter d_b , mm.	$d_b = 0.5 \frac{d}{\sqrt{n}}$
5.	Flange thickness t_f mm.	$t_f = d_b$
6.	Flange Protector width b, mm.	$b = 2.5\ d_b$
7.	Pitch circle diameter (PCD), mm.	$(PCD) = 2\ d + 50$
8.	Outer diameter, D_o	$D_o = 2.5\ d + 75$

In this table d = Shaft diameter, mm.



Rigid flange coupling



Flexible coupling

Table XII - 2

Standard Proportions For Bush Type Cast Iron Flexible Coulings

(Also refer figure on Previous page)

Coupling No.	Bore A mm		Outside dia B. mm, Min	Hub dia. C. E, mm Min	Hub length G, mm Min	Flange width H,mm Min	Bolt recess D, mm	PCD of Bolts	No. of Bolts mm	Bolt dia. F,	Bush dia.mm.
	Min	mm, Max									
B_1	12	16	80	25	28	18	10	53	3	8	20
B_2	16	22	100	30	30	20	12	63	3	10	22
B_3	22	30	112	38	32	22	12	73	3	10	22
B_4	30	45	132	55	40	30	15	90	4	12	25
B_5	45	56	170	80	45	35	15	120	4	12	25
B_6	56	75	200	100	56	40	15	150	4	12	30
B_7	75	85	250	140	63	45	22	190	6	16	40
B_8	85	110	315	180	80	50	22	250	6	16	40
B_9	110	130	400	212	90	56	28	315	8	18	45
B_{10}	130	150	500	280	100	60	28	400	8	18	45

This table assumes :

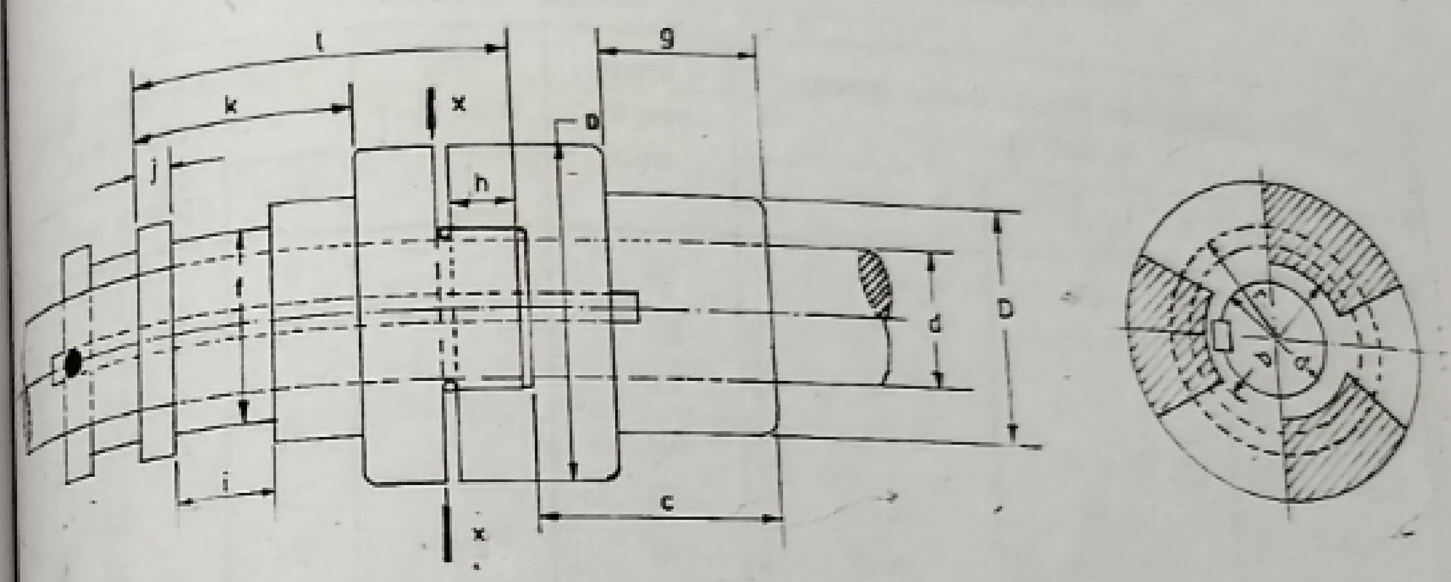
i) Material of coupling, Cast Iron I.S. 20.

ii) Maximum peripheral velocity, 30 meters/sec.

Table XII - 2A

Design of Square Jaw Clutches

A. STANDARD PROPORTIONS



a = (1.8 – 2.5) d

b = (1.5 – 2.1) d

c = (1.3 – 1.7)d

f = 1.4 d + 7.5

g = d + 5.0

h = 0.3 d + 12.5

i = 0.4 d + 6.0

j = 0.2 d + 4.0

k = 1.2 d + 20

l = 1.7 d + 55

B. DESIGN PRACTICE

S.No.	Item	Relation
1.	Number of jaws, n	n = 2, 3 or 6.
2.	Maximum shaft dia. d_{max}	d_{max} = 100 mm for cast jaws. = 250 mm for machined jaws
3.	Maximum speed of engagement	60 r.p.m.
4.	Allowable bearing pressure, P_b, MPa	P_b = 80-120, when engaged at rest = 20-30, when engaged during running.

Table XII - 3

Shaft Diameters of Three Phase Induction Motors

KW Ratings at Synchronous speeds in R.P.M.			Centre height mm	Size Tolerance	Shaft diameter, mm at speed, R.P.M.	
3000 & 1500	1000	750			3000	1500 & below
0.06 & 0.09	-	-	56		9	9
0.12 & 0.18	-	-	63		11	11
0.25 & 0.37	-	-	71		14	14
0.55 & 0.75	0.37 & 0.55	-	80		19	19
1.1	0.75	0.37	90		24	24
1.5	1.1	0.55	90		24	24
2.2	1.5	0.75 & 1.1	100		28	28
3.7	2.2	1.5	112		28	28
5.5	-	2.2	132	-0.5	38	38
7.5	3.7 & 5.5	-	132		38	38
11	7.5	3.7 & 5.5	160		48	48
15 & 18.5	11	7.5	160		48	48
22	15	11	180		55	55
30	18.5	15	180		55	55
37	22	18.5	200		60	60
45	30	22	200		60	60
55	37	30	225		60	65
75	45	37	250		65	75
90	55	45	250		65	75
110	75	55	280		65	80
125	90	75	280		65	80
160	110	90	315	-1.0	70	90
180 & 200	125	110	315		70	90

Note : Type of insulation Class B

Tolerance of shaft upto 28 mm - j6

28 - 38 mm - h6

55 - 90 mm - m6

Table XII - 4

Design Formulae & Practice For Friction Clutches

S.No.	Item	Formulae
1.	Design torque, T_d , N-mm.	$T_d = \frac{(P)_R \times 60 \times 1000\ K_1}{2N\pi}$ Where $(P)_R$ = Rated power K_1 = Load factor, Refer Table XI-5.
2.	Cone Clutch, Design Formulae	Refer Fig. of cone clutch
	i) Torque capacity, T, N-mm	$T = \frac{\mu F_a D_m}{2 \sin\alpha}$ Where, $D_m = \frac{D_o + D_i}{2}$. mm F_a = Axial force in N $= \pi D_m P_b \sin\alpha$ P = Constant pressure, Refer Table XII-6
	ii) Axial force for engagement F'_a	$F_a' = F_a (1 + \mu \cot\alpha)$
3.	Cone Clutch, Design Practice	
	i) Peripherial Speed, V	V = 600 to 1500 m/min for leather faced clutches.
	ii) Operating Speed, N	N = 300 to 1000 r.p.m. for metal to metal contact
	iii) Mean Diameter, D_m	$D_m = (5 \text{ to } 10)\ d_s$
	iv) Width b	$\frac{D_m}{b} = (4.5 \text{ to } 8)$
	v) Semi-cone angle $\propto$	$\propto = 12.5°$ For leather, asbestos, cork facing = 15° to 25° For wood facing
4.	Plate clutches, Design Formulae	
	i) Torque capacity, T, N-mm	$T = \frac{\mu F_a D_m N_f}{2}$ Where, μ = Coefficient of friction refer table XII - 6 F_a = Axial force $= \frac{\pi}{4} P (D_o^2 - D_i^2)$, N P = Contact pressure, refer table XII 6 N = Number of friction surfaces = 2 × (No. of Inner plates)

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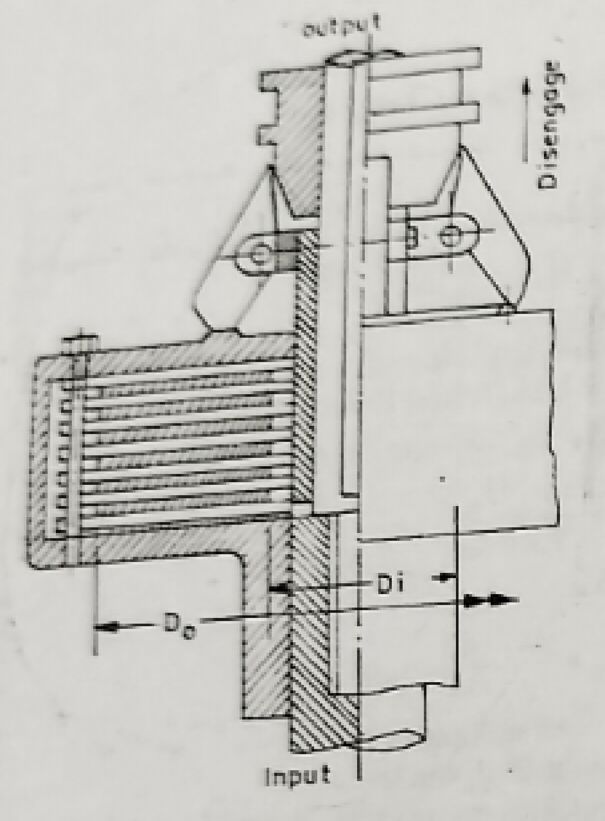
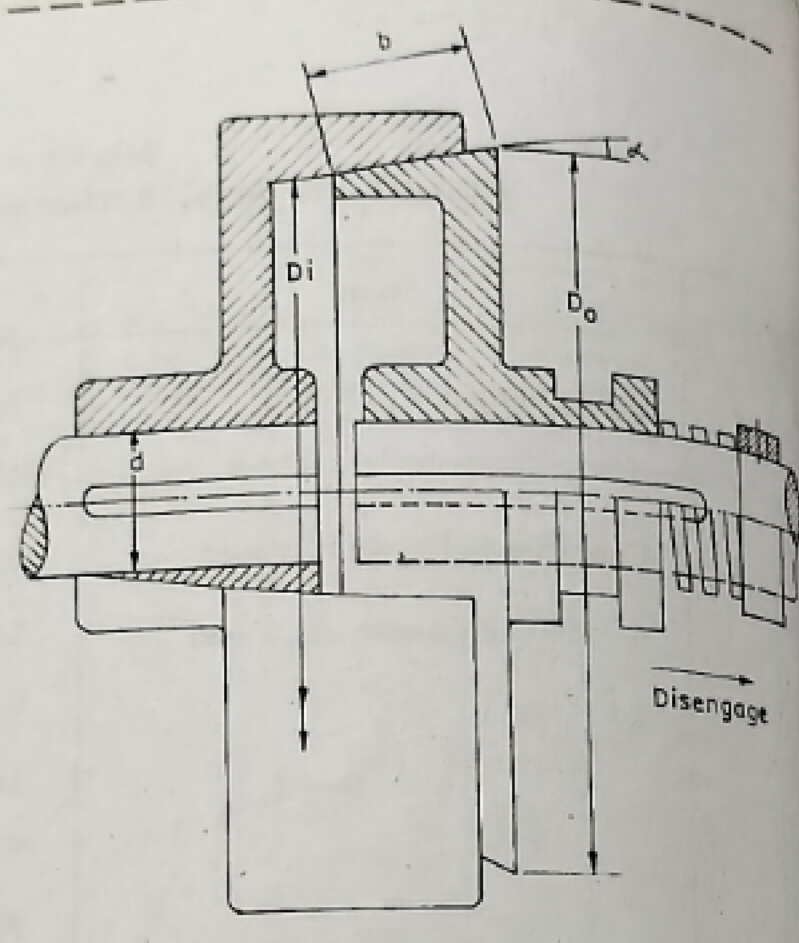
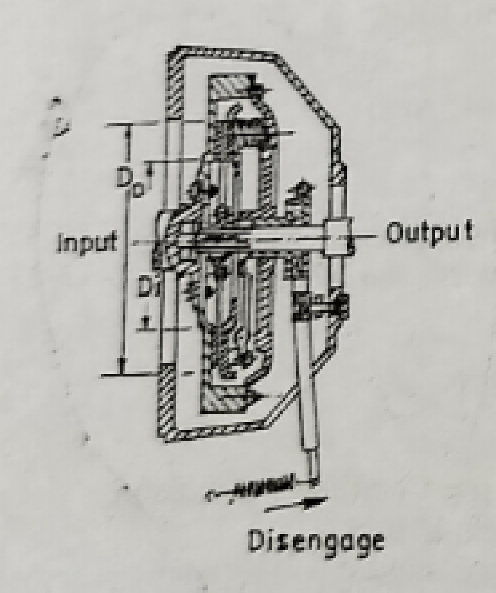


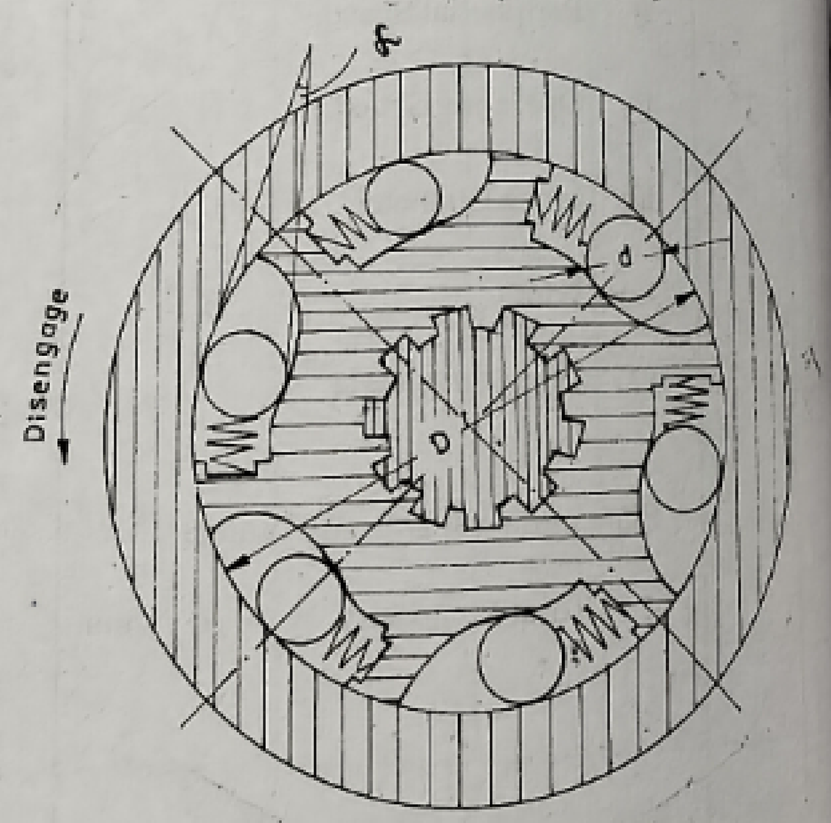
(Multiple Plate Clutch)



(Cone Clutch)



(Single Plate Clutch)



(Roller Clutch)

Table XII - 4 (Contd...)

S.No.	Item	Formulae
5.	Plate clutches, Design Practice	$D_m = (5 \text{ to } 10)\, d_s$
	i) Mean diameter, D_m	$D_m = \frac{D_i + D_o}{D_o^2}$
	ii) Ratio of inner to outer diameter	$\frac{D_i}{D_o} = (0.6 \text{ to } 0.7)$
6.	Roller Clutches, Design Formulae (over-running clutches)	Refer Fig. of roller clutches
	i) Torque capacity, T, N-mm	$T = \frac{1}{2} N_R F_r D \tan \frac{\alpha}{2}$ Where, N_R = No. of Rollers, F_r = Radial compressive force, N D = Rim diameter α = Ramp angle (Angle between tangents at contact points)
	ii) Condition for engagement	$\propto < 2 \tan^{-1} \mu$ Where, μ = coefficient of friction between rollers and rim.
7.	Roller Clutches, Design practice	
	i) Coefficient of friction, μ	$\mu = 0.03 - 0.05$ for hardened and polished surfaces.
	ii) Roller diameter, d	$d = (0.1 \text{ to } 0.25)\, D$

Table XII - 5

Design Formulae & Practice For Brakes

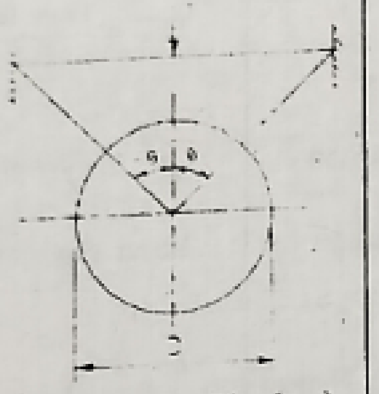
S.No.	Item	Formulae
1.	Block Brakes i) Torque capacity, T (Long shoe) 	$T = \mu F \frac{D}{2}\left[\frac{4\ \mathrm{Sin}\,\theta}{20 + \mathrm{Sin}\,\theta}\right]$
2.	Cone and Plate Brakes	Design-formulae same as clutches.
3.	Brakes Design Practice (for all types of brakes) i) P . V . limit where, P = Pressure, MPa V = Peripheral speed, m/min.	PV < 55 - For continuous operation and poor heat dissipation. PV < 110 - For intermittant operation and poor heat dissipation. PV < 165 - Continuous operation & good heat dissipation as in oil bath.
	ii) Shoe width b, mm	$b < \frac{D}{4}$, where D = Drum diameter, mm.
	ii) Maximum temperature for lining.	250° C Maximum } For asbestos on Cast iron or steel. 170° C Normal 60° C for wood or leather onsteel or Cast Iron.

Table XII - 6

Properties of Friction Lining Materials For Clutches and Brakes

S. No.	Materials		Coefficient of friction		Max. Temp °C	Max. Pressure MPa	Relative Cost	Remarks
	Lining	Drum	Wet	Dry				
1.	Bronze	Cast Iron or Steel	0.05	-	130	6	Low	Can sieze due to local heating
2.	Cast Iron	Cast Iron	0.05	0.15 - 0.2	200	1-1.6	Very low	Good at low speed
3.	Cast Iron	Steel	0.06	-	250	.9-1.2	Very low	Fair at low speed
4.	Steel	Steel	0.05	-	250	0.8	Moderate	Prone to galling
5.	Wood	Cast Iron or Steel	0.16	0.2-0.35	180	.4-.6	Lowest	Not suitable at high speed.
6.	Leather	Cast Iron or Steel	0.12-0.15	0.3-0.5	95	0.7-3	Very low	Prone to glazing there by reducing friction.
7.	Woven Asbestos	Steel	0.1-0.2	-	250	.7-1.5	Low	Good for short engagements.
8.	Impregnated Asbestos	Cast Iron or Steel	0.12	0.32	300	1.0	Moderate	For demanding applications.
9.	Carbon graphite	Steel	0.05-0.1	0.25	450	2.0	High	For critical requirements.
10.	Cork	Cast Iron or Steel	0.12-0.15	0.3-0.5	90	0.6-1	Very low	Low speed, light duty.
11.	Felt	Cast Iron or Steel	0.15-0.25	0.3-0.5	90	.03-06	Low	Low speed, light duty.

Table XIII - 1

Design Formulae For Journal Bearings

S.No.	Item	Relation
1.	Journal diameter, d mm	$d = 2R$ Where R = radius of journal, mm.
2.	Journal length, l mm	For $\frac{l}{d}$ ratio refer table XIII-4.
3.	Bearing pressure, P, MPa	$P = \frac{W}{l \times d}$ Where, W = total load on the shaft, N For allowable P, refer table XIII-4
4.	Radial clearance, C mm	$C = (kR + 0.002)$ mm For k refer table XIII-6
5.	Absolute viscosity Z, mPa	$Z = \rho_t \times Z_k \times 10^{-3}$ Where, ρ_t = density at any temp. t° C $= \rho_{15} - 0.00064\ (t-15)$ ρ_{15} = density at 15° c, refer table XIII-8 Z_K = Kinematic viscosity. m²/s refer table XIII-8
6.	Sommerfield number, S	$S = \left(\frac{Zn}{P}\right)\left(\frac{R}{C}\right)^2 \times 10^{-9}$ Where, n = speed of journal, revolutions per second.
7.	Test for type of lubrication	Refer fig. 13.1 In fig. load no. NL $NL = \frac{P}{Zn}\left(\frac{d}{L}\right)^2 10^9 \text{ for } \frac{l}{d} \leq 1$ $= \frac{P}{Zn} \times 10^9 \text{ for } \frac{l}{d} > 1$
8.	Coefficient of friction, for thick film lubrication	i) Refer fig. 13.3 ii) Approximate formulae $= .326\left(\frac{Zn}{P}\right)\left(\frac{R}{C}\right) \times 10^{-9} + f$ Where $f = 0.002$ for $0.75 < \frac{l}{d} < 2$

(Contd..next page)

Table XIII - 1 (Contd...)

S.No.	Item	Relation
9.	Coefficient of friction, for marginal lubrioation.	$= \frac{C_1 C_2}{306}\sqrt{\frac{P}{V}}$ Where C_1 = Constant, refer table XIII-2 C_2 = Constant, refer table XIII-3 V = Perpheral speed of journal, m/sec.
10.	Minimum film thickness. h_m, mm	Refer table XIII - 5
11.	P V limits for various bearings	Refer table - XIII- 4
12.	Heat generated, Hg, watts	$Hg = \mu W V$
13.	Heat dissipated, H_d, watts	$H_d = K A (t_b - t_a + 18)^2$ Where, K = Constant = 2.06 W/m² °C for still air = 3.67 W/m², °C for good ventilation. t_b = Housing temp, °C , t_a = Ambient temp, °C A = Projection area of journal, m²
14.	Oil temperature, $t_{(oil)}$ °C	$t_{(oil)} = 2(t_b - t_a) + t_a$

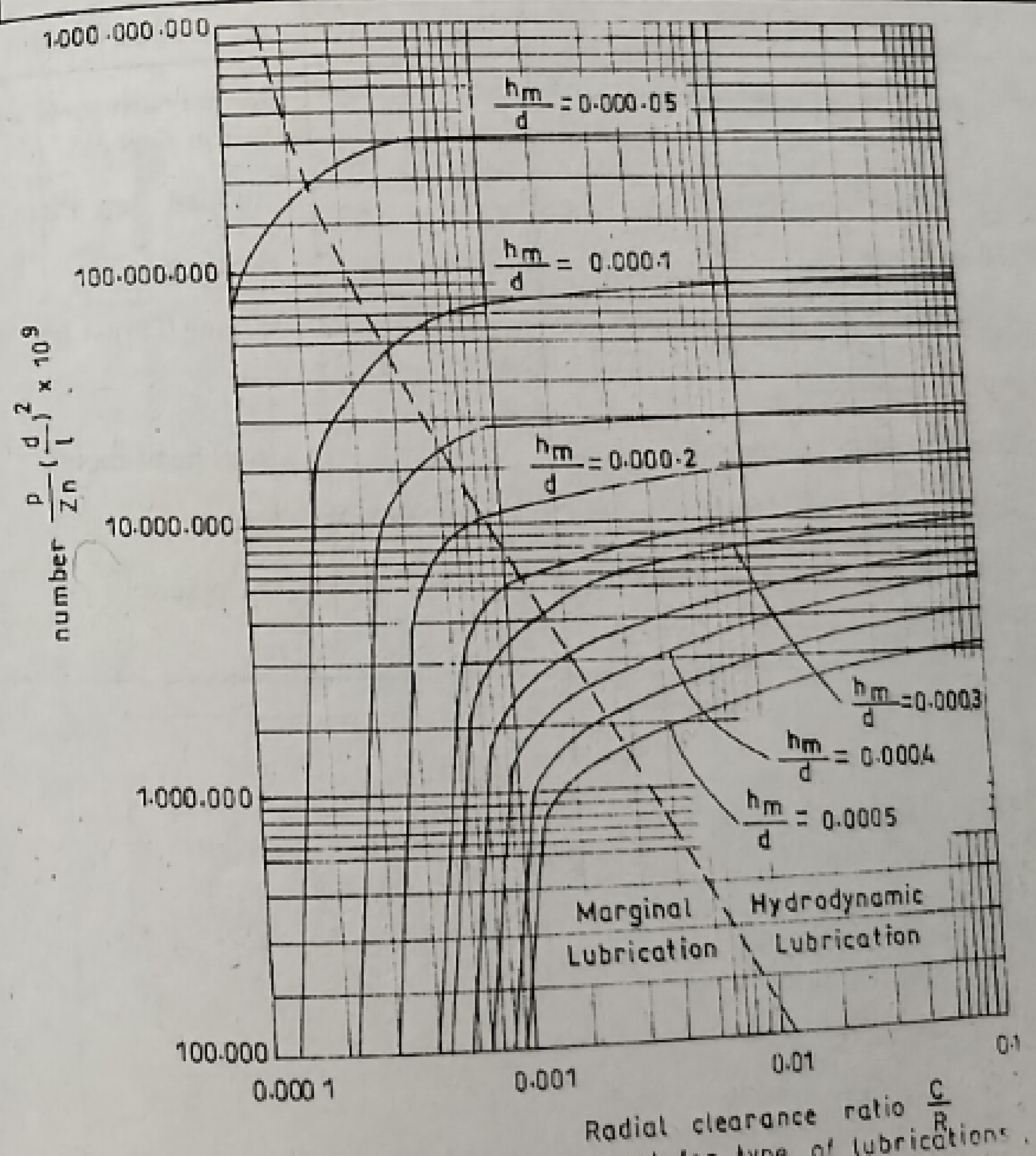


Fig. 13-1. Test chart for type of lubrications.

Table XIII - 2
Constant C_1 In Table XIII - 1

S.No.	Type of lubrication	C_1
1.	Oil Bath. Good workmanship. Good maintainance, Clean and protected bearing.	1
2.	Drop feed continuous lubrication. Ordinary conditions	2
3.	Intermittent lubrication. Poor maintainance, Exposed to dirt.	4

Table XIII - 3
Constant C_2 In Table XIII-1

S.No.	Bearing Type	C_2	Application
1.	Rotating or oscillating journals	1	Main bearings, Crankpins Piston pins etc.
2.	Rotating flat surface, lubricated		Annular step, Pivot
	i) From centre	2	Bearings
	ii) Lubricated from outer periphery	3-4	Marine Thrust bearing.
3.	Reciprocating surfaces		
	i) Long (length > 2.5 breadth)	2	Cross head shoes
	ii) Short	3	Guides
4.	Long surfaces, Not having uniform Pressure distribution	4-6	Power screws etc.

Table XIII - 4
Design Data For Journal Bearings

S. No.	Machinery	Bearing	Bearing pressure P mPa	Suitable viscosity Z mPa-s	Minimum $\frac{Zn}{P}$	$\frac{c}{R}$	$\frac{l}{d}$	Maximum (P × V) MPa × m/min.
1.	Transmission Shafting	Light fixed	1.5	25-60	230	0.001	2.0 - 3.0	40
		Self aligning	1.5	25 - 60	70	0.001	2.5 - 4.0	40 - 120
		Heavy	1.5	25-60	70	0.001	2.0 - 3.0	40 - 120
2.	Machine Tools	Main	2	25 - 60	80	0.001	1.0 - 4.0	5 - 200
3.	Punching and Shearing M/c	Main	30	100	-	0.001	1.0 - 2.0	80 - 240
		Crank-pin	60	100	-	0.001	1.0 - 2.5	160 - 480
4.	Aircraft, Automobile Engines	Main	5.5-12	7 - 8	3.5	-	0.8 - 1.8	1200 - 10000
		Crank-pin	10 - 5.25		2.3	-	0.7 - 1.4	1200 - 10000
		Piston pin	16 - 35		2.0	-	1.5 - 2.2	-
5.	Gas & oil Engines 4 strokes	Main	5 - 8.5	20 - 65	4.6	0.001	0.6 - 2.0	1200 - 10000
		Crank pin	10 - 13		23	0.001	0.6 - 1.5	1200 - 10000
		Piston pin	13-15		11	0.001	1.5 - 2.0	-
6.	Gas & Oil Engines 2 strokes	Main	3.5 - 5.5	20 - 65	59	0.001	0.6 - 2.0	1200 - 10000
		Crank pin	7 - 10.5		29	0.001	0.6 - 1.5	1200 - 10000
		Piston pin	8.5 - 13		23	0.001	1.5 - 2.0	-

Table - XIII - 4 (Contd...)

S. No.	Machinery	Bearing	Bearing pressure P mPa	Suitable viscosity Z mPa-S	Minimum $\frac{Zn}{P}$	$\frac{c}{R}$	$\frac{l}{d}$	Maximum (P × V) mPa × m/min.
7.	Marine steam engines	Main	3.5	30	46	0.001	0.7-1.5	80-400
		Crank pin	4	40	35	0.001	0.7-1.2	80-400
		Piston Pin	10.5	30	23	0.001	1.2-1.7	-
8.	Steam Locomotives	Main	4	100	70	0.001	1.6-18	220
		Crank pin	15	40	11	0.001	0.7-1.1	280
		Piston pin	30	30	11	0.001	0.8-1.3	-
9.	Railway cars	Axles	3	100	110	0.001	1.8-2.0	200-300
10.	Steam Turbines	Main	0.7-2	2-16	23	0.001	1.0-20	400-4000
11.	Reciprocating pump and compresses	Main	2	30-80	70	0.001	1.0-2.2	40-160
		Crank pin	4		40	0.001	0.9-1.7	40-160
		Piston pin	7		23	0.001	1.5-2.0	-
12.	Generator, Motors centrifugal pumps	Rotor	0.7-1.4	25	46	0.001	1.0-2.0	100-180
13.	Gyroscope	Rotor	6	30	130	0.0013	-	-
14.	Rolling Mill	Roll Necks	10-20	50	25	0.0015	1.1-1.5	320-8000

Table XIII - 6

Bearing Materials & Their Characteristics

S.No.	Bearing Material	Composition in %		Recommended values			K (constant) in radial clearance crelation table XIII-1	Fatigue Strength	Remarks & uses
				Max.p MPa	Min.	Min Sommer fled No. S. (dimen-nsions			
1.	Lead Base Babbits	Pb 75.0 Sb 15.0 Sn 10.0	without steel back	4-6	45	.05	0.0005	Bad	Light pressure, High speed
			with steel back	15	23	.025	0.0005	Medium	With steel back for moderate duty in automobiles, Electric motors.
2.	Tin Base Babbits	Sn 83.4 Sb 8.3 Cu 8.3		5-7	60	.06	0.0005	Medium with steel backing	Lining with steel or brass back. For light & medium loads for automobiles, diesel engines, turbines pumps.
3.	Cadmium Base	Cd 98.6 Ni 1.4		13-27	9	.009	0.0008	Medium with steel backing	Very thin lining on steel back for heavy loads, needs hard shaft, good for intermittent lubrication.
4.	Copper lead alloys	Cu 61.5 Zn 35.5 Pb 3.0		4.5	12	.12	0.001	High	General purpose material for brasses & bushes.
5.	Bronzes	Cu 80.0 Sn 10.0 Pb 10.0		55	120	.12	0.001	High	General purpose material for bushes. Good for less maintenance & expose to dirt.

Table XIII - 9

Design of Flat Plate Thrust Bearings (Collar Type)

S.No.	Item	Relation
1.	Ratio of diameters	$\frac{D_o}{D_i} < 1.5$ Where, D_o = Outside diameter of collar D_i = Inside diameter of collar
2.	Allowable pressure P, MPa i) For single collar bearings.	$P\,Vm < .72$, for $0.25 < Vm < 1.0$ Where, Vm = Mean peripheral velocity, m/sec $P = .7$ for $Vm > 1.0$ P_{max} = 14'0 for very low speed.
	ii) Multicollar bearings	If watercooled, use same values as for single collar bearing. If uncooled, divide the value of single collar bearings by 2. For calculating P use only 80 percent of available area,because rest is required for oil grooves.
3.	Coefficient of friction, μ	$\mu = \frac{.008\sqrt{V_m}}{P^{0.66}}$
4.	Power lost in friction $(P)_F$, W	$(P)_F = \mu\, W\, Vm$
5.	Heat dissipated, H_d	Same relations as given in the Table XIII-1.

Table XIII-11 Design of Titled Pad Thrust Bearings

S.No.	Item	Relation
1. emp.	Load carrying capacity, W,N	$W = \frac{6ZULB^2}{h_2^2} \times 10^{-6} \times K_w \eta_w$ Where, Z = Viscosity of lubricating oil at operating mPa-s, U = Peripheral velocity, m/sec. $\left.\begin{matrix} L = \\ B = \end{matrix}\right\}$ Refer. Fig. 13.9, mm h_2 = minimum oil film thikness, mm. K_w & η_w = constants, refer fig. 13.9.
2.	Coefficient of friction,	$\mu = \frac{h_2}{n_f B} K_f$, Where, n_f & K_f = constants refer fig. 13.9
3.	Film thickness ratio, m	$m = \frac{h_1}{h_2} - 1$ Where, h_1 = oil film thickness at entry, mm.

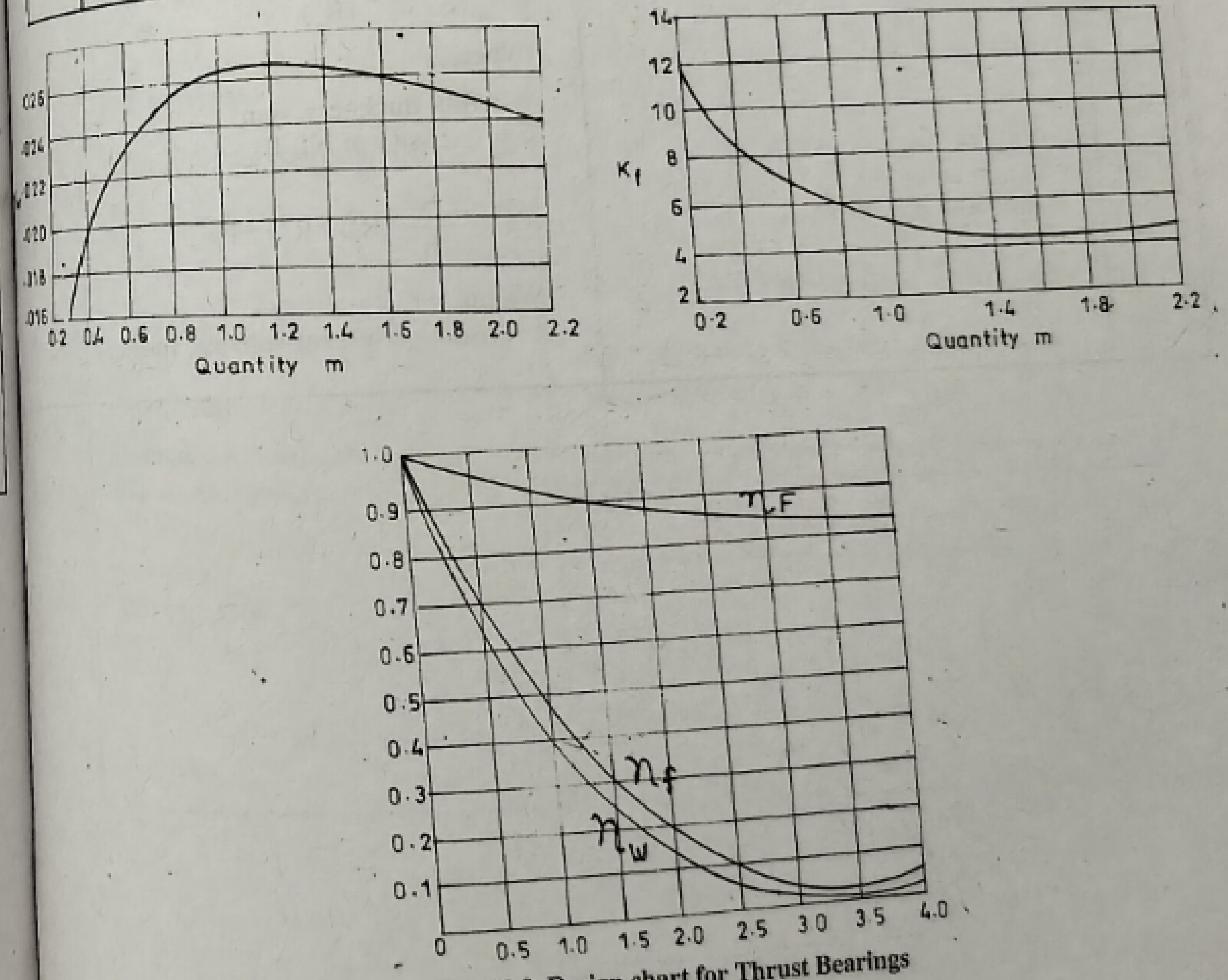


Fig. 13.9 Design chart for Thrust Bearings

Table XIII - 12

Design of Externally Pressurised Thrust Bearings

(Pocket Type)

S.No.	Item	Relation
1.	Pressure in the pocket, P_p, MPa	$$P_P = \frac{2\,W \log \frac{(R_o)}{(R_i)}}{(R_o^2 - R_i^2)}$$ Where, W = load on the bearing, R_o = outside radius of pocket, mm R_i = inside radius of pocket, mm.
2.	Oil flow from the pocket Q, m^3/sec.	$$Q = \frac{\pi P_P h^3}{6 z \log \frac{(R_o)}{(R_i)}}$$ Where, h = film thickness, mm. z = viscosity of oil, Pa - s
3.	Power loss, P_f , W	$$P_f = \frac{\pi z v^2}{2h}(R_o^2 - R_i^2) * 10^{-3}$$ Where, v = mean peripheral velocity, m/sec.

Table XIII - 13

Formulae For Rolling Element Bearings

<table>
<tr><th>S.No.</th><th>Item</th><th>Relation</th></tr>
<tr><td>1.</td><td>Equivalent load coming on bearing, Fe, N</td><td>$Fe = (XF_r + YF_a) K_s K_o K_p K_r$
Where, F_r = Radial load, N
F_a = Axial load, N
X,Y = Constants, refer table XIII-14.
$K_s K_o K_p K_r$ = Constants, refer table XIII-15</td></tr>
<tr><td>2.</td><td>Mean load, F_m ,N, on bearing when load is changing.</td><td>$$F_m = \left(\frac{F_1^3 t_1 + F_2^3 t_2 + \ldots\ldots}{\Sigma t}\right)^{1/3}$$
$$F_m = \left(\frac{F_1^3 N_1 + F_2^3 N_2 + \ldots\ldots}{\Sigma N}\right)^{1/3}$$
Where, F_1 , F_2 loads acting, N
t_1 , t_2 = hours or N_1 , N_2 = revoluations</td></tr>
<tr><td>3.</td><td>Life of bearing
L (in millon revolu-tions)
life)</td><td>$$L = \left(\frac{C}{Fe}\right)^n K_{rel}$$
Where, C = Dynamic load capacity, N, refer table
(Refer table XIII-17 for desirable XIII-21.
n = 3 for Ball bearings.
$= \frac{10}{3}$ for Roller bearings
K_{rel} = Reliablity factor table XIII-15</td></tr>
<tr><td>4.</td><td>Coefficient of friction, μ</td><td>Refer table XIII-16.</td></tr>
<tr><td>5.</td><td>Decrease in tabulated load capacity due to temprature</td><td><table><tr><td>Temp °C</td><td>125</td><td>150</td><td>175</td><td>200</td><td>225</td></tr><tr><td>Decrease %</td><td>5</td><td>10</td><td>15</td><td>25</td><td>35</td></tr></table></td></tr>
<tr><td>6.</td><td>Bearing Fits.</td><td>For
Shaft, stationary g 6
Shaft, rotating k 5
Housing, stationary H 7
Housing, rotating M7</td></tr>
</table>

Table XIII-14

Factors X and Y For Rolling Element Bearings

S.No.	Bearing Type	e	$\frac{F_a}{F_r} \leq e$		$\frac{F_a}{F_r} > e$	
			X	Y	X	Y
1.	Deep grove ball bearings	0.25	1	0	0.56	1.6
2.	Angular contact ball bearings	1.14	1	0	0.35	0.57
3.	Self aligning ball bearings	0.65	1	2.3	0.65	2.8
4.	Spherical roller bearings	0.30	1	2.5	0.67	3.5
5.	Taper roller bearings	0.35	1	0	0.40	2.0

For exact values refer manufacturers catalogue.

Table XIII-15

Various Factors in Bearing Load Calculation

S.No.	Item	Constant	
1.	Oscillation factor, K_o		
	i) Constant rotational speed of races	1.0	
	ii) Sinusoidal oscillations of the races	0.67	
	iii) Other non constant rotational conditions	0.5 - 1.5	
2.	Preloading factor, K_p		
	i) Non preloaded bearings	1.0	
	ii) Preloaded bearings	1.05 - 1.2	
3.	Rotational factor, K_r		
	i) Outer race fixed Inner race rotating	1.0	
	ii) Outer race rotating Inner race fixed	1.4 - 1.6	
	iii) Outer & Inner races rotating in opposite direction.	2.0	
4.	Service factor, K_s	Ball bearings	Roller bearings
	i) Uniform & steady load	1.0	1.0
	ii) Light shock' load	1.5	1.0
	iii) Moderate shock load	2.0	1.3
	iv) Heavy shock load	2.5	1.7
5.	Reliability factor, K_{ref}		

Reliability %	K_{ref}	Reliability %	K_{ref}	Reliability %	K_{ref}
90	1.0	60	4.0	30	8.0
80	2.0	50	5.0	20	10.5
70	2.9	40	6.4	10	14.5

Table XIII - 18

Design Formulae For Needle Bearings

S.No.	Item	Relation
1.	Load capacity, F, N	$F = K_h K_L P. L. D.$ Where, L = length of needles, mm D = Diameter of inner race, mm K_h , K_l , P = as defined below
2.	Hardness factor, K_h	$K_h = 1 - \frac{63-(R_C)}{4000}$ Where, R_C = Rock well hardness of race way.
3.	Life load factor, K_l	$K_l = \frac{14}{\sqrt[3]{L_h}}$ Where, L_h = Expected life, hours.
4.	Allowable pressure, P, MPa	$P = \frac{110}{\sqrt[3]{D_1 N}}$ Where, D_1 = Diameter of revolving race, mm. N = Revolutions per minute. P = 35 MPa for oscillating bearings.

Table XIII - 19

Design Practice For Needle Bearings

S.No.	Item	Practice		
1.	Needle diameters for various shafts	Shaft diameter, mm	Needle diameter, mm	Radial clearance, mm.
		10 - 20	1.5	0.0125 - 0.04
		20 - 30	2.0	0.0175 - 0.05
		30 - 50	3.0	0.0200 - 0.055
		50 - 75	3.0	0.0250 - 0.0650
		75 - 125	4.0	0.0300 - 0.0750
		125 - 175	4.0	0.0350 - 0.850
2.	Available needle lengths	Needle diameter, mm	Available needle lengths, mm	
		1.5	9, 12, 15	
		2.0	10, 20, 25	
		3.0	20, 25, 28	
		4.0	20	
		5.0	12, 20, 25, 30, 35	
		7.0	30	

Table XIII - 21

Specific Dynamic Capacities of Ball & Roller Bearings

S. No.	Bore		Deep groove Ball Bearing			Cylindrical Roller bearings		
	No.	mm	01 xx	02xx	03xx	04xx	N2xx	N3xx
1.	00	10	3600	4000	6300	-	-	-
2.	01	12	4000	5400	7650	-	-	-
3.	02	15	4400	6100	8800	-	-	-
4.	03	17	4650	7500	10600	18000	-	-
5.	04	20	7350	10000	12500	24000	-	-
6.	05	25	7800	11000	16600	28000	11100	23600
7.	06	30	10400	15300	22000	33500	18300	30800
8.	07	35	12500	20000	26000	43000	26500	39900
9.	08	40	13200	22800	32000	50000	34700	47100
10.	09	45	16300	25500	41500	60000	36200	64800
11.	10	50	17000	27500	48000	68000	37600	77000
12.	11	55	22000	34000	56000	78000	46000	94200
13.	12	60	22800	40500	64000	85000	56600	10.5100
14.	13	65	24000	44000	72000	93000	66100	117800
15.	14	70	30000	48000	81500	112000	66100	138200
16.	15	75	31000	52000	90000	1,20,000	81500	165300
17.	16	80	37500	57000	96500	127000	88800	165300
18.	17	85	39000	65000	104000	134000	101500	199300
19.	18	90	45500	75000	112000	146000	129100	215200
20.	19	95	47500	85000	120000	160000	142700	244600
21.	20	100	47500	96500	137000	176000	155300	285400
22.	21	105	54300	102400	145000	193400	-	-
23.	22	110	63400	-	145000	193400	-	-
24.	24	120	66100	-	-	-	226500	444000
25.	26	130	79700	-	-	-	-	525000
26.	28	140	86000	-	-	-	-	584000
27.	30	150	97800	-	-	-	-	634000

Table XIV - 1

Design Formulae For Roller Chain Drives

S.No.	Item	Formula
1.	Design power $(P)_d$	$(P)_d = (P)_R \; K_l$ Where, $(P)_R$ = Rated power K_l = load factor (refer Table XIV - 2)
2.	Tooth load, F_t, N	$F_t = \dfrac{(P)_d}{V}$ Where, V = Pitch line velocity, m/sec.
3.	Power capacity of roller chain as per A.S.A. standards, W	
	i) Exact equation	$P = p^2 \left[\dfrac{V}{104} - \dfrac{V^{1.41}}{526}\left(26 - 25 \times \cos\dfrac{180}{T}\right)\right] 10^3$ Where, p = chain pitch. mm T = No. of teeth on smaller sprocket.
	ii) Approximate equation	$F_t < p^2$ for $V > 1.25$ m/sec.
	iii) Exact charts	Refer Fig. 14.1
4.	Length of chain in pitches, Lp	$Lp = \dfrac{T_1 + T_2}{2} + \dfrac{2C}{p} + \dfrac{p\,(T_1 - T_2)^2}{40\,C}$ Where, C = centre distance, mm T_1, T_2 = no. of teeth on sprockets
5.	Pitch diameter of sprocket, Dp	$Dp = \dfrac{P}{\text{Sin}\left(\dfrac{180}{T}\right)}$
6.	Recommend Wear load, Fw, N	$Fw = (0.35 \times p^2)$ to be suitably modified for velocity effects.

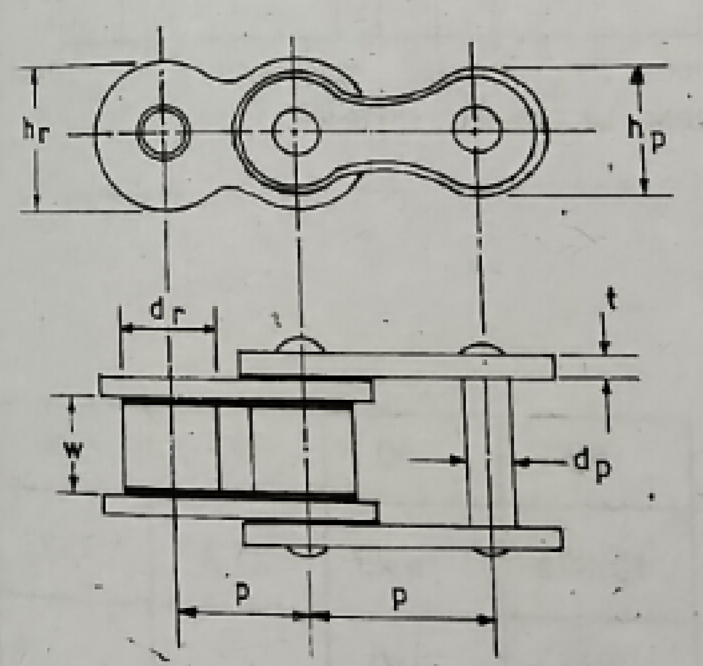
Table XIV - 2

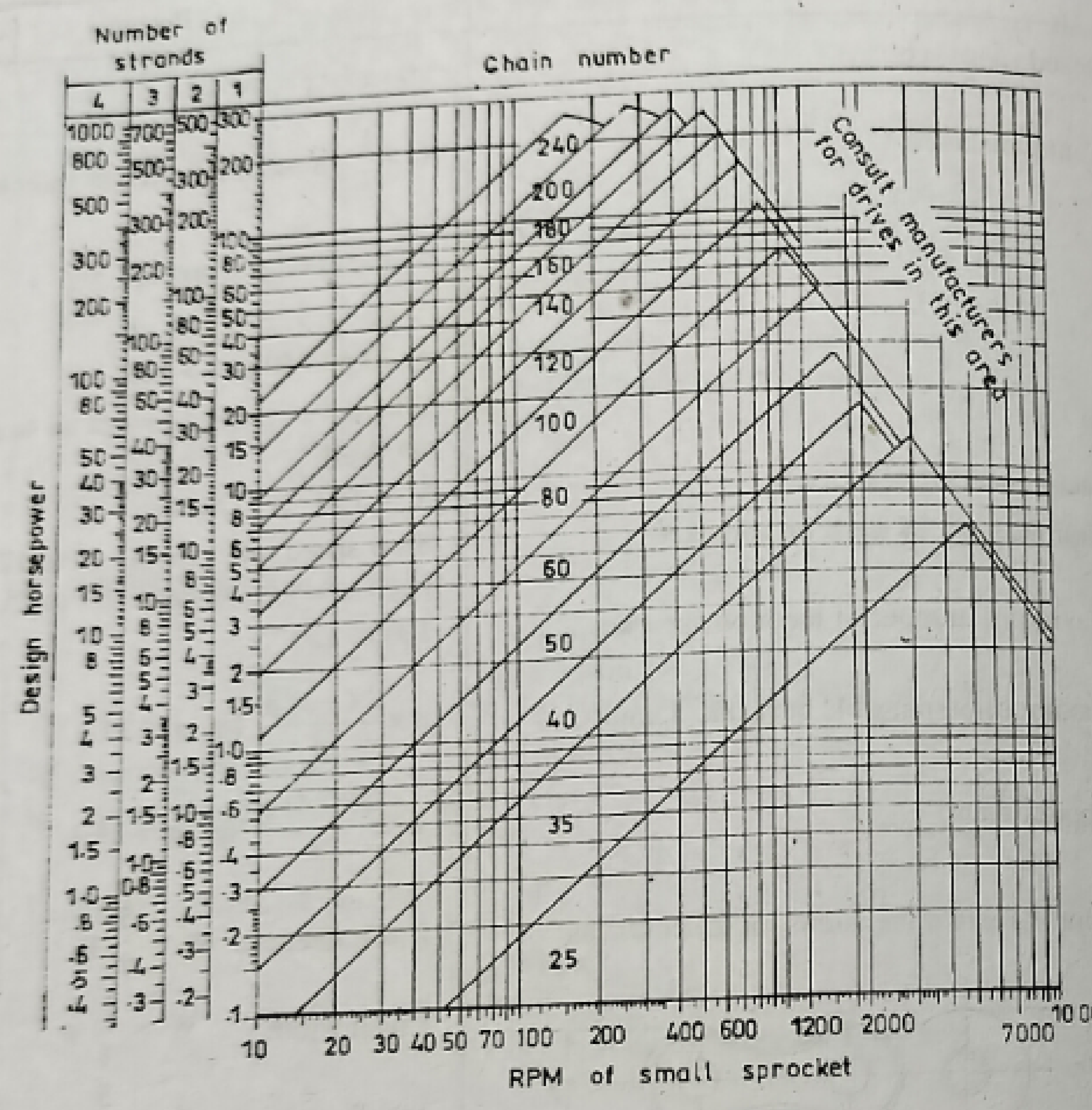
Load Factor K_l For Chain Drives

S.No.	Type of load	Service	
		10 Hrs-day	24 Hrs-day
1.	Uniform	1.0	1.2
2.	Moderate shock	1.2	1.4
3.	Heavy shock	1.4	1.7

Table XIV - 3

Design Practice For Roller Chain Drives

S.No.	Item	Practice
1.	Speed ratio	Maximum upto 10 : 1
2.	Centre distance, C	$C > \dfrac{1}{2}$ (Sum of the O.D. of sprockets) = Dia. of larger sprocket + $\dfrac{1}{2}$ dia. of smaller sprocket = 30 p to 50 p, $\theta > 120°$. Where, θ = ange of contact on smaller sprocket.
3.	Recommended velocity, V	$V = 150 - 450$ m/min.
4.	Minimum no. of teeth on sprocket	Slow speed roller chain - 12 to 17. High speed roller chain - 17 to 25.
5.	Maximum number of teeth on sprocket	100 to 120
6.	Maximum permissible bore dia. d, mm (approximate)	$d < \dfrac{(T-5)\,p}{4}$ for $p \leq 25.4$ $d < \dfrac{(T-4)\,p}{4}$ for $p > 25.4$
7.	Approximate dimensions for roller chains 	Roller diameter $d_r = \dfrac{5}{8}\,p$ Chain width $W = \dfrac{5}{8}\,p$ Pin diameter $d_p = \dfrac{5}{16}\,p$ Thickness of link plate $= \dfrac{1}{8}\,p$ Max. height of pin link plates $H_p = 0.82\,p$ Max. height of roller link plate $H_p = 0.95\,p$
8.	Standard pitches used, p mm	6.25, 8, 9.525, 12.7, 15-875, 18.75, 25.4, 31.75.

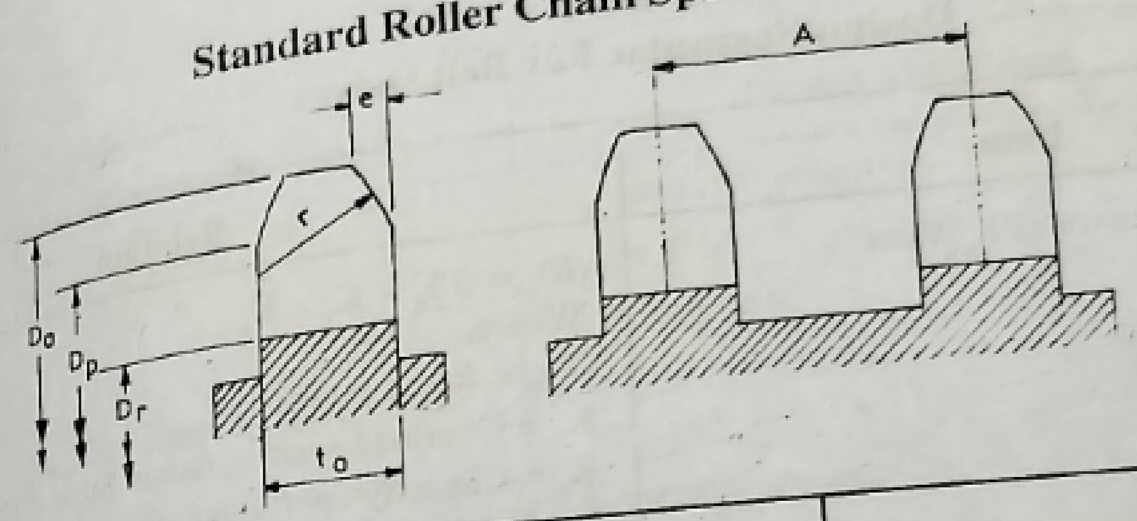


Chain No.	25	35	40	50	60	80	100	120
Pitch, mm	6.25 *	9.525	12.7	15.875	18.75	25.4	31.75	38.10
Chain No.	140	160	180	200	240			
Pitch, mm	44.35	50.8	56.65	63.50	76.20			

Fig. 14.1 Chain drive power ratings

Table XIV - 4

Standard Roller Chain Sprocket Dimensions



S.No.	Item	Relation
1.	Width of sprocket teeth, t_o mm. i) For single strand chains ii) For double & triple stand chains iii) For quadruple strand chains	 $t_o = .58P - 0.15$ $t_o = .56P - 0.15$ $t_o = .54P - 0.30$
2.	Transverse pitch for multiple strand chains, A, mm.	$A = 1.1525P.$
3.	Corner relief, e, mm.	$e = 0.125P.$
4.	Chamfer radius, r, mm.	$r = 0.54$ P.
5.	Outside diameter, Do, mm.	$D_o = P\left[0.6 + \cot\left(\frac{180}{T}\right)\right]$
6.	Root or Bottom diameter, Dr, mm.	$Dr = Dp - 0.625$ P.

Table XV - 1
Design Formulae For Belt Drives

S.No.	Item	Relation
1.	Design Power $(P)_d$ Watts	$(P)_d = (P)_R \times K_l \times K_\theta$ Where, $(P)_R$ = Rated power K_l = Overload service factor Table XV-2. K_θ = Capacity coefficient of inclination Table XV-3.
2.	Belt tensions : F_1, F_2, N	$(F_1 - F_2) = \frac{(P)_d}{V_p}$ V_p = Belt velocity, m/sec.
3.	Belt tension ratio :	$\frac{F_1}{F_2} = e^{\mu\theta}$ Where, μ = Coefficient of friction between belt and pully. (Table XV-4) θ = Angle of lap on smaller pully in radians. $\theta = \pi \pm \frac{D_2 - D_1}{C}$ approx D_1, D_2 = Smaller & bigger pulley diameter C = Centre distance ; – For open belt, + For cross belt
4.	Belt section bxt, mm^2 (b = width, mm & t = thickness, mm)	$b.t = \frac{F_1}{(S_d - S_{cf})}$ Where, S_d = Design stress, MPa S_{cf} = Centrifugal stress; MPa
5.	Centrifugal stress, S_{cf}, MPa	$S_{cf} = \rho V^2 \times 10^{-6}$ ρ = Density of belt material, Kg/m^3, Table XV-5.
6.	Initial tension, F_i, N	$2\sqrt{F_i} = \sqrt{F_1} + \sqrt{F_2}$

Table XV - 2
Load Factor K_l For Belts

S.No.	Driving Machinery	Driven Machinery	K_l
1.	Steam turbines	Centrifugal pump	1.05
		Line shaft	1.10
		Rolling mill	1.20
2.	Electric motor	Lathe, Milling M/c	1.05
		Blower Centrifugal pump	1.10
		Line shafts, propellors Reciprocating pumps	1.15
		Cranes, Elevators	1.30
		Rolling mill	1.35
3.	Steam or I.C. engines	Blowers, Centrifugal pumps Line shafts, propellors Reciprocating pumps	1.45
		Cranes, Elevators Rolling mill	1.60

Table XV - 3
Capacity Coefficient of Inclination K_θ

Angle with horizontal	K_θ	Angle with horizontal	K_θ
0	1.00	70	1.40
50	1.1	80	1.65
60	1.25	90	2.00

Table XV - 4
Practice In Flat Belt Drives

S.No.	Item	Practice
1.	Peripheral speed, V_p	
	i) For max power & economy	1500 – 1800 m/min
	ii) For longer belt life	900 – 1200 m/min
2.	Belt thickness, t, mm	t = 0.02 D Normally t = 0.03 D Maximum } For leather belts No. of plies = 0.013 D Low speed = 0.01 D High speed } For rubber belts
3.	Centre distance, C	i) Min = 3.5 × Larger pulley dia. ii) Max = 7.5 Meters.
4.	Maximum belt width, b_{max}	$b_{max} \equiv (1.33) \times$ Smaller pulley dia.
5.	Stress due to initial tension Si, MPa	i) Leather = 1.4 – 1.6 ii) Rubber = 0.65 – 1.05
6.	Slip	1.5 to 2 percent
7.	Coefficient of friction, μ	

	Pulley material		Cast Iron	Wood	Paper
	Belt	i) Leather	0.35	0.38	0.50
	Material	ii) Rubber	0.30	0.33	0.42

Table XV - 5
Belts, Materials & Their Properties

S No.	Material	Grade of belt	Approx. thickness, mm.			Ultimate strength MPa	Allowable stress MPa	Modulus of Elasticity E MPa	Specific gravity
			Single	Double	Triple				
1.	Leather	Light	3.0	6.0	-	Oak Tanned 21 - 31 Chrome Tanned 28 to 38	Oak Tanned 2.0-3.0 Chrome Tanned 2.8 - 3.8	140 to 315	0.97
		Medium	4.0	8.0	12.0				
		Heavy	4.5	9.750	13.75				
2.	Rubber	No. of plies 3 to 15	Thickness per ply 1.2 to 2 mm			6.3 to 10.5	0.8 to 1.7	900 to 1500	1.25
3.	Cotton	-	-			35 to 50	3.5 to 5.0	500 to 1500	0.8 to 1.5

Table XV - 7

Flat Belt Pulley Construction & Properties

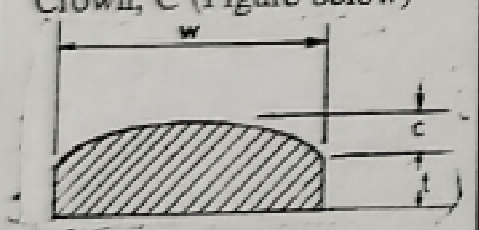
<table>
<tr><th>S.No.</th><th>Item</th><th>Proporsions</th></tr>
<tr><td>1.</td><td>Materials</td><td>i) Cast - Iron, Normally preferred
ii) Pressed Steel</td></tr>
<tr><td>2.</td><td>Type of construction</td><td>Diameter below 150 mm - Web construction
Diameter above 150 mm - Arm construction
Number of arms as given below
<table>
<tr><th rowspan="2">Width / diameter</th><th colspan="2">upto 300 mm</th><th colspan="2">above 300 mm</th></tr>
<tr><th>No. of Arms</th><th>No. of Sets</th><th>No. of Arms</th><th>No. of Sets</th></tr>
<tr><td>150 - 450</td><td>4</td><td>1</td><td>4</td><td>2</td></tr>
<tr><td>450 - 1500</td><td>6</td><td>1</td><td>6</td><td>2</td></tr>
<tr><td>1500 & above</td><td>8</td><td>1</td><td>8</td><td>2</td></tr>
</table></td></tr>
<tr><td>3.</td><td>Moment on each arm</td><td>$M = \frac{(F_1 - F_2)(D - D_h)}{n}$; (only half arms share the load)
Where, n = no. of arms
D = pulley diameter
D_h = hub diameter
Allowable stress for arms refer table XI - 6.</td></tr>
<tr><td>4.</td><td>Face width, W</td><td>W = 1.1 b</td></tr>
<tr><td>5.</td><td>Crown, C (Figure below)

 </td><td>C = 1 mm in 100 mm. face width</td></tr>
<tr><td>6.</td><td>Rim thickness, t, mm</td><td>$t = 0.25\sqrt{D} + 15$ Light duty pulleys
$= 0.375\sqrt{D} + 3$ Heavy duty pulleys</td></tr>
<tr><td>7.</td><td>Hub proportions
i) Diameter, D_h, mm
ii) Length L_h, mm</td><td>
$D_h = 1.5\, d_s + 25$ mm
$L_h = 1.5\, d_s$ where, d_s = diameter of shaft, mm.</td></tr>
</table>

Table XV - 8

'V' Belt - Available Sizes & Details

Designation	Nominal width W		Nominal thickness, t		Recommen-ded min. pulley dia. D, mm	Max. No. of strands used	Suggested range of power kW	Bending stress factor K_b	Centrifugal tension factor K_c
	mm	in.	mm	in.					
A	13	1/2	8	5/16	75	6	.35 - 3.5	17.6×10^3	2.52
B	17	21/32	11	13/32	135	9	1.5 - 15	45.6×10^3	4.34
C	22	7/8	14	17/32	225	14	10 - 75	125.1×10^3	7.72
D	32	5/4	19	3/4	325	14	35 - 150	434×10^3	15.75
E	38	3/2	23	29/32	540	20	75 & above	821×10^3	22.5

Table XV - 9

Design Formulae For 'V' Belt Drives

S.No.	Item	Formulae
1.	Design Power $(P)_d$	$(P)_d = (P)_R \times K_l$ Where $(P)_R$ = Rated Power K_l = Load factor table XV - 2.
2.	Power rating per belt, W	$(\text{Power/Belt}) = (F_W - F_C)\dfrac{e^{\mu\,\theta/\sin(\alpha/2)} - 1}{e^{\mu\,\theta/\sin(\alpha/2)}} \times V_P$ Where, F_w = Working load, N $= w^2$ w = Nominal width of belt, mm. Table XV - 8. F_C = Centrifugal tension, N $= K_C\left(\dfrac{V_P}{5}\right)^2$ K_C = Centrifugal tension factor. Table XV - 1. θ = Arc of contact on smaller pulley, radians, table XV - 1. μ = Coefficient of friction (usually 0.3) $\propto$ = Cone angle, table XV - 11. V_P = Periferal speed of belt, m/sec.
3.	Bending load, F_b, N	$F_b = \dfrac{K_b}{D}$ Where, K_b = Bending stress factor, Table XV - 8. D = Pulley diameter, mm.
4.	Fatigue life of belt	Refer fig. 15.1. Max. total force, $F_1 = F_i + F_C + F_{b\,(max)}$

Table XV - 10

Practice For 'V' Belt Drives

S.No.	Item	Relation
1.	Coefficient of friction, μ	$\mu = 0.3$ usually
2.	Velocity range, V_P	V_P = 300 to 1500 m/min.
3.	Centre distance, C	$C = D_1$ or $(D_1 + D_2)$; D_1 = Biger pulley diameter D_2 = Smaller pulley diameter

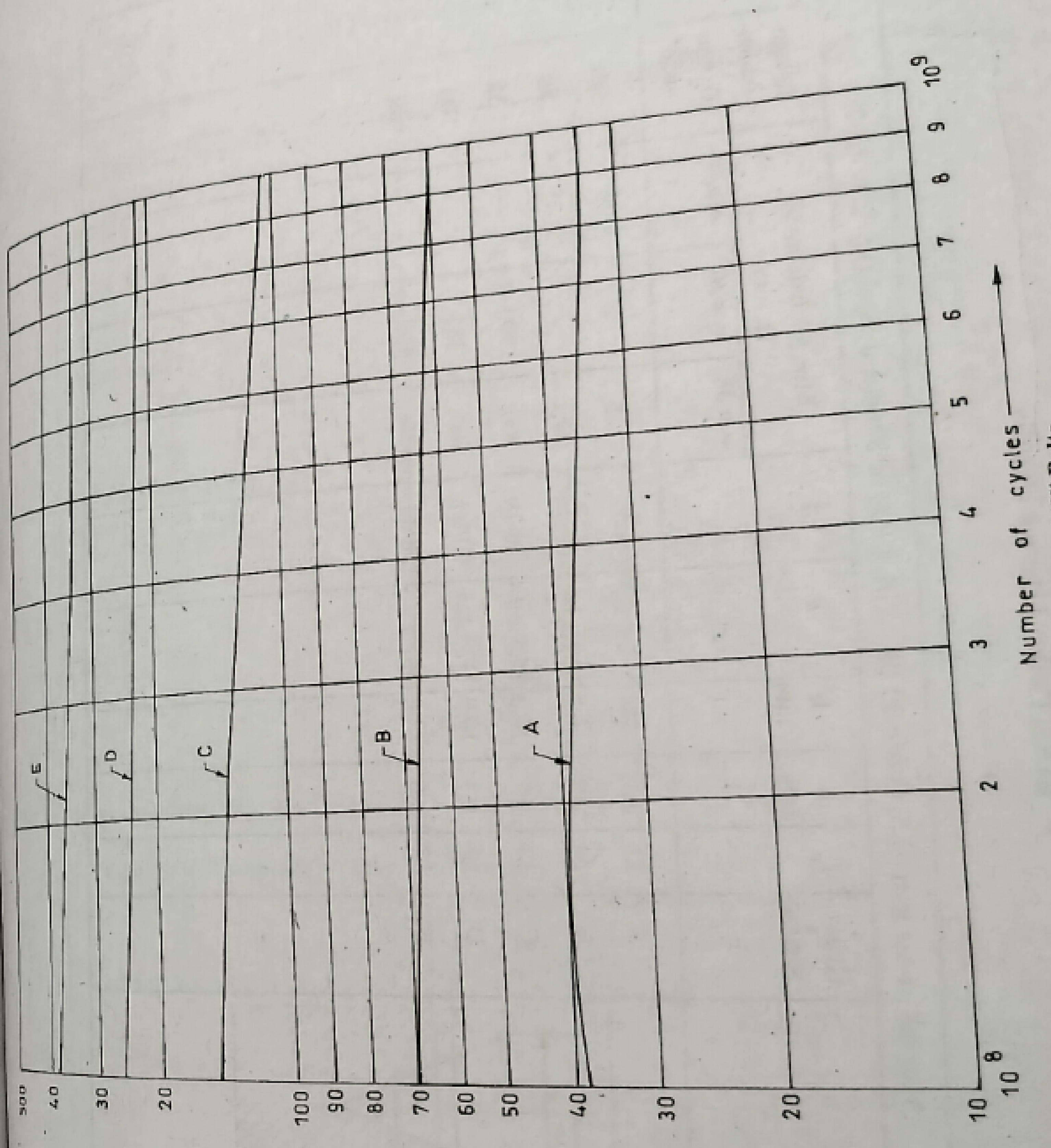


Fig. 15.1 Fatigue life of 'V' Belts

Table XV - 11

'V' Belts, Pulley Groove Details

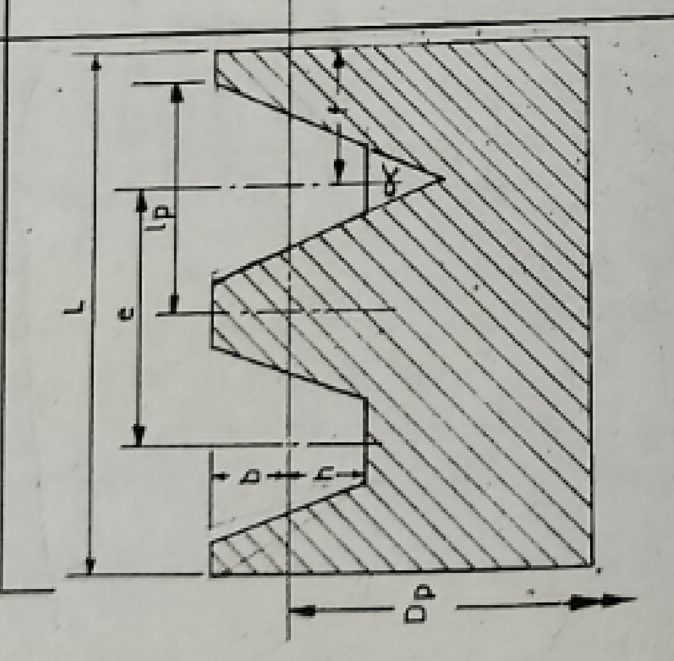
Material : Cast Iron : I.S. Grade 20 (IS . 210, 1962) or Structural Steel (I.S. 226, 1962)											
	Groove section	l_p	b mm	h mm	e	f	Min. pitch diameter D_p mm			Recommen D_p mm $\propto = 38°$	Tolerance in groove angle
							$\propto = 38°$	$\propto = 36°$	$\propto = 34°$		
	A	11	3.3	8.7	15 ± 0.3	9-12	125	-	75	200	± 1°
	B	14	4.2	10.8	19 ± 04	11.5-14.3	200	-	125	280	± 1°
	C	18	5.7	14.3	25.5 ± 0.4	16-19	300	200	-	355	± 3°
	D	27	8.1	19.9	37 ± 0.5	23-27	500	355	-	500	± 3°
	E	32	9.6	23.4	44.5 ± 0.7	28-33	630	500	-	630	± 3°

Table XVI - 1

Design Formulae For Spur Gears

S.No.	Item	Formulae
1.	Design Power $(P)_d$, W	$(P)_d = (P)_R * K_1$ Where, $(P)_R$ = Rated power K_1 = Load factor, table XVI - 2
2.	Tooth load F_t , N	$F_t = \frac{(P)_d}{V_p}$ Where, V_p = Pitch line velocity, m/sec.
3.	Bending strength by Lewis equation, F_B , N.	$F_B = S_o \cdot C_V \cdot b \cdot Y \cdot m$ Where, S_o = Basic strength, MPa, table XVI - 10. C_V = velocity factor table XVI - 3. b = Face width of gears, mm. Y = Modified Lewis form factor, table XVI-5. m = Module, mm.
4.	Dynamic load, F_d , N (Based on Buckingham's equ.)	$F_d = F_t + \frac{21\,V_p\,(Ceb + F_t)}{21\,V_p + \sqrt{Ceb + F_t}}$ Where, C = Deformation factor, table XVI - 4 e = Error in profile, mm, fig. 16.1
5.	Limiting Wear strength F_W , N	$F_W = D_p\,b\,K\,Q$ K = Load stress factor Table XVI - 6 $Q = \text{Size factor} = \frac{2t_g}{t_g + t_p}$ For external gears $= \frac{2t_g}{t_g - t_p}$ For internal gears t_g, t_p = No. of teeth on gear & pinion respectively.
6.	Endurance strength, F_{en}, N	$F_{en} = S_{eb} \cdot b * Y * m$ Where, S_{eb} = Endurance strength, table XVI - 9

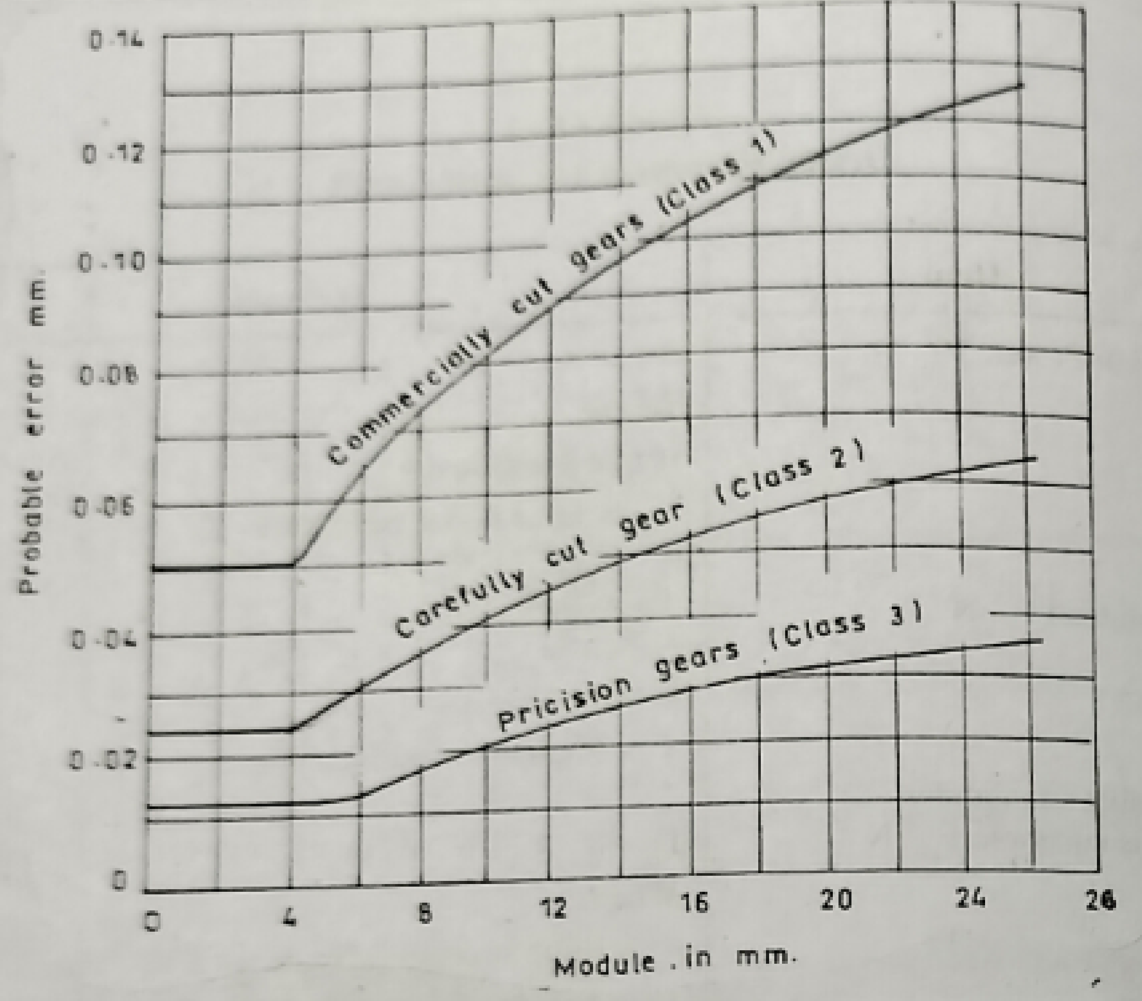


Fig. 16.1 Probable error is spur gears

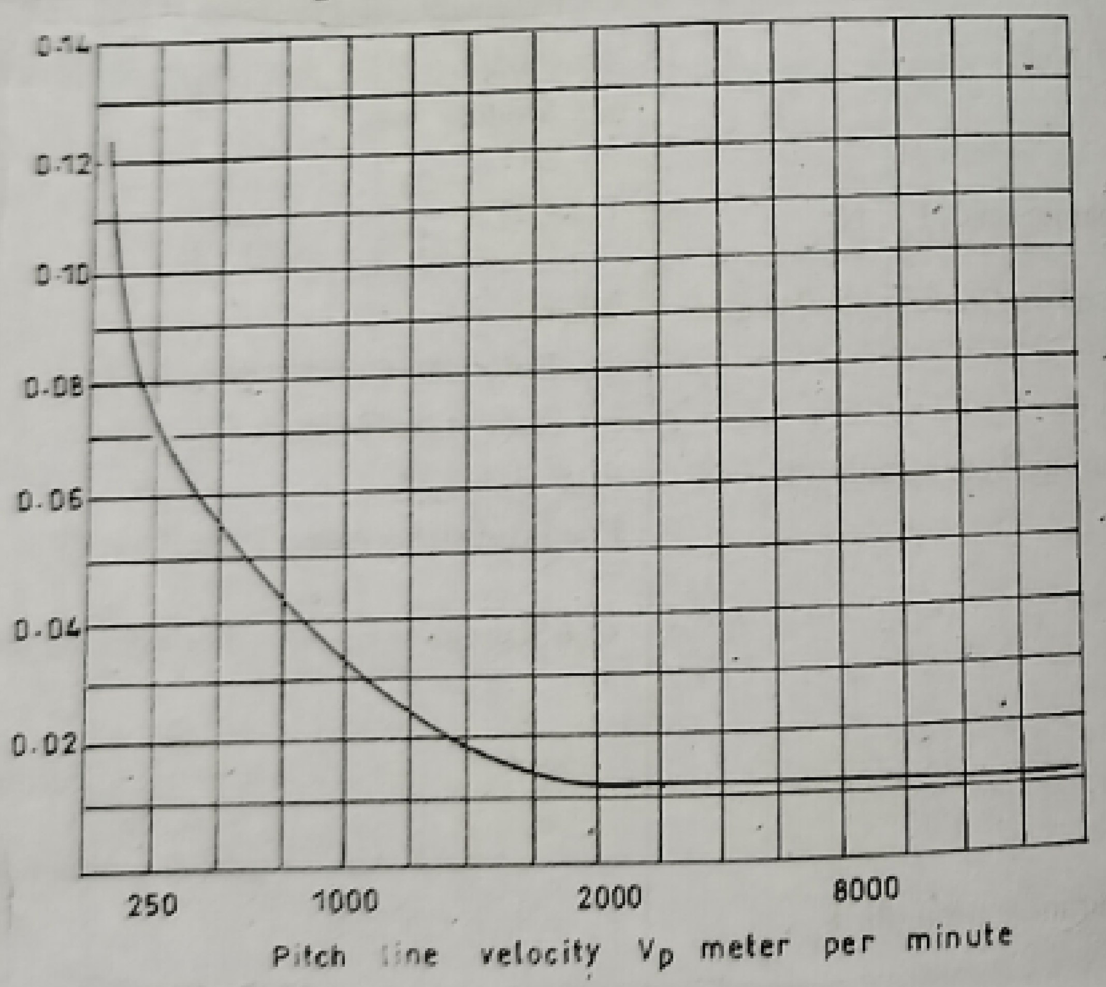


Fig. 16.2 Permissible error is spur gears

Table XVI - 2
Load Factors K_1 For Gear Drives

S.No.	Type of load	Type of service		
		Intermitant or 3 Hrs/day	8 to 10 hrs per day	Continuous 24 Hrs/day
1.	Steady	0.80	1.00	1.25
2.	Light shocks	1.00	1.25	1.50
3.	Medium shocks	1.25	1.50	1.80
4.	Heavy shocks	1.50	1.80	2.00

Table XVI - 3
Velocity Factor C_V For Spur Gears

S.No.	Class of gear	C_V	Velocity m/sec	Accuracy Grade
1.	Commercially cut gears, class 1	$\frac{3}{3+V_p}$	2.5 - 5.0 5.0 - 7.5	8 - 9 7 - 8
2.	Carefully cut gears, class 2	$\frac{4.5}{4.5+V_p}$	7.5 - 15	5 - 6
3.	Precision gears, class 3	$\frac{6}{6+V_p}$	15 - 20	4 - 5
4.	Ground and lapped gears	$\frac{5.5}{5.5+\sqrt{V_p}}$	above 20	3 - 4
5.	Non metallic gears	$\frac{0.75}{1.0+V_p}+0.25$	0 - 15	

Table XVI - 4
Deformation Factor 'C' For Gears

Relation	Nomenclature		
$C = \left(\frac{a}{1/E_p + 1/E_g}\right)$ MPa	E_p & E_g = Young's modulus for pinion and gear, MPa a = 0.107 for 14.5° Full depth a = 0.111 for 20.0° Full depth a = 0.115 for 20.0° Stub		
Materials of gear in mesh	Values of 'C'		
	14.5° Involute	20° Full depth	20° Stub teeth
i) Gray Cast Iron and Gray Cast Iron	5700	5900	6100
ii) Gray Cast Iron and Steel	7800	8100	8500
iii) Steel and Steel	11500	11800	12300

Table XVI - 5
Modified Lewis Form Factor, Y, For Involute Gears

S.No.	Type of profile	Relation of Y
1.	14.5° Full depth (Addendum = m)	$Y = 0.39 - \frac{2.16}{t}$
2.	20° Full depth (Addendum = m)	$Y = 0.485 - \frac{2.87}{t}$
3.	20° Stub (Addendum - 0.8 m)	$Y = 0.551 - \frac{2.90}{t}$ Where, t = No. of teeth.

Table XVI - 6
Load Stress Factor K

Gear		Pinion		K	
Material	BHN core	Material	BHN core	14 1/2° involute	20° involute Full depth & stub
Steel	150	Steel	150	.210	.288
	150		200	.302	.407
	150		250	.407	.555
	200		200	.407	.555
	200		250	.534	.721
	200		300	.674	.920
	250		250	.674	.920
	250		300	.835	-
	250		350	1.011	1.376
	300		300	1.011	1.376
	300		350	1.197	1.636
	300		400	1.306	1.783
	350		350	1.411	1.920
	350		400	1.636	2.232
	350		450	1.755	2.401
	450		450	2.415	3.219
	450		500	2.555	3.489
	450		600	2.705	3.489
	500		500	3.019	4.123
	600		600	4.423	6.044
Cast Iron	180	Steel	150	.309	.421
	180		200	.611	8.35
	180		250	1.011	1.376
Gear Bronze	100	Steel	150	.323	.435
	100		200	.639	.70
	100		250	.948	1.292
Cast Iron	180	Cast Iron	180	1.355	1.853
Metal	-	Any Non-Metal (See - 2.24 MPa)	34	1.327	1.811

Table XVI - 7

Design Practice For Gears

S.No.	Item	Practice
1.	Recommended Module, mm	
	i) Preferred	1.0, 1.25, 1.5, 2.0, 2.5, 3.0, 4.0 5.0, 6.0, 8.0, 10.0, 12.0, 16.0, 20, 25, 32, 40, 50.
	ii) Second Choice	1.125, 1.375, 1.75, 2.25, 3.50, 4.50 5.50, 7.0, 9.0, 11.0, 14.0, 18.0, 22.0, 28, 36, 45.
2.	Compact design	Dp = (2d + 25) mm, Keyed pinion Dp = (d + 25) mm, Integral pinion Where Dp = Pitch diameter of pinion, mm d = pinion shaft diameter, mm.
3.	Economical design	A) Choose $V_p = 3 - 5$ m/sec. B) $Dp = \frac{23}{r+1}\sqrt{\frac{Q}{K_1 * b}}$ Where, Dp = Pitch diameter of pinion, mm r = Gear ratio $= \frac{Dg}{Dp}$ Dg = Pitch diameter of gear, mm. $Q = \frac{P}{\text{pinion R.P.M}} \times \frac{(r+1)^3}{r}$; K_1 = Table XVI - 8
4.	Preffered number of teeh on pinion	7-9 Requires long addendum to aviod undercutting, 25° pressure angle preffered, Poor wear characteristics 10 Smallest No. of teeth with 20° pressure angle, requires long addendum, contact ratio critical. 12 Minimum No. of teeth for fractional H.P drive. 14 Minimum No. for 20° stub teeth 19 Minimum No. of teeth for 20° full depth teeth. 25 Good balance in strength & wear for hard steel. 35 Good wear resistance even for medium hardness (R_c-30) quiet in operation 50 Excellent wear resistance, preferred for quietness
5.	Face width, b, mm.	$8.5\,m < b < 12.5\,m$ or b = Dp to 2 Dp

Table XVI - 9

Endurance Strength of Gear Materials

S.No.	Material	BHN	Reversed Bending Endurance Limit S_{eb}, MPa	Surface Endurance Limit S_{ec}, MPa
1.	Gray C.I.	160	84	631
2.	Semisteel	200	126	-
3.	Ph-Bronze	100	168	-
4.	Steel	150	252	350
	"	200	350	490
	"	240	420	617
	"	300	526	772
	"	350	596	910
	"	400	700	1050
	"	450	700	1270
	"	500	700	1420
	"	550	700	1570
	"	600	700	1720

Table XVI - 10

Gear Material & Their Strengths

Basic stress for gears, $S_o = S_{yt} / 1.5$

S.No.	Material	S_o ; MPa
1.	Cast Iron, ordinary	56
2.	Cast Iron, medium grade	70
3.	Cast Iron, high grade	105
4.	Cast steel, 0.20%, carbon, untreated	140
5.	Cast steel, 0.20% carbon, heat-treated	196
6.	Forged carbon steel	
	a) SAE 1020, Case hardened	126
	b) SAE 1030, untreated	140
	c) SAE 1035, untreated	161
	d) SAE 1040, untreated	175
	e) SAE 1045, untreated	210
	f) SAE 1045, heat treated	210
	g) SAE 1045, heat treated	245
7.	Alloy steel	
	a) SAE 2320, case hardened	350
	b) SAE 3245, heat treated	456
	c) SAE, 6145, heat treated	473
8.	Bronze SAE 62	70
9.	Ph Bronze SAE 65	84
10.	Meehanite Metal grade GA	88
11.	Rawhide Fabroil	42
12.	Bakelite, Micarta, Celoron	56

Table XVI - 11

Design Formulae & Practice For Helical Gears

All symbols in this table have same meaning & relation as in table XVI-1 except otherwise mentioned

S.No.	Item	Relation
1.	Normal module, m_n (use standard modules of table XVI - 7)	$m_n = m_t \cos \Psi$ Where, Ψ = Helix angle m_t = Tangential module.
2.	Normal Pressure angle, φ_n	$\tan \phi_n = \tan \phi * \cos \Psi$, Where, ϕ = Pressure angle in tangential plane
3.	Formative Number of teeth t_f	$t_f = t / \cos^3 \Psi$, Where t = no. of teeth.
4.	Design Power, $(P)_d$	$(P)_d = (P)_R * K_l * K_w$ Where, K_l = Load factor, Table XVI - 2. K_w = Lubrication & wear factor, Table XVI - 12
5.	Tangential load : F_t, N	$F_t = \frac{(P)_d}{V_p}$ Where V_p = Pitch line velocity m/sec.
6.	Axial load F_a, N	$F_a = F_t \tan \Psi$
7.	Beam strength; F_B, N	$F_B = S_o C_v Y b m_n$ C_v = velocity factor, refer Table XVI - 13 Y = Modified Lewis form factor Table XVI - 5 (use $t = t_f$)
8.	Dynamic load based on Buckingham's equation F_d, N	$F_d = F_t + \frac{21 V_p \left[C_{eb} . \cos^2 \psi + F_t\right]}{21 V_p + \sqrt{C_{eb} \cos^2 \psi + F_t}} \times \cos \psi$
9.	Limiting wear strength F_w, N	$F_w = \frac{K_b D_p Q}{\cos^2 \psi}$
10.	Endurance strength F_{en}, N	$F_{en} = S_{eb} b . Y . m_n$
11.	Practice	$\psi = 15° \text{ to } 45°$ $b = (4.5 m_t) / \tan \psi$ to $(20 m_t) / \tan \psi$

Table XVI - 12
Lubrication Factor K_W

S.No.	Type of lubrication	K_W
1.	Continuous lubrication as in oil bath	1.15
2.	Scant lubrication	1.25
3.	Indifferent lubrication	1.35

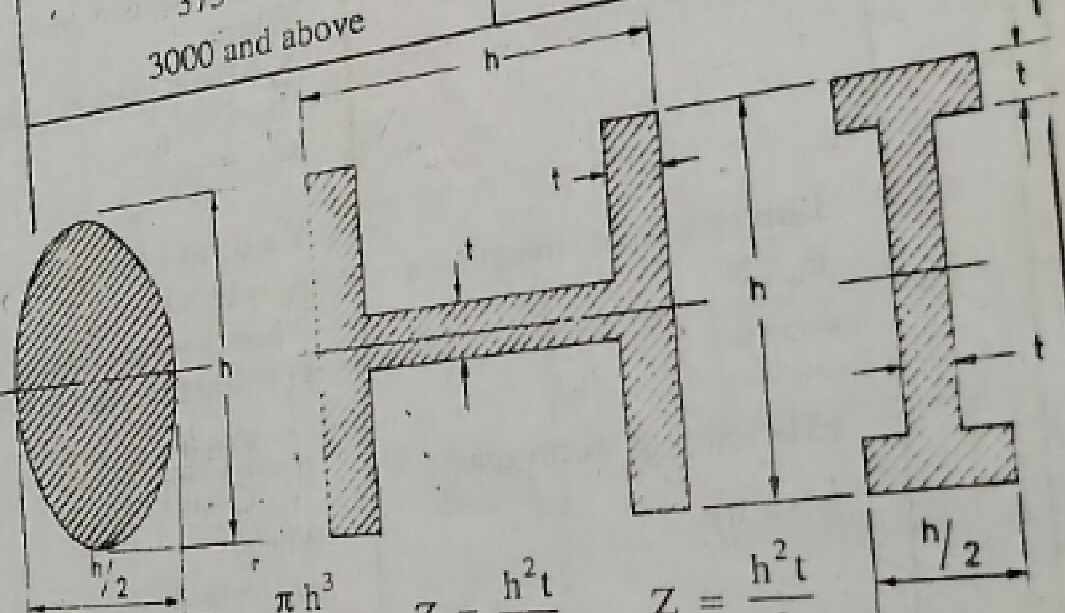
Table XVI - 13
Velocity Factor C_v For Helical Gears

S.No.	C_v	Velocity Range m/sec	Accuracy grade
1.	$\frac{4.5}{4.5+V_p}$	Low helix angle and below 5	8 - 9
2.	$\frac{6}{6+V_p}$	5 - 10	6 - 7
3.	$\frac{15}{15+V_p}$	10 - 20	4 - 5
4.	$\frac{5.5}{5.5+\sqrt{V_p}}$	Above 20	3 - 4
5.	$\frac{.75}{1.0+V_p}+0.25$	0 - 15 Non metallic gears	

Table XVI - 14
Design Data For Gear Blanks

S.No.	Item	Relation
1.	Solid construction	For $D_p \leq 15\ m + 60$, mm
2.	Web construction	For $D_p \leq 24\ m + 80$, mm.
3.	Web thickness, w mm.	$w = 1.5\ m + 3$, mm
4.	Arm construction	For $D_p > 24\ m + 80$, mm.
5.	No. of Arms, n	

Pitch diameter, mm	No. of Arms, n
Upto 375	4
375 - 3000	6
3000 and above	8

S.No.	Item	Relation
6.	Commonly used sections for arms	
7.	Rim thickness, t, mm	$t = 1.5 \times m$ to $2.0 \times m$.
8.	Hub proportions	d_s = Shaft diameter, mm

$Z = \frac{\pi h^3}{64}$ $Z = \frac{h^2 t}{3}$ $Z = \frac{h^2 t}{3}$

(Taper in arms, 1 in 12)
for allowable stress in arms, table XI - 6.

Type of service	Hub diameter, Dh		Hub length, L_h
	Cast Iron	Cast Steel	
Light shock	1.75 d_s	1.6 d_s	1.5 d_s
Medium shock	1.85 d_s	1.7 d_s	1.75 d_s
Heavy shock	2.0 d_s	1.8 d_s	2 d_s

Table XVI - 15
Design Formulae For Worm Gears

S.No.	Item	Formula
1.	Design Power $(P)_d$ Watts	$(P)_d = (P)_R \times K_1$ Where $(P)_R$ = Rated Power, watts K_1 = Load Factor, Table XI - 5.
2.	Tooth load, F_t, N	$F_t = \frac{(P)_d}{V_p}$ Where V_p = Pitch line velocity of gear, m/sec.
3.	Beam strength by Lewis equation	$F_B = S_o . C_v\, b . Y . m$ Where S_o = Basic strength, table XVI - 10. $C'_V = \frac{6}{6 + V_p}$ $Y = 0.314 + 0.0151\,(\phi_n - 14.5°)$
4.	Limiting wear strength F_W, N	$F_W = D_g\, b\, K_2$; Where, b = Face width of gear, mm. K_2 = load stress factor, table XVI - 17.
5.	Efficiency of worm gear drive η	$\eta = \frac{\cos\theta - \mu \tan\lambda}{\cos\theta' + \mu \cot\lambda}$ Where, $\tan\theta = \tan\phi_n \cos\lambda$ μ = coefficient of friction
6.	Heat generated in worm gears, Q_1. Watts	$Q_1 = (1 - \eta)(P)_R$
7.	Heat dissipated; Q_2 watts	$Q_2 = KA\,\Delta t$ Where A = heat dissipation area (projected area of worm gear), mm² $K = 0.4 * 10^{-3}$ w/mm², °C Δt = Temp rise (max oil temp 60-70° C)

Symbol :

λ = Pitch lead angle, degrees

ϕ_n = Normal pressure angle, degrees

P_c = Circular pitch, mm

w, g = suffix for worm and gear

L = Lead of worm $= t_w P_c$

λ = Lead of worm $= \tan^{-}\frac{L}{\pi D_W}$

t = number of teeth

N = speed in R.P.M.

Pa = Axial pitch, mm

D = Pitch diameter, mm

R = velocity ratio $\frac{N_W}{N_g}$

C = center distance, mm

Table XVI - 16
Design Practice For Worm Gears

<table>
<tr><th>S.No.</th><th>Item</th><th>Practice</th></tr>
<tr><td>1.</td><td>No. of teeth on worm, t_w</td><td>
<table>
<tr><th>Velocity Ratio</th><th>t_W</th></tr>
<tr><td>4 - 10</td><td>6</td></tr>
<tr><td>6 - 12</td><td>4</td></tr>
<tr><td>8 - 12</td><td>3</td></tr>
<tr><td>12 - 36</td><td>2</td></tr>
<tr><td>36 & Above</td><td>1</td></tr>
</table>
</td></tr>
<tr><td>2.</td><td>Lead angle, λ</td><td>6° per worm tooth, Min - 9°, Max - 45°
For compact design $\lambda = \tan^{-1}\sqrt[3]{N_g/N_W}$</td></tr>
<tr><td>3.</td><td>No. of teeth on gear, t_g</td><td>$t_g > 29$, Normally</td></tr>
<tr><td>4.</td><td>Pitch Diameter of worm, D_W</td><td>$\frac{C^{0.875}}{1.132} > D_W > \frac{C^{0.875}}{2.0}$</td></tr>
<tr><td>5.</td><td>Choice of lead angle ; λ and pressure angle, ϕ_n</td><td>
<table>
<tr><th>λ</th><th>ϕ_n</th><th>Remarks</th></tr>
<tr><td>upto 12°</td><td>14.5°</td><td>Difficult to cut by hob</td></tr>
<tr><td>12° – 20°</td><td>20.0</td><td>-</td></tr>
<tr><td>20° – 25°</td><td>22.5</td><td>Normally used</td></tr>
<tr><td>above 25°</td><td>25.0</td><td>-</td></tr>
<tr><td>above 25°</td><td>30.0°</td><td>For reversible drives</td></tr>
</table>
</td></tr>
<tr><td>6.</td><td>Coefficient of friction, μ</td><td>
<table>
<tr><th>Rubbing velocity, V_R, m/min.</th><th>μ</th></tr>
<tr><td>Low speed - below 2.</td><td>0.15</td></tr>
<tr><td>Medium speed - 10 - 170</td><td>$0.132 / V_R^{0.28}$</td></tr>
<tr><td>High speed - 170 above</td><td>$0.025 + \frac{V_R}{20,000}$</td></tr>
</table>
</td></tr>
<tr><td>7.</td><td>Rubbing velocity, V_R m/min.</td><td>$V_R = \frac{\pi . D_W . N_W}{\cos\lambda . 1000}$, usual range
$400 < V_R < 1000$ for precision gears
$V_R < 400$ for ordinary gears</td></tr>
</table>

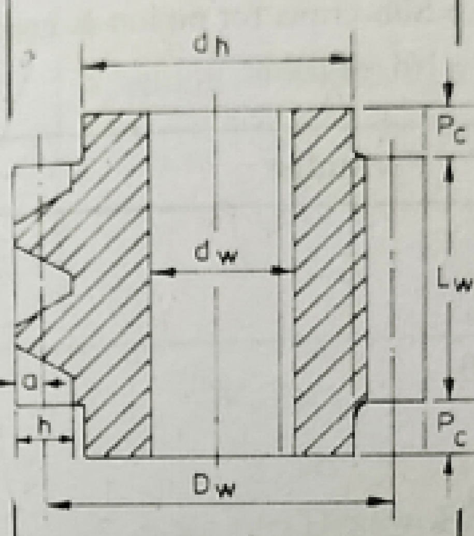
Table XVI - 17
Load Stress Factor K_2

Material of worm	Material of gear	K_2, MPa
Steel BHN 250 or more	C - I or semi steel	0.35
	Mn - Bronze	0.56
	Ph - Bronze	0.70
	Bakelite or other non metals	0.87

Table XVI - 18
Material of Construction For Worm Gears

Type of Service	Worm	Gear
Light	Steel, SAE 1040	C-I or Bronze SAE 63
Medium	Steel SAE 2320 or 3120 Case hardened	Phosphor-Bronze SAE 65
Heavy	Mo or Cr - V Steel	Phospher Bronze SAE 65

Table XVI - 19
Dimensions of Worm Gears : For Worm

S. No.	Figure	Symbol	Dimensions	Single & Double threads	Tripple or Quadruple threads
1.		ϕ_n	Normal pressure angle	14.5°	20°
2.		D_w	Pitch dia. of worm bored for shaft, mm	$2.4\ P_c + 27.5$	$2.4\ P_c + 27.5$
3.		D_w	Pitch dia. of worm integral with shaft, mm	$2.35\ P_c + 10$	$2.35\ P_c + 10$
4.		L_w	Face length, mm	$(4.5 + 0.02t)\ P_c$	$(4.5 + 0.02t)\ P_c$
5.		h	Depth of tooth	$0.686\ P_c$	$0.623\ P_c$
6.		a	Addendum	$0.318\ P_c$	$0.286\ P_c$
7.		d_h	Hub diameter	$1.66\ P_c + 25$	$1.726\ P_c + 2.5$
8.		d_w	Minimum bore of shaft	$P_c + 16$	$P_c + 16$

FOR GEAR

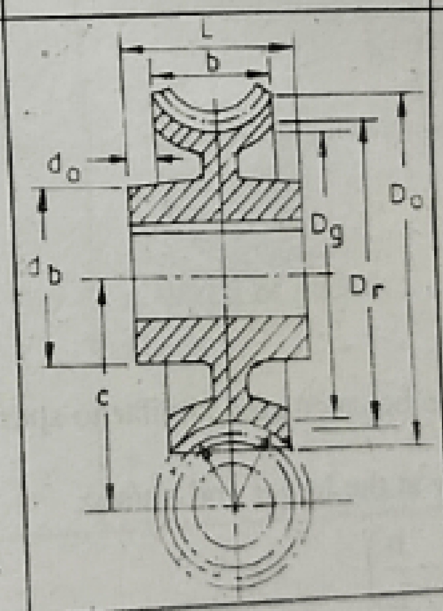
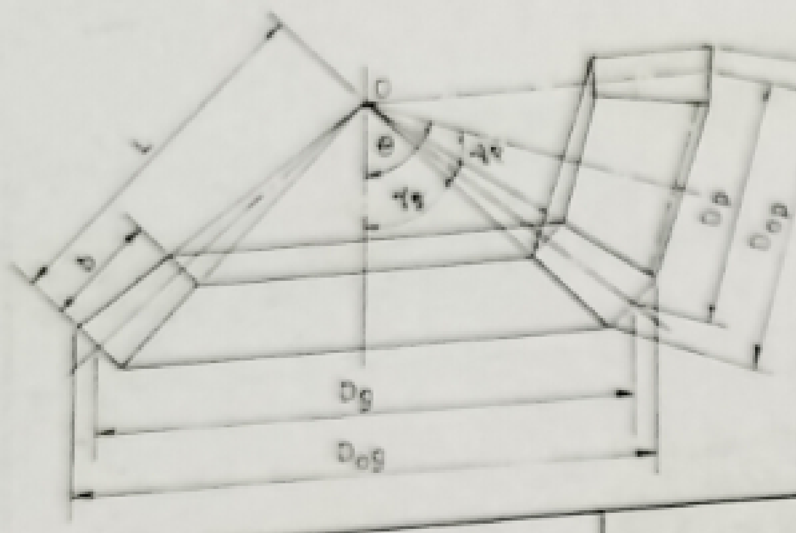
S. No.	Figure	Symbol	Dimensions	Single & Double threads	Tripple or Quadruple threads
1.		ϕ_n	Normal pressure angle	14.5°	20°
2.		D_o	Outside dia, mm	$D_g + 1.0315\ P_c$	$D_g + 0.8903\ P_c$
3.		Dr	Throat dia, mm	$D_g + 1.0636\ P_c$	$D_g + 0.572\ P_c$
4.		b	Face width, mm	$2.38\ P_c + 6.25$	$2.15\ P_c + 5$
5.		r	Radius of gear face, mm	$0.882\ P_c + 13.75$	0.914 P+13.75
6.		r_b	Radius of gear rim, mm	$2.2\ P_c + 13.75$	$2.10\ P_c + 13.75$
7.		r_r	Radius of edge	$0.25\ P_c$	$0.25\ P_c$
8.		d_h	Hub diameter	$1.875\ d_s$	$1.875\ d_s$

Table XVI - 20



Design Formulae For Straight Tooth Bevel Gears

θ = Angle between axes of the shafts
γ = Pitch angle or center angle
L = Cone distance
b = Width of the face
p or g = Subscripts for pinion & gears
t = No. of teeth

S.No.	Item	Formulae
1.	Pitch Angles, γ a) For acute angle gears	$\tan\gamma_p = \dfrac{\sin\theta}{\dfrac{t_g}{t_p} + \cos\theta}$ $\tan\gamma_g = \dfrac{\sin\theta}{\dfrac{t_p}{t_g} + \cos\theta}$
	b) For Obtuse angle gears	$\tan\gamma_p = \dfrac{\sin(180-\theta)}{\dfrac{t_g}{t_p} - \cos(180-\theta)}$ $\tan\gamma_g = \dfrac{\sin(180-\theta)}{\dfrac{t_p}{t_g} - \cos(180-\theta)}$
2.	Cone distance, L	$L = 0.5\sqrt{Dg^2 + Dp^2}$
3.	Formative no. of teeth t_f	$t_f = \dfrac{t}{\cos\gamma}$
4.	Tooth load, F_t, N	$F_t = \dfrac{(P)_d}{V_p}$ Where, $(P)_d$ = Design Power, to be calculated similar to spur gears. V_p = Pitch line velocity at the larger end, m/sec.
5.	Beam strength, F_B, N	$F_B = S_o C_V Y b m \left(1 - \dfrac{b}{L}\right)$ Y to be calculated for formative no. of teeth.

(Contd..next page)

Table XVI - 20 (Contd...)

S.No.	Item	Formulae
6.	Dynamic load, F_d, N	$F_d = F_t + \dfrac{21 V_p (Ceb + F_t)}{21 V_p + \sqrt{Ceb + F_t}}$
7.	Limiting wear load, F_W, N	$F_W = \dfrac{K b D_p Q}{\cos\gamma_p}$ Where, $Q = \dfrac{2 t_{fg}}{t_{fg} + t_{fp}}$

Table XVI - 21

Design Practice For Straight Tooth Bevel Gears

S.No.	Item	Formulae
1.	Number of teeth, t	t (min) $= 12$ for $\dfrac{tg}{tp} = 4$ $= 18$ for $\dfrac{tg}{tp} = 1$
2.	Pitch line velocity, V_p m/sec	$V_p = 5 - 7$ for ordinary cut teeth $= 10$ for generated teeth
3.	Width of gear face, b, mm	$b \le \dfrac{L}{3}$ $b = 6\,m - 7\,m$ for $L < 30$ m $= 7m - 10\,m$ for $L > 30$ m

* * *

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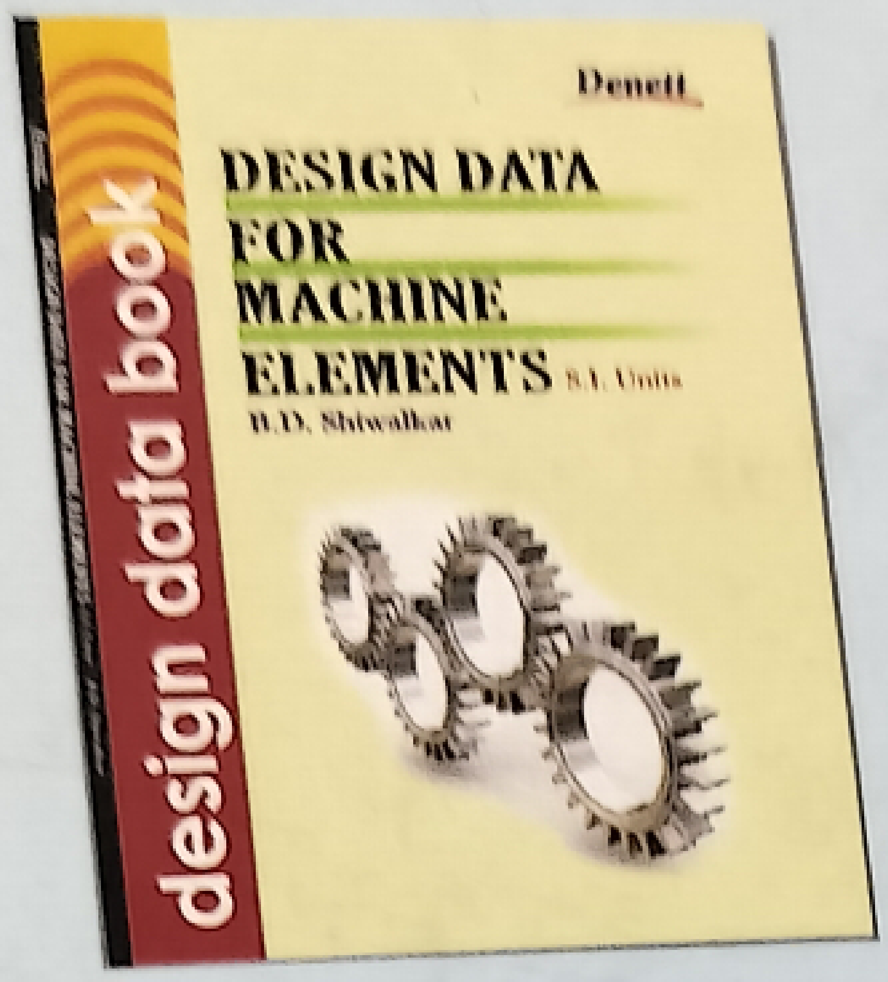
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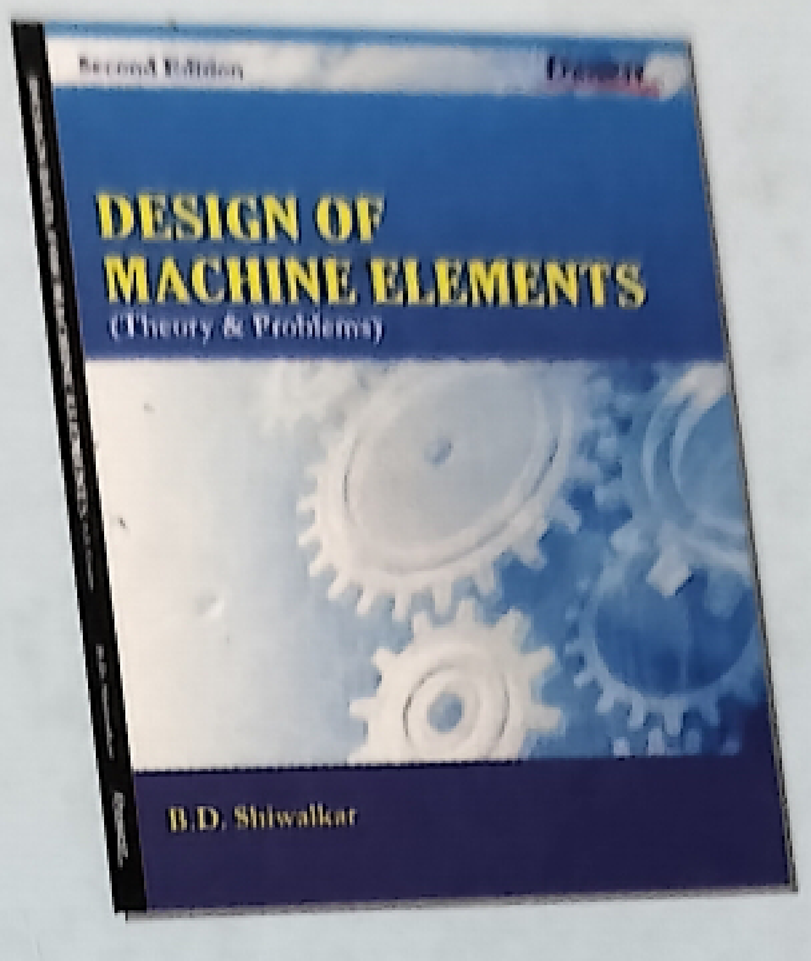
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B.D. Shiwalkar

Prof. B.D. Shiwalkar has retired as Prof. & Head of Mechanical Engineering Department from Visvevaraya Regional College of Engineering (Now Visvesvaraya National Institute of Technology), Nagpur in 1992. He passed B.E. (Mech) examination from Sagar University in 1954 standing first in order of merit. Later he obtained M.S. Degree from School of Mines and Metallurgy, Rolla, U.S.A. in 1960 with specialization in Dynamics of Machinery and Machine Design. He has taught Machine Design for about Thirty five years at graduate and postgraduate level. He has guided two students for Ph.D. and about a dozen of students for M. Tech. thesis. He has been co-ordinator for two winter schools sponsored by I.S.T.E. in the fields of:

1. Reliability, Maintainability and Wear Considerations in Machine Design.
2. Utilization of Solar Energy.

He has published number of papers in national and international journals. In addition to this teaching and research experience he has worked in the design, manufacture and erection of various machines in the following industries:

1. Department of Atomic Energy, Mumbai.
2. The National Newsprint and Paper Mills, Nepanagar.
3. Larsen & Tourbo Ltd., Alucapsules Division, Pawai, Mumbai.
4. The Associated Cement Companies (ACC) Ltd., Shahabad, Kymore and Krishna Cement Plants.

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Denett & Co.
23 Farmland, New Ramdaspeth,
Central Bazar Road, Nagpur - 440010 (MS) India.
Ph.: 0712 - 6611102, 6612660 Fax: 0712 - 2555213
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